# CS-E4850 Computer Vision
# Exercise Round 12

The following instructions are for the Matlab version. The instructions for the Python version are in Github. The solutions should be returned via the MyCourses page. Upload two files: (1) a PDF file containing your written answers to tasks 2-4, (2) a ZIP file containing the source codes. The answers to the feedback task are collected separately through the course feedback system.

**Exercise 1.** Course feedback. (Worth of 1 bonus point in similar manner as other tasks.)
Fill in the official course feedback through the link which you should receive by December 6 to your email. The feedback collection is anonymous. The system separately reports the emails of students, who have returned the feedback, but they are not associated with the answers.

**Exercise 2.** Epipolar geometry. (Pen & paper problem)
Let's assume that the camera projection matrices of two cameras are $\mathbf{P} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$ and $\mathbf{P}' = \begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$,where $\mathbf{R}$ is a rotation matrix and $\mathbf{t} = (t_1, t_2, t_3)^\top$ describes the translation between the cameras. Hence, the cameras have identical internal parameters and the image points are given in the normalized image coordinates (the origin of the image coordinate frame is at the principal point and the focal length is 1).

The epipolar constraint is illustrated in Figure 1 below and it implies that if $p$ and $p'$ are corresponding image points then the vectors $\overrightarrow{Op}$, $\overrightarrow{O'p'}$ and $\overrightarrow{O'O}$ are coplanar, i.e.

$$\overrightarrow{O'p'} \cdot \left(\overrightarrow{O'O} \times \overrightarrow{Op}\right) = 0 \tag{1}$$

Let $\mathbf{x} = (x, y, 1)^\top$ and $\mathbf{x}' = (x', y', 1)^\top$ denote the homogeneous image coordinate vectors of $p$ and $p'$.

Show that the equation (1) can be written in the form

$$\mathbf{x}'^\top \mathbf{E} \mathbf{x} = 0, \tag{2}$$

where matrix $\mathbf{E}$ is the essential matrix $\mathbf{E} = [\mathbf{t}]_\times \mathbf{R}$ (as defined on slide 21 of Lecture 11).

*Hint*: When the camera calibration matrix $\mathbf{K}$ is an identity matrix, then the homogenous image coordinate vectors directly represent the direction vectors of the incoming light rays in the camera coordinate frame. The proof follows in a relatively straightforward manner by writing the vectors of Equation (1) in the coordinate frame of the second camera.

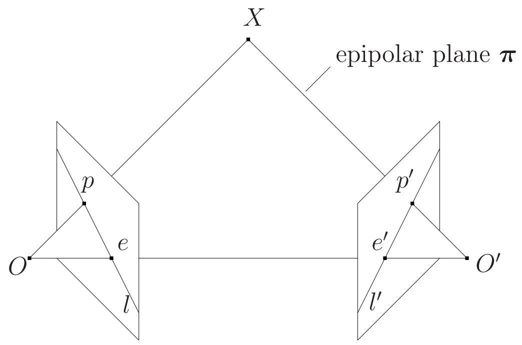


**Figure 1:** Epipolar geometry. Given a point $p$ in the first image its corresponding point in the second image is constrained to lie on the line $l'$ which is the epipolar line of $p$. Correspondingly, the line $l$ is the epipolar line of $p'$. Points $e$ and $e'$ are the epipoles.

**Exercise 3.** Stereo vision. (Pen & paper problem)

In Figure 2 below there is a picture of a typical stereo configuration, where two similar pinhole cameras are placed side by side. The focal length of the cameras is $f$ and the distance between the camera centers is $b$. The point $P$ is located in front of the cameras and its disparity $d$ is the distance between the corresponding image points (i.e. $d = |x_l - x_r|$). The disparity depends only on the parameters $b$ and $f$ and the $Z$-coordinate of $P$.

*a)* Assume that $d = 1\ cm$, $b = 6\ cm$ and $f = 1\ cm$. Compute $Z_P$.

*b)* Assume that the smallest measurable disparity is 1 pixel and the pixel width is 0.01 $mm$. What is the range of $Z$-coordinates for those points for which the disparity is below 1 pixel?

*c)* In the configuration illustrated in Figure 2 the camera matrices are $\mathbf{P}_l = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}$ and $\mathbf{P}_r = \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{t} = (-6, 0, 0)^\top$. The point $Q$ has coordinates $(3, 0, 3)$. Compute the image of $Q$ on the image plane of the camera on the left and the corresponding epipolar line on the image plane of the camera on the right. (Hint: The epipolar line is computed using the essential matrix as shown on slide 43 of Lecture 11.)

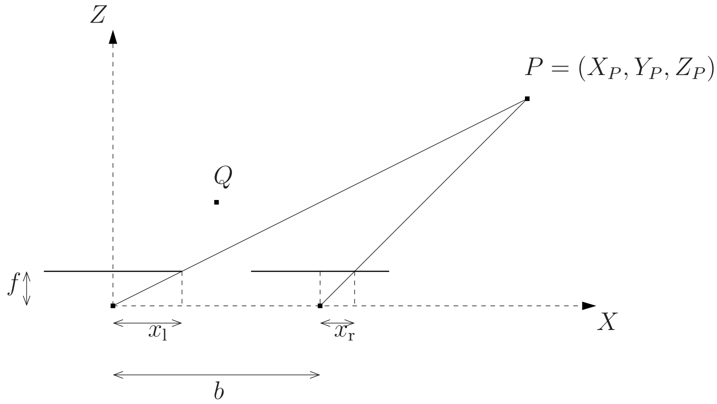


**Figure 2:** Top view of a stereo configuration where two pinhole cameras are placed side by side.

**Exercise 4.** Fundamental matrix estimation. (Matlab exercise)

Get the example `m`-files by downloading `Exercise12.zip` from the MyCourses page. See the comments in `Fmatrix_example.m` and implement the two missing functions:

*a)* Implement the eight-point algorithm as explained on slide 28 of Lecture 11.

*b)* Implement the normalized eight-point algorithm as explained on slide 31 of Lecture 11 (see also Algorithm 11.1. in Hartley & Zisserman).

The epipolar lines obtained with both F-matrix estimates should be close to those visualized by the example script.