Statistical Mechanics
E0415

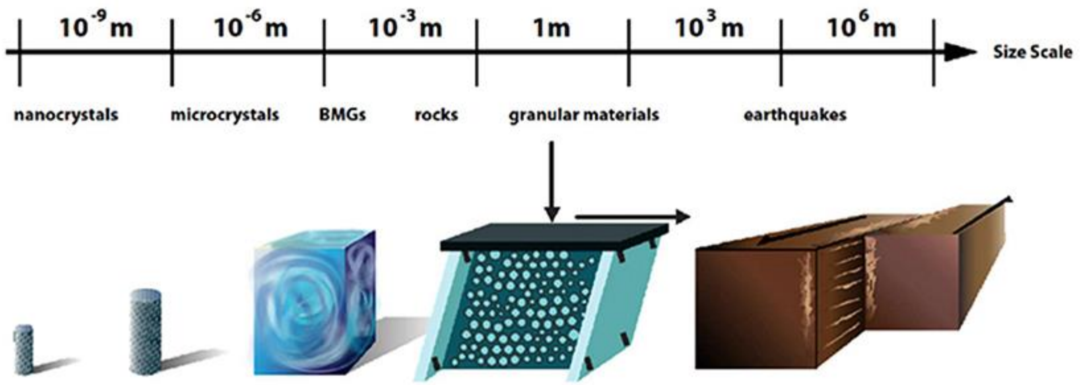
Fall 2021, lecture 10
Out-of-equilibrium

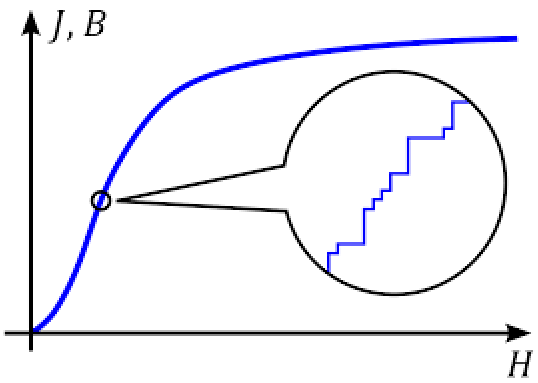
… previous take home

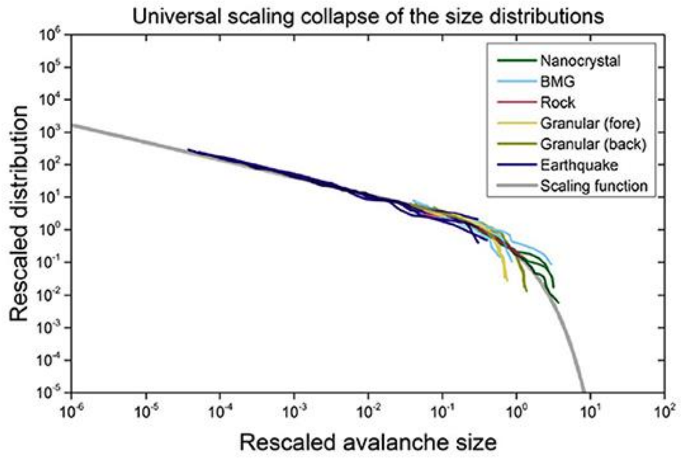
… examples…

Statistical mechanics for complex systems

Here are a few examples of "avalanching" systems: deformation up to earthquakes , Barkhausen noise (magnets), a real snow avalanche. How to understand such statistics?







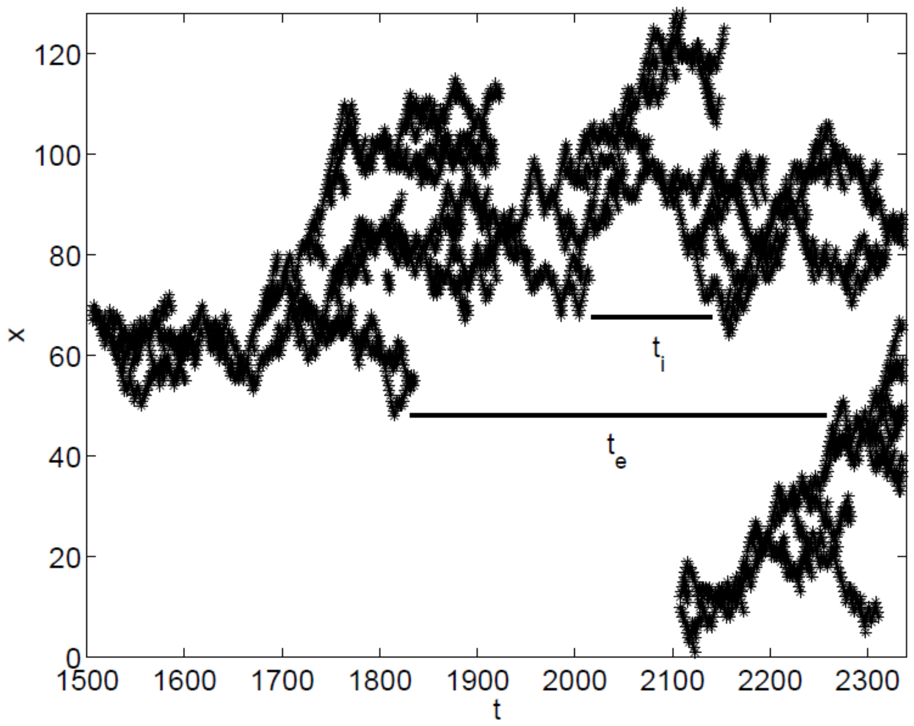
Activity map

A 1D model of activated random walkers.

Plot the locations where there is activity.

The pattern is (here) a self-affine fractal, since the system is in a (self-organized) critical state.

Correlation functions….
Avalanche distributions.

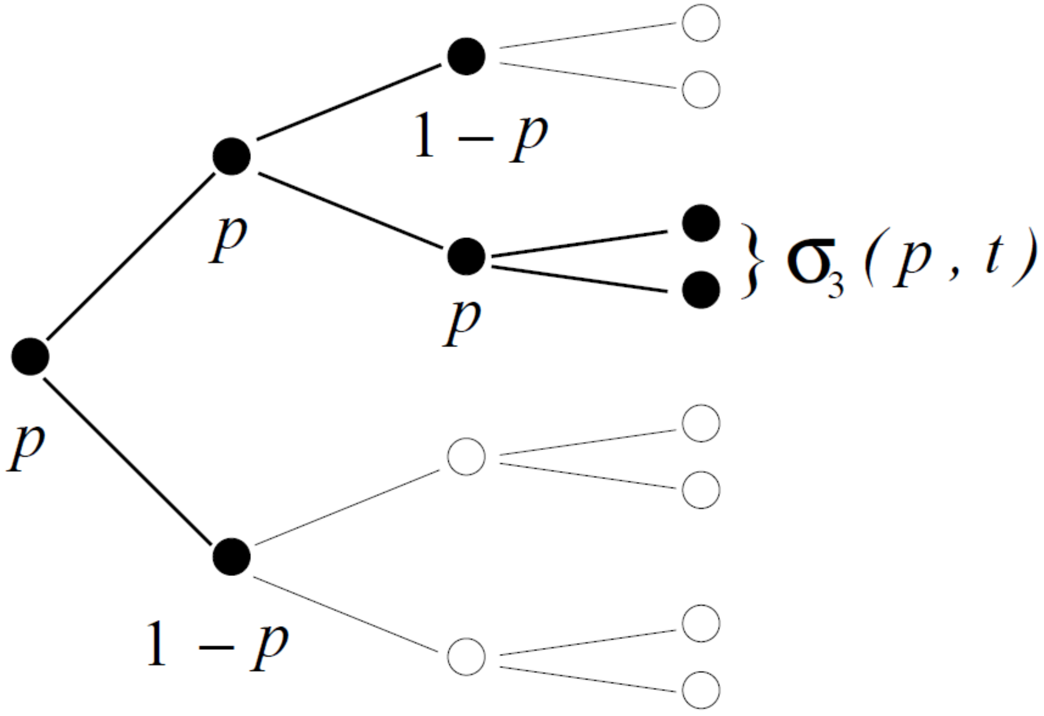


Branching process on a tree

Size of the avalanche? ”7”

Number of generations? 4 (0…3 counting also the seed).

Limit of infinite dimensions: the avalanche never re-visits the same location.

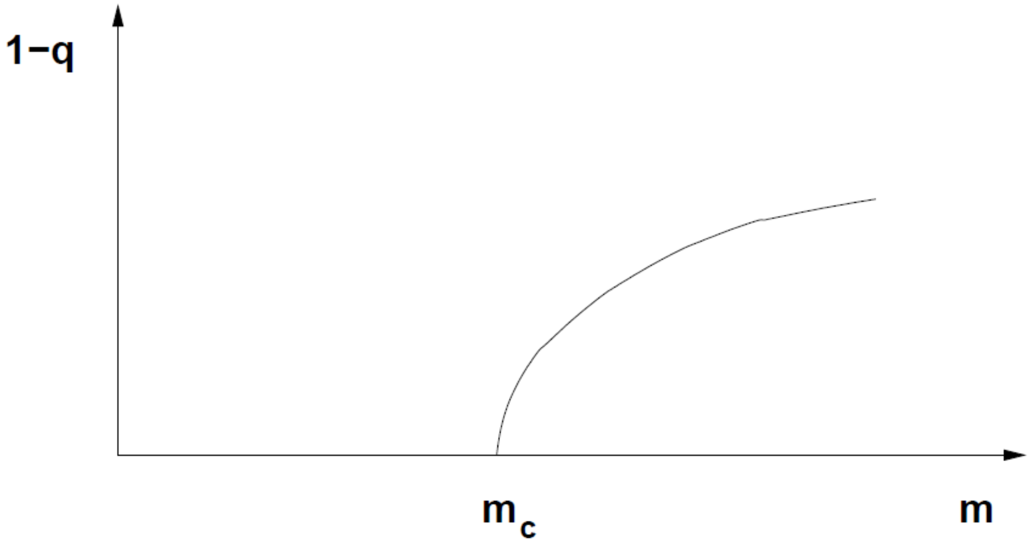


Critical properties

$$\text{Prob(survival)} = \begin{cases} 0, & m \le m_c \\ (m - m_c)^\beta, & m > m_c. \end{cases}$$

Order parameter: survival probability.

Critical point value, order parameter exponent β.



Properties of critical branching processes

The statistics can be solved with the aid of the generating functions of the stochastic process as a function of generation n, and looking at the scale-free, Fixed-Point solution.

$$f(x,p) = \frac{1-\sqrt{1-4x^2p(1-p)}}{2xp}.$$

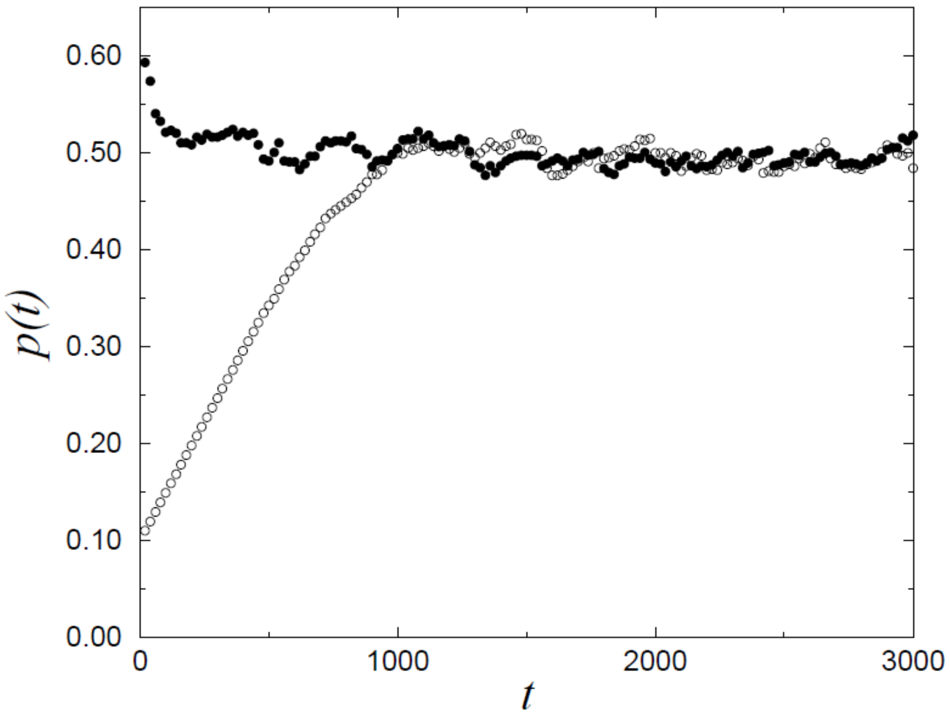
This shows that indeed, the avalanches are scale-free apart from a cut-off function.

$$P_n(s,p) = \frac{\sqrt{2(1-p)/\pi p}}{s^{3/2}} \exp\left(-s/s_c(p)\right).$$

Self-organized Branching Process

At a fixed n, we study the effect of the initial condition on the time-dependent branching probability p(t) (above and below the critical value).

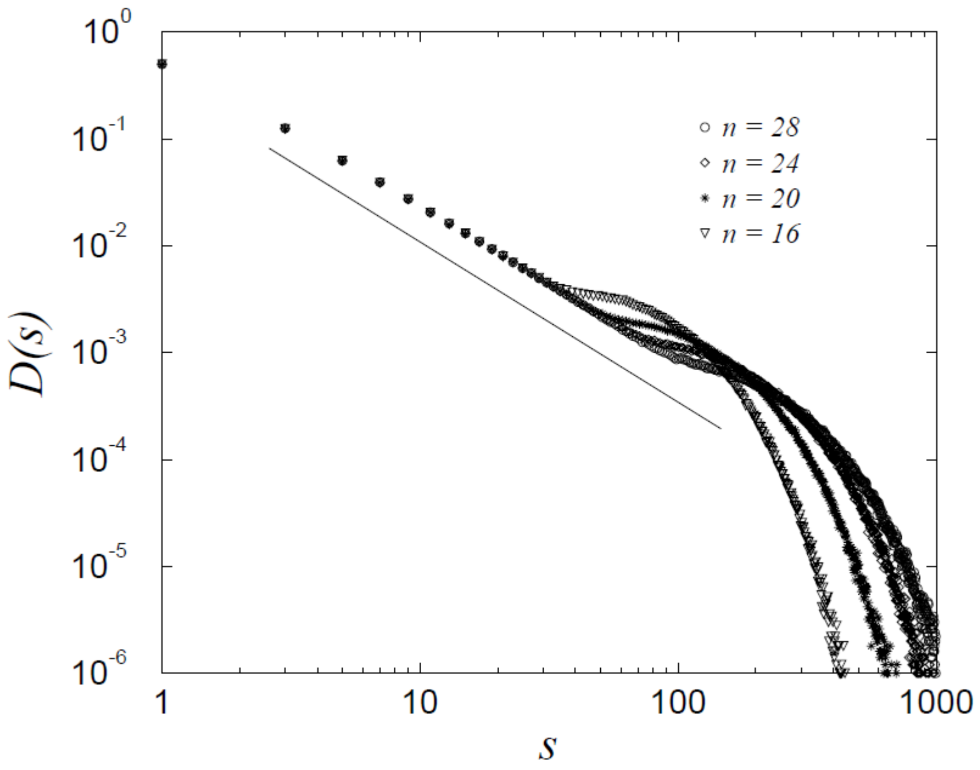
After transient periods, a fluctuating steady-state is reached.



SOBP: avalanches

At finite n, the avalanches follow the theoretical prediction (exponent -3/2, exponential cut-off).

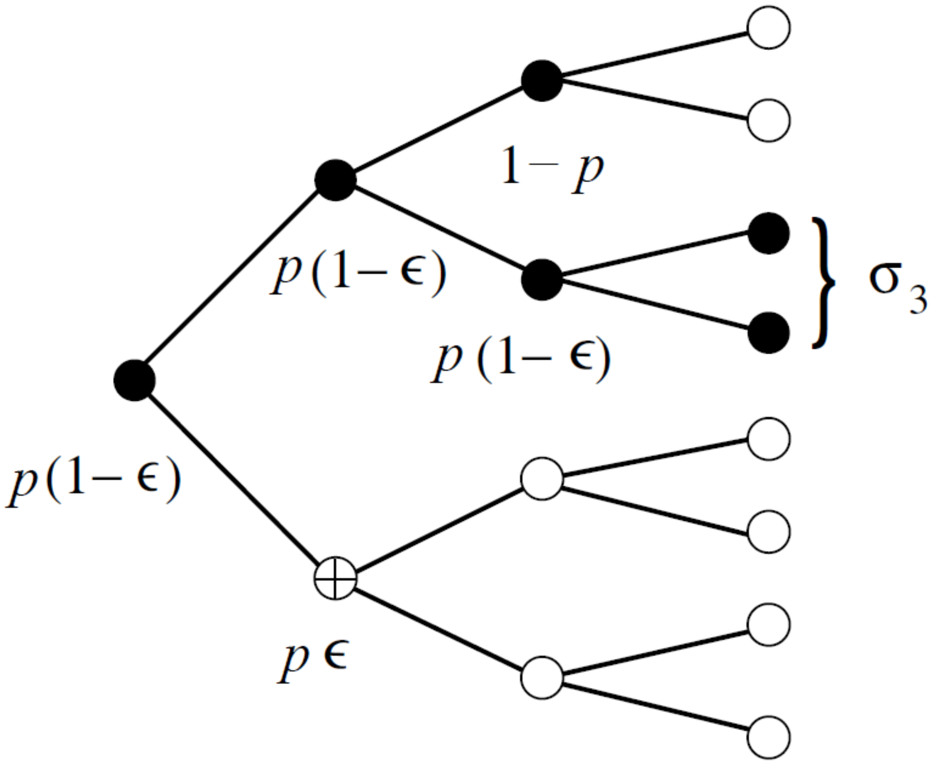
Similar results for durations T.



SOBP with dissipation

How does dissipation (of particles, energy…) influence the critical properties.

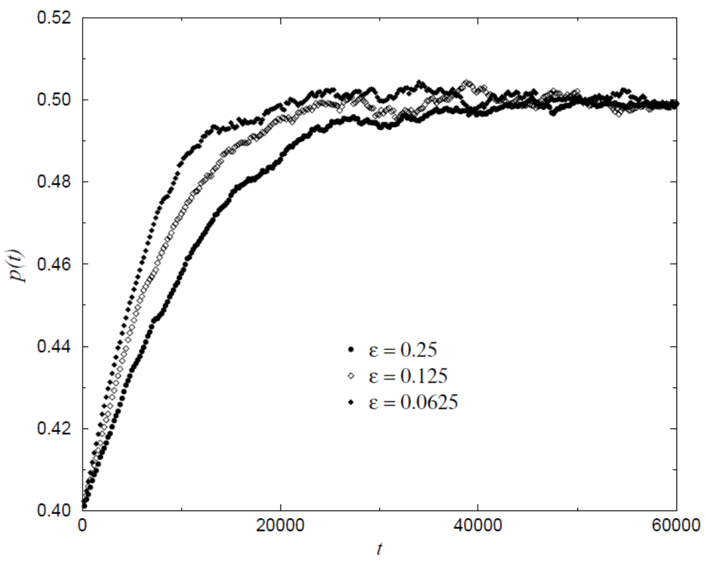
In the tree/mean-field picture this, “ε”, is easy to add to the process dynamics.

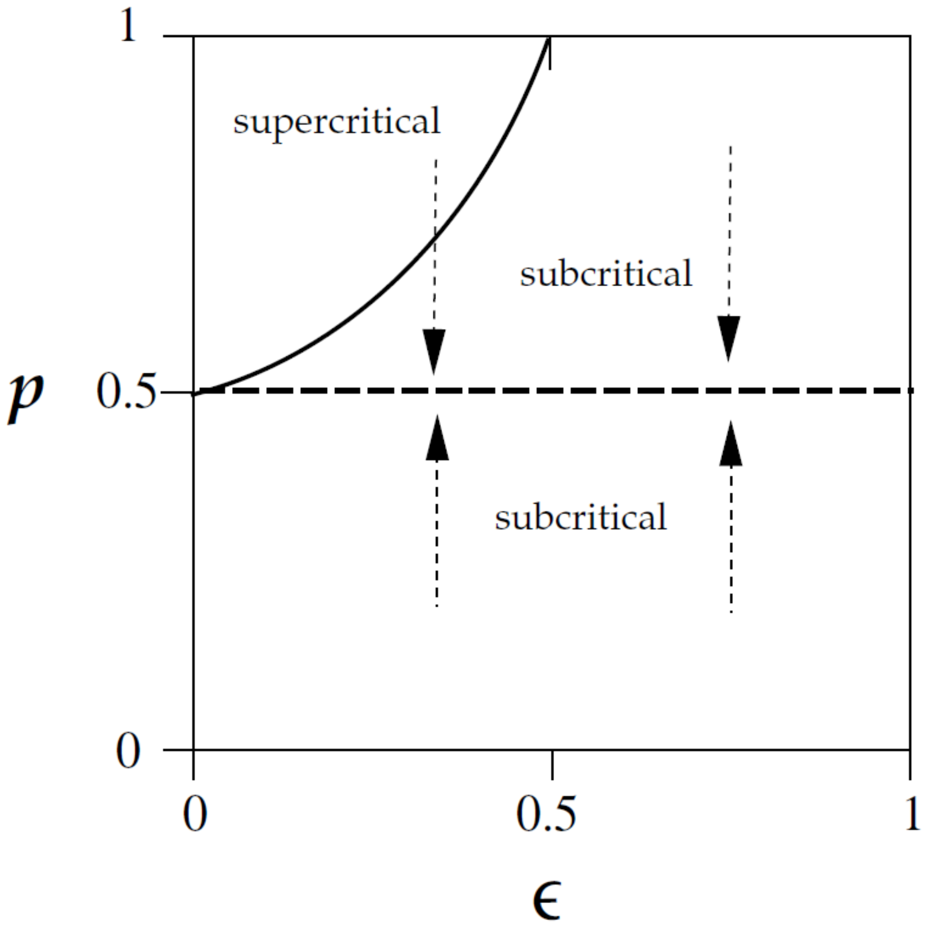


Phase diagram in the presence of dissipation

Dynamical equation for p(t): steady-state.

Resulting phase diagram.

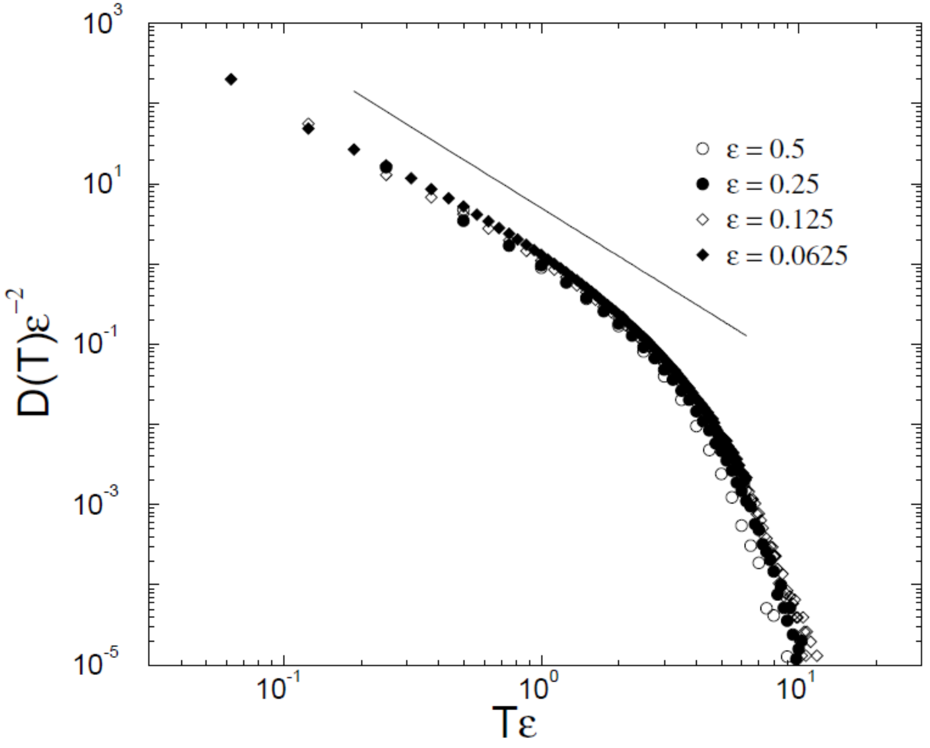


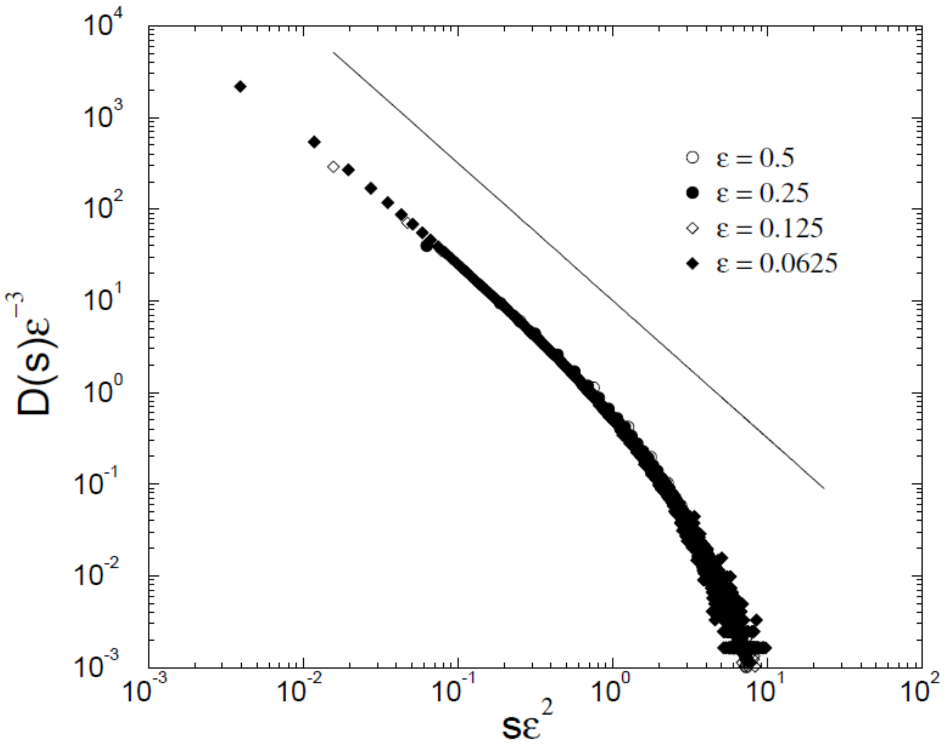


Avalanche distributions with dissipation

Distributions of durations, sizes with varying dissipation.

Rescaling/collapse of statistics (cf. x/y-axis).





Last take home

This time we study out-of-equilibrium systems that exhibit what is called crackling noise or bursts of activity. Such phenomena arise in many contexts (materials, astrophysics, geophysics - earthquakes, neuroscience, biology)…. and so forth. The material for this lecture is a set of lecture notes.

The key points are: understand some mechanisms (there are more) by which systems in nature produce such behavior. If you are really interested and want more depth you may have a look at the very recent review article in https://www.frontiersin.org/articles/10.3389/fphy.2020.00333/full

To finish off the take homes, we have again then a pick of THREE recent papers for you. These illustrate (all from 2020) the applications of such ideas to various fields.

We start from neuroscience

https://arxiv.org/abs/2011.03263

... move over to the deformation of materials....

https://advances.sciencemag.org/content/6/41/eabc7350

... and finish with earthquake (prediction) in a laboratory.

https://arxiv.org/abs/2011.06669

And your task is like the previous time "2+8" sentences on the selection and main points.

… end of the course…

On the 7th of December presentations of the computational projects.

We will that week let you know of your projected total score for the course.

Enjoy!