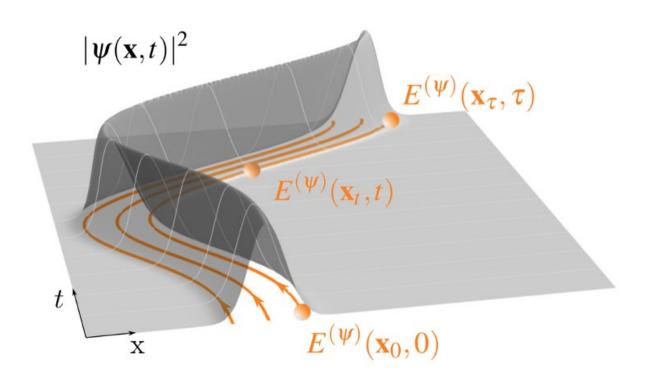
PHYS-C0252 - Quantum Mechanics Part 2 Section 5.3

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5.3 Annihilation and Creation Operators

• Consider first the case of fermions (antisymmetric WFs). The creation operator C_{α}^{\dagger} is defined by the relations

$$egin{aligned} C_{lpha}^{\ \ \dagger}|0
angle &= |lpha
angle \ , \ \ C_{lpha}^{\ \ \dagger}|eta
angle &= C_{lpha}^{\ \ \dagger} C_{eta}^{\ \ \dagger}|0
angle &= |lphaeta
angle = -|etalpha
angle \ , \ \ C_{lpha}^{\ \ \dagger}|eta\gamma
angle &= C_{lpha}^{\ \ \ } C_{eta}^{\ \ \ } C_{\gamma}^{\ \ \ \dagger}|0
angle &= |lphaeta\gamma
angle \ , \ \ \mathrm{etc} \ . \end{aligned}$$

The Pauli exclusion principle requires that

$$C_{lpha}{}^{\dagger}|lpha\cdots
angle=0$$

• The adjoint operator $C_{\alpha}\equiv (C_{\alpha}^{\dagger})^{\dagger}$ is defines the annihilation operator

$$egin{aligned} C_lpha |lpha
angle = |0
angle \ C_lpha |0
angle = 0 \end{aligned}$$

 It is easy to show (homework?) that these fermionic operators obey an anticommutation relation

$$\{C_{\alpha}, C_{\beta}^{\dagger}\} \equiv C_{\alpha}C_{\beta}^{\dagger} + C_{\beta}^{\dagger}C_{\alpha} = \delta_{\alpha\beta}I$$

and the number operator
$$N=\sum_{lpha} \ {C_{lpha}}^{\dagger} \ {C_{lpha}}$$

• Similarly, for the case of bosons (symmetric WFs) the creation operator a_{α}^{\dagger} is defined by the relations

$$a_{lpha}{}^{\dagger}|0
angle = |\phi_{lpha}
angle = |0,0,\ldots,n_{lpha}=1,0,\ldots
angle,$$
 $a_{lpha}{}^{\dagger}|n_1,n_2,\ldots,n_{lpha},\ldots
angle \ \propto \ |n_1,n_2,\ldots,n_{lpha}+1,\ldots
angle.$ and the annihilation operator $a_{lpha} \equiv (a_{lpha}^{\dagger})^{\dagger}$

$$a_{lpha}|\phi_{lpha}
angle=|0
angle\,, \ a_{lpha}|n_{1},n_{2},\ldots,n_{lpha},\ldots
angle \;\; \propto \;\; |n_{1},n_{2},\ldots,n_{lpha}-1,\cdots
angle\,, \;\; (n_{lpha}>0)\,,$$

$$a_{\alpha}|n_1,n_2,\ldots,n_{\alpha}=0,\ldots\rangle=0$$
.

The number operator is given by

$$N = \sum_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$$

and

$$egin{aligned} a_lpha |n_1,n_2,\ldots,n_lpha,\ldots
angle &= (n_lpha)^{1/2} |n_1,n_2,\ldots,n_lpha-1,\ldots
angle \ a_lpha^\dagger |n_1,n_2,\ldots,n_lpha,\ldots
angle &= (n_lpha+1)^{1/2} |n_1,n_2,\ldots,n_lpha+1,\ldots
angle \end{aligned}$$

These were proven for the QHO already. The bosonic operators obey a commutation relation

$$\left[a_{\alpha}, a_{\beta}^{\dagger}\right] \equiv a_{\alpha} a_{\beta}^{\dagger} - a_{\beta}^{\dagger} a_{\alpha} = \delta_{\alpha\beta} I$$