L12 Current Topics

Friday, December 3, 2021 1:25 AM

Lecture notes by Ethan Minot, visiting from Oregon State University
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Nanoelectronics Class at Aalto University, Autumn, 2021.

Philosophy of the course - examine electron transport phenomena in small conducting structures.
The structures could be made out of

1. Normal metal wires
2. Superconducting wires
3. Semiconductors
4. Molecules
5. Graphene

From 2000-2010 researchers were happy to call this Nanoelectronics

From 2010 ⟶ 2020

"Nanoelectronics" morphing into "Hybrid quantum systems"
or "Quantum Materials"

distinguish from "mature" nanoelectronics research topics that have shifted from physics departments into electrical engineering departments and companies.

(e.g. making a CPU for a computer using CNTs).

Hybrid quantum system

superconducting circuit — coupled to ⟷
- nanomechanical resonator
- light-emitting diode.
- CNT quantum dots

Adding functionality that isn't possible in a single system

Quantum Materials

Semiclassical descriptions can't capture the phenomena
e.g. m_{eff}, $\frac{p^2}{2m_{eff}}$, $\tau_{scatter}$ (parameterizing quantum effects to map onto classical intuition)

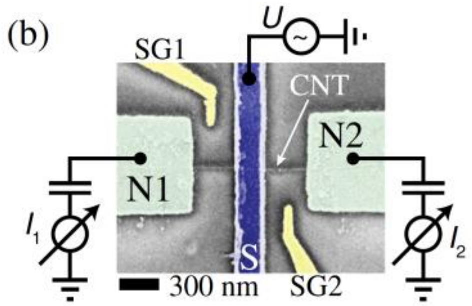
- Strong electron-electron interactions & quantum phase transitions
- Non-trivial topology of the band structure
- Exotic quasi particles in the system

Table of contents for the lecture

(choose some recent research that uses ideas taught in this class, and are examples of exciting/promising future research)

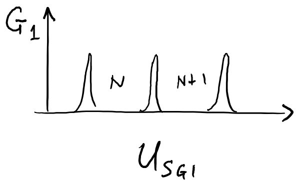
- Hybrid Quantum System : Cooper Pair Splitter
- Quantum Materials : Magic-Angle bilayer graphene (interactions)
  Quantum Spin Hall Materials (topology)
  Majorana zero modes in nanowires with Al shell (exotic quasi particles)
- Outlook..

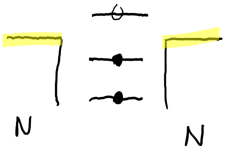
COOPER PAIR SPLITTER from PRL (2012) Schönenberger's group in Basel



20mK , $kT \approx 2\,\mu eV$

If normal-normal contacts

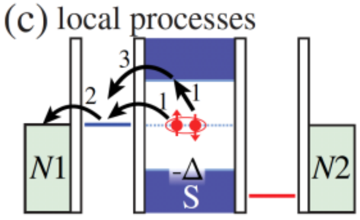




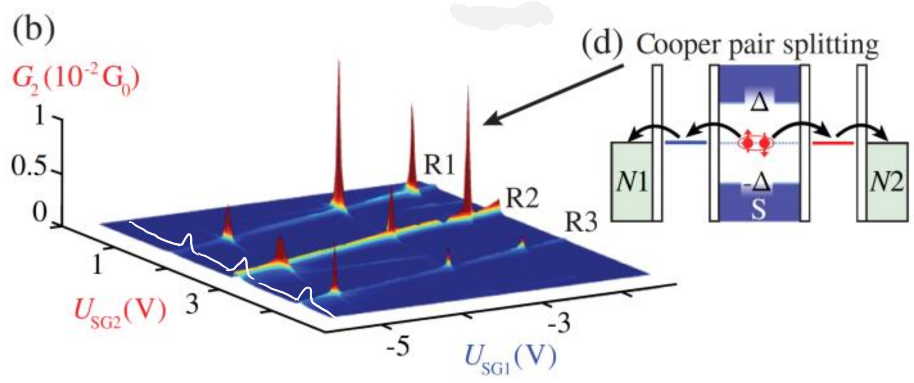
design experiment so E_c is large ($E_c \approx 5\,meV$), no virtual state with double occupancy.

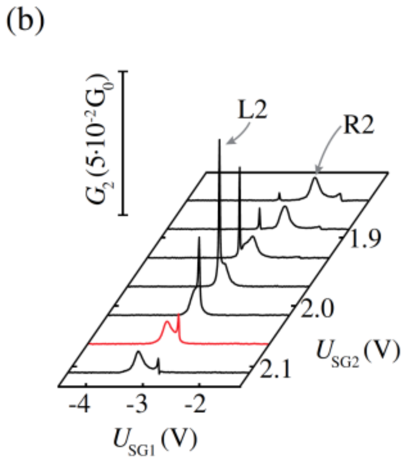
Transport is suppressed when only one QD is tuned to resonance with E_F in S.

(The local process involves virtual excited state)

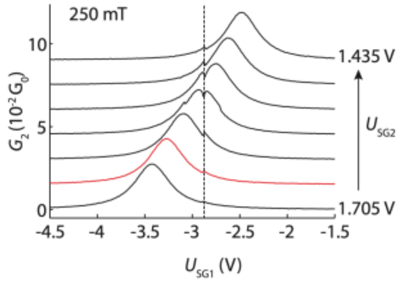


$\Delta \sim 0.1\,meV$





The effect goes away when superconductivity is destroy.



90% efficient at splitting CPs.

The CNT provided
- two normal QDs (not superconducting)
- large charging energy.
- Tuned by individual gates

Hybrid system, needs the S.C. metal & the nanomaterial

MAGIC ANGLE GRAPHENE (First experiments reported in 2018).

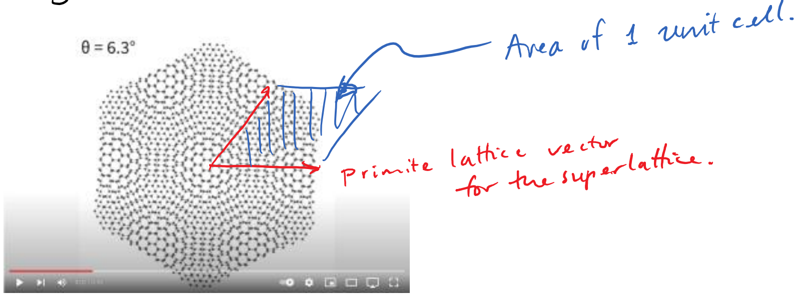
Cao et al. "Correlated insulator behavior..." Nature (2018)
Cao et al. "Unconventional superconductivity..." Nature (2018)

(Moire materials, flat band physics, enhanced e-e interactions)

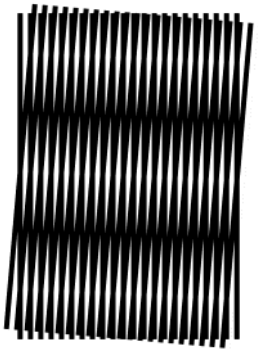
Recommended introduction/overview
YouTube: March meeting 2018 Magic Angle Graphene Superlattices

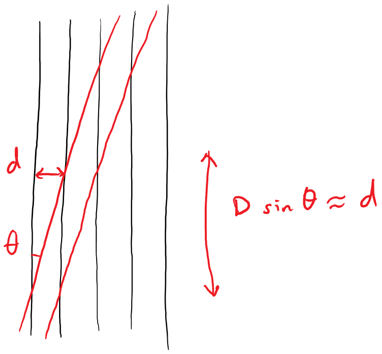
YouTube:

To introduce topic, lay 2 pieces of graphene on top of each other



Mathematics of Moiré patterns...





Periodicity of the superlattice $\boxed{D = \frac{d}{\theta}}$ for small angles

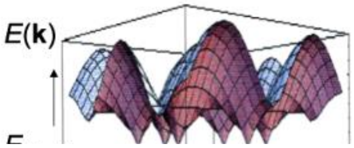
In graphene, length of primitive lattice vector 0.25nm

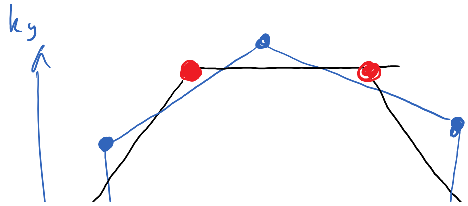
$\theta = 0.018\,\text{rad}$ (1.1°) ← magic angle.

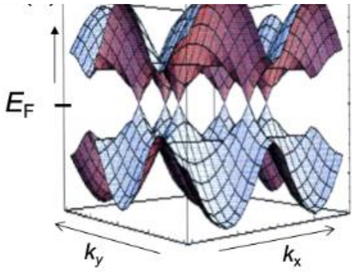
$D = \frac{0.25\,\text{nm}}{0.018} = \underline{13\,\text{nm}}$

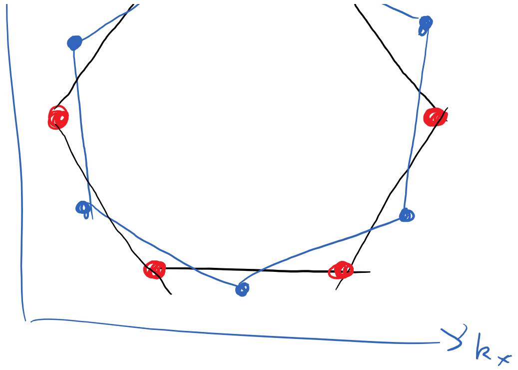
Why pick 1.1° ?

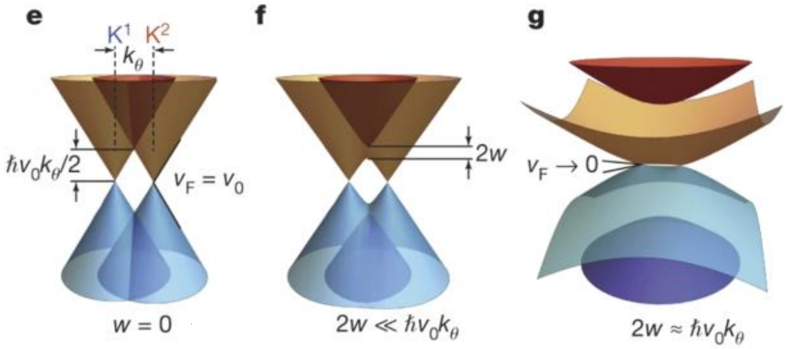
Bistritzer and McDonald "Moire bands in twisted double-layer graphene" PNAS (2011)



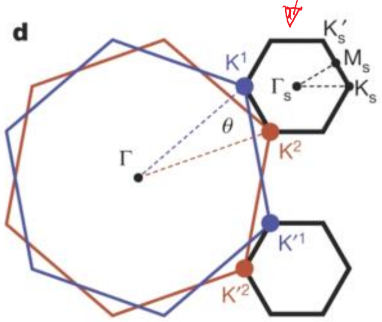


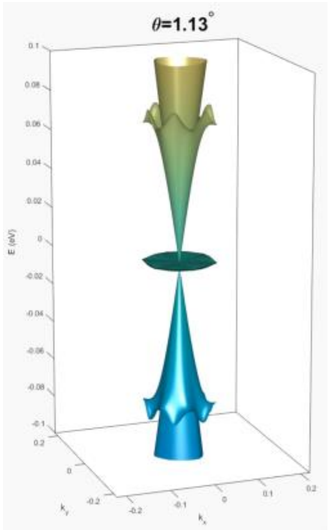






Next step in analysis, fold this band structure into the Brillioun zone of the super lattice.

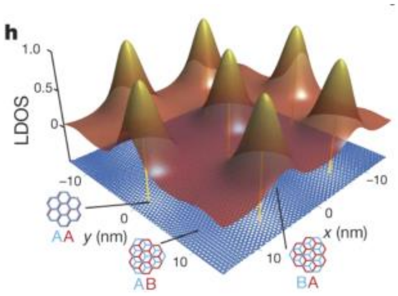

Black hexagon is BZ of superlattice



video of BZ getting smaller and bands reshaping.

See video in Supp Info of

Cao et al. "Unconventional superconductivity..." Nature (2018)



Electron density concentrated over the AA stacked areas.

Area of superlattice unit cell

$$A \approx \frac{\sqrt{3}}{2}\frac{a^2}{\theta^2} \, , \quad a = 0.25\,\text{nm}$$

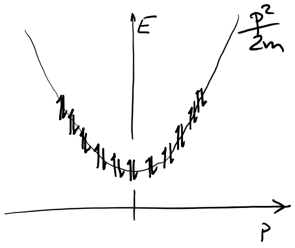
Why go for flat bands?

If the bands are flat, the electronic ground state may not be the standard Fermi sea that minizes K.E.

Ground state might be determined by some other term in the Hamiltonian.

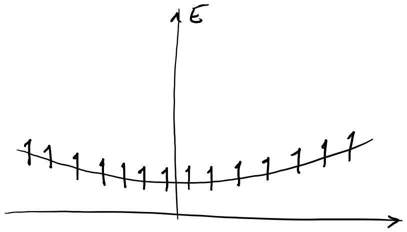
Examples:

STEEP BAND



Pack electrons into the lowest momentum states. Minimize $\sum \frac{p_i^2}{2m}$

FLAT BAND WITH EXCHANGE INTERACTION



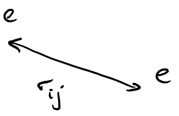
Spread out momentum to allow spin alignment.

$\sum_i \frac{p_i^2}{2m}$ is not in big enough to determine structure of ground state.

$$-J S_i \cdot S_j$$

Other interesting ground states can happen when K.E. is insignificant. For example, the domininate energy scale might be the Coulomb interactions between electrons:

$$E_c = \sum_{ij} \frac{e^2}{4\pi\epsilon r_{ij}}$$



[Side Note: One way to make E_c dominate is to use low electron densities

E_c dominates if the avg spacing between electrons, r_{e-e}, is very big

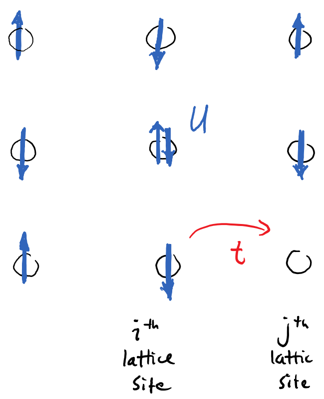
Kinetic energy in system $E_k \sim N \frac{\hbar^2}{2m_{eff} r_{e-e}^2}$ regardless if electrons are localized or delocalized.

If localized, then $\Delta x \Delta p \sim \hbar \Rightarrow p \sim \frac{\hbar}{r_{e-e}}$ $\qquad E_k = N\frac{p^2}{2m}$

If delocalized, then $\lambda_F \sim r_{e-e} \Rightarrow k_F \sim \frac{1}{r_{e-e}}$ $\qquad E_k = N\frac{\hbar^2 k^2}{2m}$]

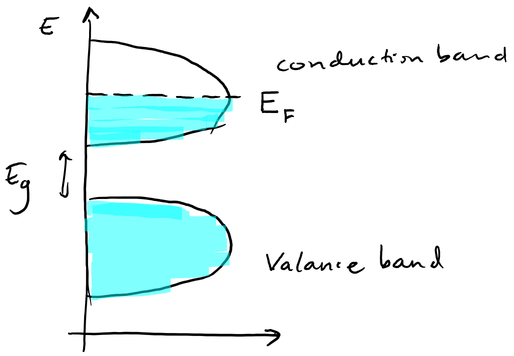
Hubbard model is a minimalistic model for systems that have large E_c

$$\hat{H} = t \sum_{\substack{\text{nearest}\\\text{neighbors}\\ \sigma}} \left(c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma}\right) + U \sum n_{i\uparrow} n_{i\downarrow} \qquad (1963)$$



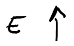
When U is sufficiently large, the model predicts insulating state if there is one electron per lattice site (½ filled band).

NON-INTERACTING ELECTRONS

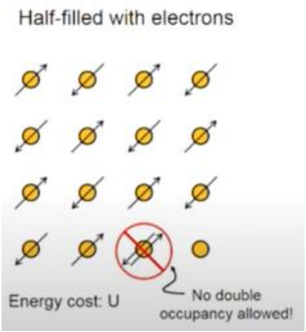


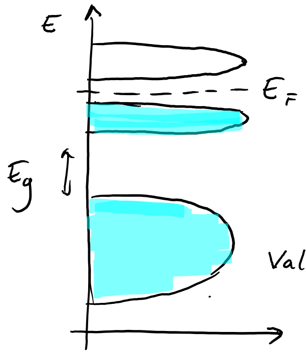
Electrons on a hubbard lattice with U big, t small

Half-filled with electrons



with U big, t small

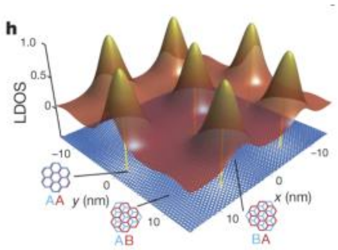




Interaction driven energy gap

→ Insulating state.

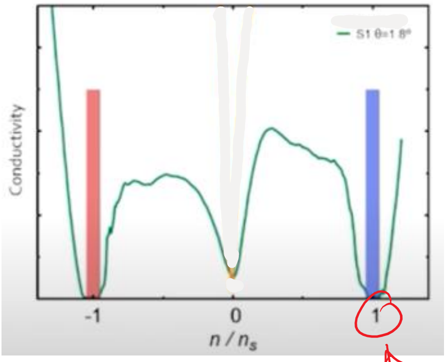
High T_c superconductors show this type of behavior.



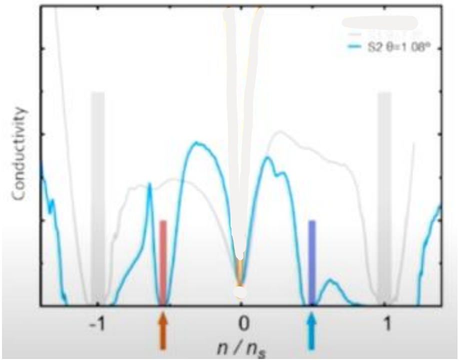
Excitement about magic angle graphene, created a new system to study electrons on a Hubbard lattice.

Transport measurements on two different twist angles.

1.8° Not flat band



1.08° Flat band



Fill the first miniband.

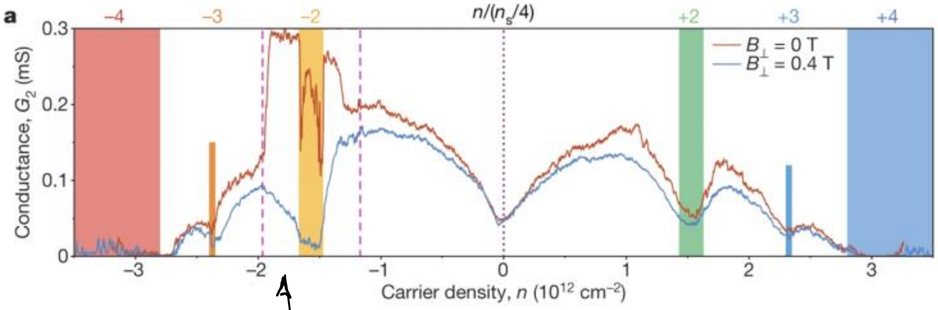
correlated electron gaps.

concentration of electrons, n_{2d}, that fills the first miniband.

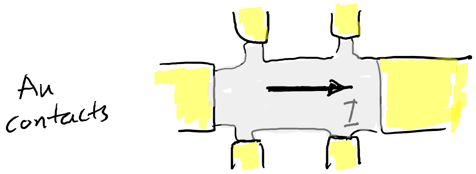
Results from another device...

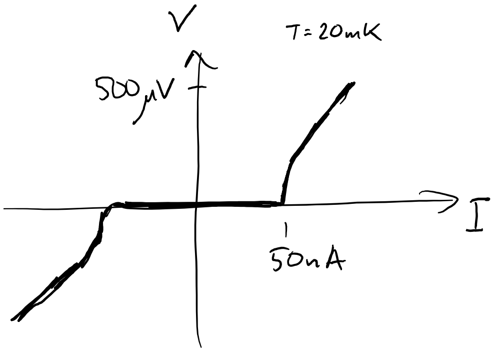
2-probe measurements, with and without B-field.

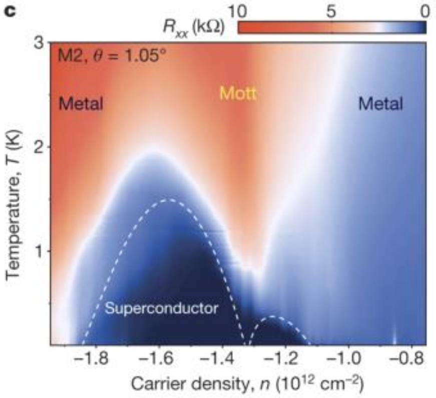
$B = 0.4T$ will destroy superconductivity.



4-probe measure here at $B=0$







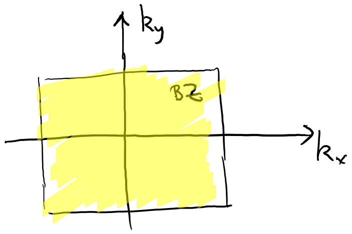
Summary... engineer a quantum material.

TOPOLOGY OF AN ELECTRONIC BAND

Reference: Ramirez & Skinner Physics Today (2020)

We typically compute electronic wavefns on the way to to finding $E(\vec{k})$ and then discard the wavefns and move forward with $E(\vec{k})$

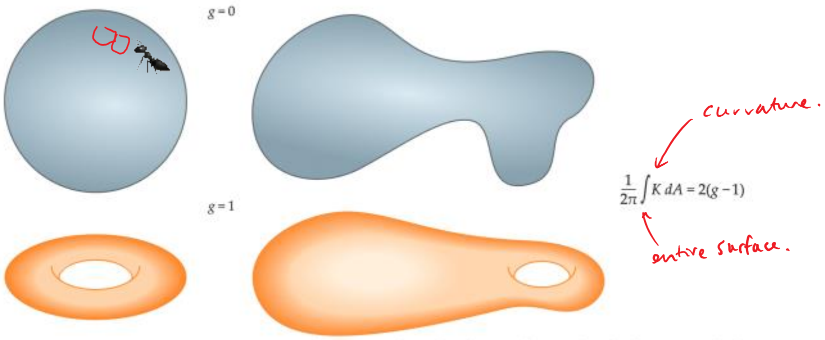
There is more information in the shape of the electronic wavefns.



To find this "topological" information, we should examine every wave function in the Brillioun Zone.

Analogy... examine every piece of this rope to decide if a knot will form when I pull the ends

(I could make many adjustments to the rope without affecting number of knots)



$$\frac{1}{2\pi}\int K\,dA = 2(g-1)$$

FIGURE 1. A TOPOLOGICAL INVARIANT is a property of a geometric shape that does not change when the shape is stretched or distorted. One such invariant is the genus *g*, which is given by the number of holes in the surface and is related to the integral of the Gaussian curvature *K* over the surface of the shape. Shapes with no holes in them ($g = 0$) all give the same value of this integral, as do shapes with one hole in them ($g = 1$). (Image by Donna Padian.)

Bloch wavefunction $e^{i\vec{k}\cdot\vec{r}}\, u_{\vec{k}}(\vec{r})$ for cystalline material.

We can construct the Hamiltonian for electrons in the crystal so that the Hamiltonian is a function of $\vec{k}$ (Like we did for graphene). So we have the situation $H(\vec{k})$ with eigenstates $|u_{\vec{k}}\rangle$. We can gradually change $\vec{k}$ (by applying a force the the electron) and the eigenstate of $H(\vec{k})$ will gradually evolve.

The property we now examine at every point in k-space is Berry's connection:

$$\vec{X} = i \langle u_{\vec{k}} | \nabla_{\vec{k}} | u_{k} \rangle, \qquad ①$$

(the projection of the eigenstate on $\frac{\partial}{\partial k_x} |u_{\vec{k}}\rangle$, $\frac{\partial}{\partial k_y} |u_{\vec{k}}\rangle$, and $\frac{\partial}{\partial k_z} |u_{\vec{k}}\rangle$)

Note that $\vec{X}$ is a vector with same size as $\vec{k}$, and the components of $\vec{X}$ have dimensions of length.

Brian Skinner (author of the Physics Today article) likes to think of $\vec{X}$ as the "centroid position" of the $u_{\vec{k}}(\vec{r})$ function. See figure below

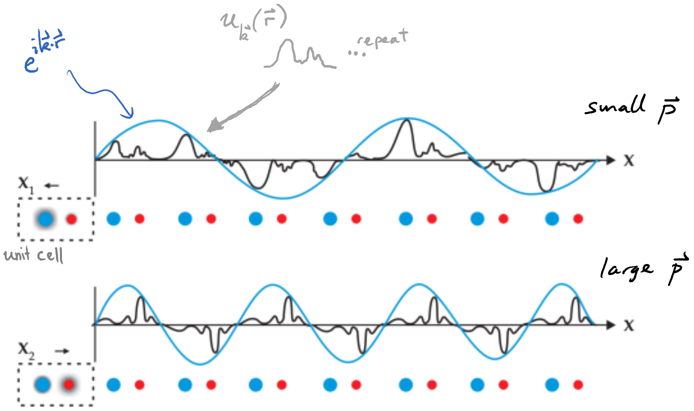
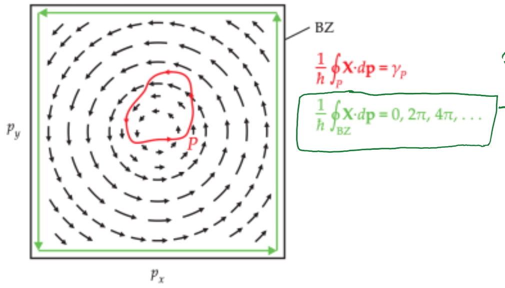


FIGURE 2. IN A CRYSTAL, AN ELECTRON STATE (black curve) is described by a slowly oscillating plane wave (blue curve) whose wavelength corresponds to the electron momentum that is modulated by a periodic Bloch function describing the electron's attraction to the atoms (red and blue circles) in the crystal's repeating unit cell. The electron probability density is shared among the atoms in the unit cell, as indicated by the shaded black areas in the outlined unit cells. The Berry connection **X**, shown above those unit cells, is a vector that can be thought of as the center of the state's density distribution. (Image by Donna Padian.)

Don't take the centroid analogy too literally. $\vec{X}$ is not the same thing as $\int_{\text{unit cell}} \vec{r} \, |u_{\vec{k}}(\vec{r})|^2 \, d^3\vec{r}$



Path integral of $\vec{X}$ around the BZ boundary.

FIGURE 3. THE BRILLOUIN ZONE (BZ) is defined by the set of all possible momenta **p** for electrons in a crystal. This illustration shows the BZ for a two-dimensional crystal. The Berry connection **X** is a vector field (black arrows) in the BZ that indicates the electron wavefunction's center in the crystal's unit cell as a function of **p**. If an electron is accelerated and decelerated along some closed path P (red loop), its wavefunction acquires an overall phase γ_P, whose sign depends on the direction of the path, clockwise or counterclockwise. But if that closed path runs along the BZ boundary (green loop), the phase must be a multiple of 2π.

Equivalently

$$\int_{BZ} \nabla\times\vec{X} \; d^2\vec{p} = \underline{0}, \underline{2\pi}, \underline{4\pi} \ldots$$

topologically trivial.

$\nabla\times\vec{X}$ is called Berry Curvature

If an electron is accelerated and decelerated along some closed path P (red loop), its wavefunction acquires an overall phase γ_P whose sign depends on the direction of the path, clockwise or counterclockwise. But if that closed path runs along the BZ boundary (green loop), the phase must be a multiple of 2π. (Image by Donna Padian.)

topologically trivial.

$$\text{Chern number} = \underline{0}, \underline{1}, \underline{2} \ldots$$

$$= \frac{\int \nabla \times X \, d^2\vec{p}}{2\pi}$$

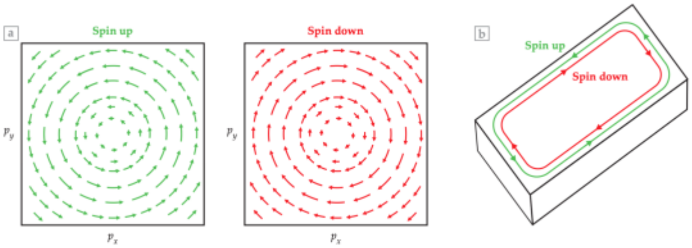
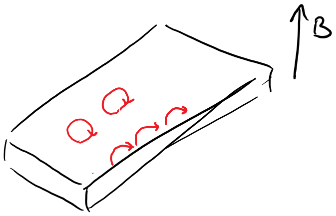


FIGURE 4. A QUANTUM SPIN HALL SYSTEM has equal and opposite Chern numbers, which describe electron wavefunction winding, for its two spin species. **(a)** The Berry connection **X**, shown here for a two-dimensional quantum spin Hall material, winds counterclockwise for up spins and clockwise for down spins, which gives Chern numbers of +1 and −1 respectively. The system remains symmetric under time reversal, which simultaneously changes **p** to −**p** and spin up to spin down. **(b)** The boundary of a quantum spin Hall material features edge states in which one spin species moves clockwise around the sample while the other moves counterclockwise. (Image by Donna Padian.)



Spin up band has same topology as a QH state.

Spin down band has same topology as a QH state.

Another way of saying it...

The effective magnetic field from spin-orbit interaction generates skipping orbits around the edge of the sample.

Why the excitement....

Some electronic bands have non-trivial topology.

QHE is incredibly robust, making it the standard for electrical resistance.

Can "topological protection" be used for other tasks?

MAJORANA ZERO-MODES IN NANOWIRES WITH SUPERCONDUCTING SHELL

(Exotic quasi-particles)

References

- Sergey Frolov lecture on youtube "Majoranas in nanowires" (2016)
- RETRACTED ARTICLE: Quantized Majorana conductance Nature (2018)
- Aguado & Kouwenhoven Physics Today (2020)

- WIRED:

TOM SIMONITE BUSINESS 02.12.2021 07:00 AM

Microsoft's Big Win in Quantum Computing Was an 'Error' After All

In a 2018 paper, researchers said they found evidence of an elusive theorized particle. A closer look now suggests otherwise.

Background:

In graphene, the Hamiltonian for the electron states has same mathematical structure as the Dirac eqn.

$$H_{\vec{q}} = \hbar v_F (q_x \hat{\sigma}_x + q_y \hat{\sigma}_y)$$

$$= \hbar v_F \, \vec{q} \cdot \vec{\hat{\sigma}}$$

Like the Dirac Hamiltonian for an ultrarelativistic electron

$$H = \hbar c (\partial_x \hat{\sigma}_x + \partial_y \hat{\sigma}_y)$$

A similar mathematical mapping was found between quasiparticle excitations in superconductors and the Majorana eqn.

Alternative representation of Dirac Eq in terms of real wavefns.

Cautionary Tale

- 2018 Nature Paper Zhang et al. "Quantized Majorana Conductance" Nature (2018)

  claimed strong evidence of "Majorana zero modes". (MZM)

- Jan 2021 authors release more data in an arxiv publication

  Zhang et al., arXiv:2101.11456

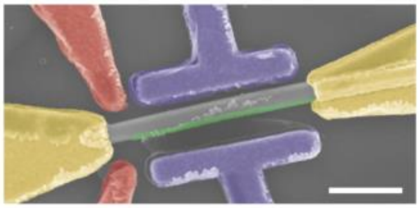
  admitting that they don't have great evidence

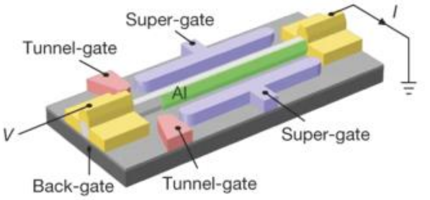
  "A dedicated search [for a specific feature predicted by theory] has the potential to lead to lead to confirmation bias and effectively yield false-positive evidence for majorana zero modes."

- March 2021 Nature paper is retracted.

Examine the controversial paper in some more detail...

Zhang et al. "Quantized Majorana Conductance" Nature (2018)

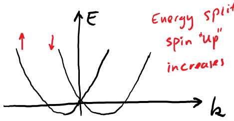




Antimony

InSb nanowire chosen because

$g \approx 50$ & strong spin-orbit $\vec{\sigma} \times \vec{k}$



Energy splitting between spin "up" & "down" increases with k.

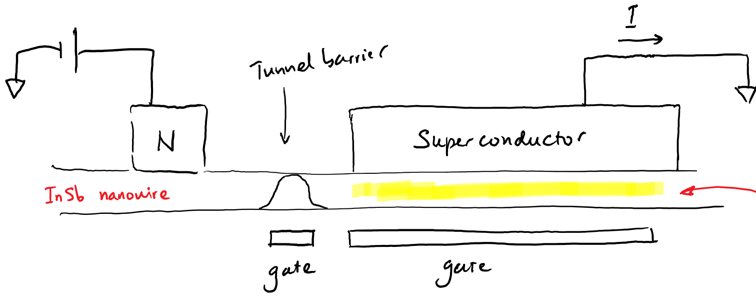
$\pm 1/2$

$E_z = g\mu_B m_s B$ Zeeman energy in InSb

$= \left[1.5\ \frac{\text{meV}}{\text{T}}\right] B$

But B doesn't effect Al so much.

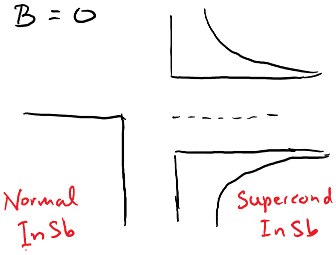
$g \approx 1$



MZM predicted to form here when B field is tuned correctly.

At zero B field... simple tunneling spectrum for NS junction.

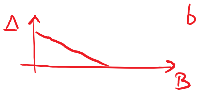
$B = 0$



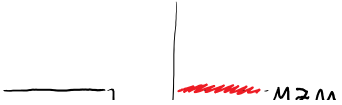
Proximity induced super cond gap

$\Delta = 0.2$ meV

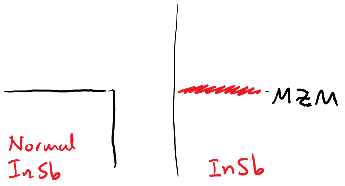
Expect Δ to shrink as function of B because E_z breaks cooper pairs in the InSb.



When $\frac{g\mu_B B}{2} \gtrsim \Delta$, theory predicts interplay between superconductivity, spin orbit and Zeeman, such that MZM forms.

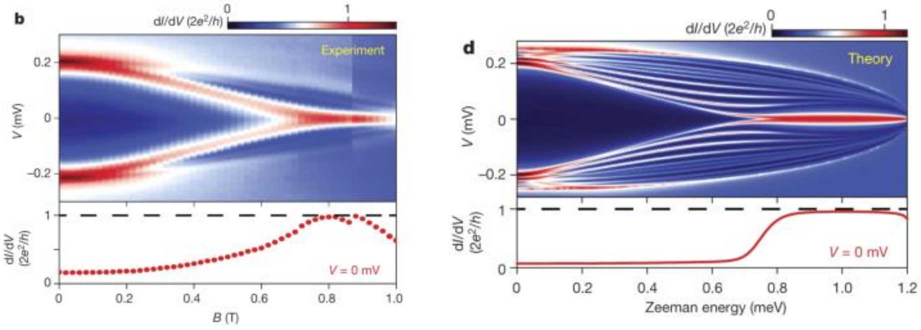


The transparancy of tunnel

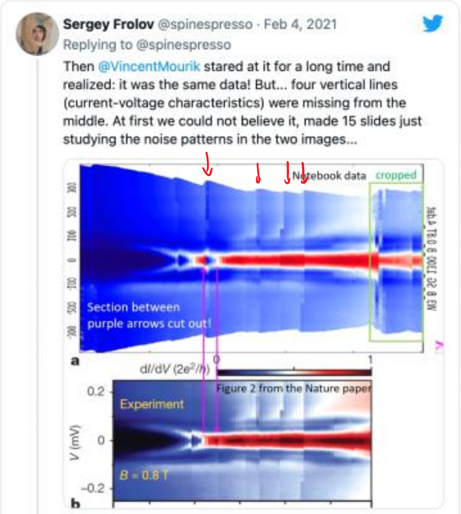


The transparancy of tunnel barrier shouldn't matter...

Theory predicts resonant tunneling and therefore $G = \frac{2e^2}{h}$.



Controversy with the 2018 Nature paper is related to experimental evidence for a plateau at $\frac{2e^2}{h}$.



Sergey Frolov @spinespresso · Feb 4, 2021

Replying to @spinespresso

Then @VincentMourik stared at it for a long time and realized: it was the same data! But... four vertical lines (current-voltage characteristics) were missing from the middle. At first we could not believe it, made 15 slides just studying the noise patterns in the two images...

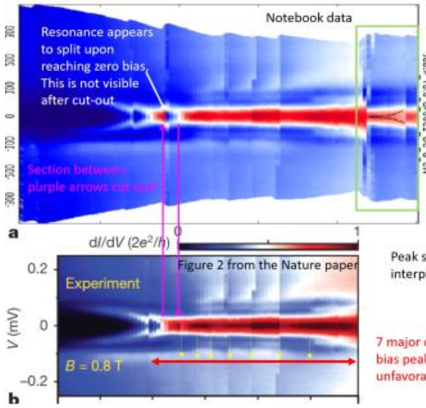


Figure 2 – splittings cut, plateau generated by charge jumps

Zero-bias peak continues, but dramatic charge switches and peak splitting cropped.

A segment on the far right of raw data has been cut (green rectangle). It contains zero bias peaks below quantized value and a peak splitting.

Peak splitting favors trivial Andreev bound state interpretation and goes against 'robust plateau' claim

7 major charge jumps are left uncut to extend zero bias peak and generate an apparent '**plateau**', while unfavorable charge jumps were cut.

Moving forward.

"There is good science to do with reasonable expectations, not magical expectations." — Sergey Frolov

SUMMARY: A good example of "quantum materials" because the interplay between superconductivity, spin orbit and Zeeman is uncharted territory with exciting possibilities. The experiments use many techniques from nanoelectronics.

OUTLOOK

An important lens to use when examining these research directions: Applications.

"Should we concentrate on basic or applied science? is like asking 'is light a particle or a wave? The answer is yes." — Marvin Cohen

- Hybrid Quantum System eg. Cooper Pair Splitter
- Quantum Materials eg. Magic-Angle graphene
  Quantum Spin Hall Materials
  Majorana zero modes

Quantum computing.
Dissipationless transport
Computing with less carbon footprint.
Quantum sensing.