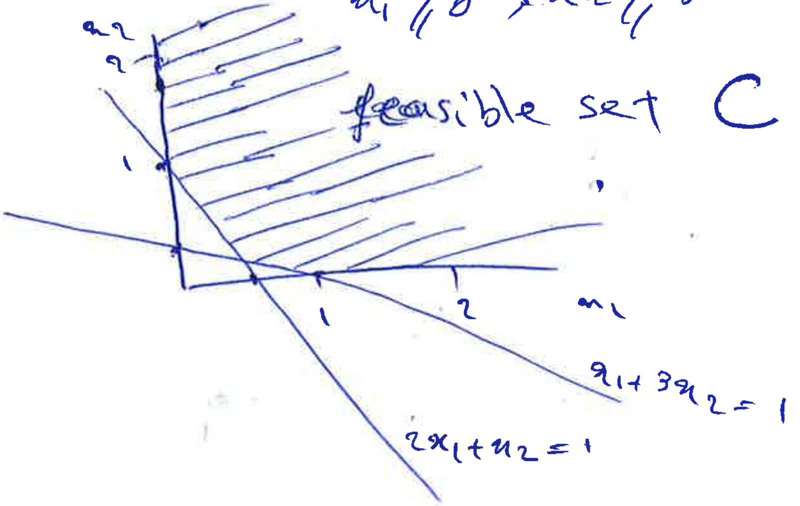
Convex optimization Exercise 3

2) minimize $f_0(x_1, x_2)$

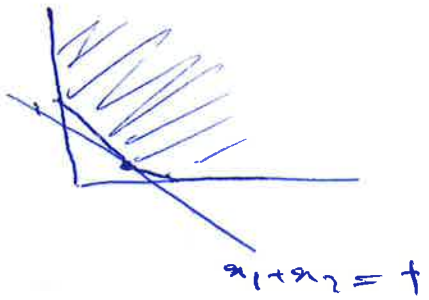
subject to $2x_1 + x_2 \geq 1$

$x_1 + 3x_2 \geq 1$

$x_1 \geq 0$, $x_2 \geq 0$



a)



minimize t

$x_1 + x_2 \leq t$

$x_1, x_2 \in C$

$$\left.\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 + 3x_2 &= 1 \end{aligned}\right\} \Longrightarrow x_2 = \frac{1}{5}, \quad x_1 = \frac{2}{5}$$

$f^* = \frac{3}{5}$ $\quad x^* = \left(\frac{1}{5}, \frac{2}{5}\right)$

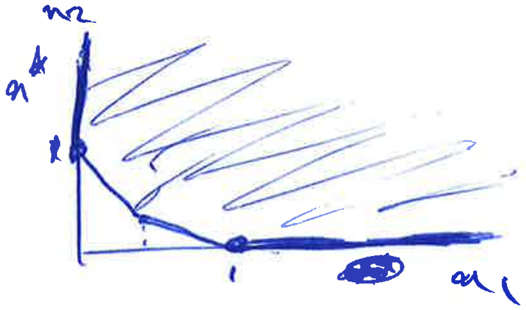
b)

$f_0(x_1, x_2) = -x_1 - x_2$

$\lim_{x_1 \to \infty} f_0(x_1, x_2) = -\infty$ unbounded below

$f_0^* = -\infty$

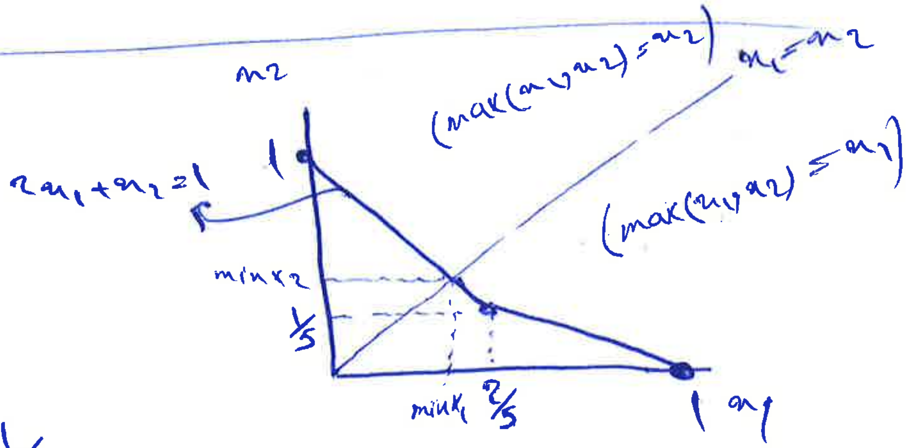
c) $f_0(x_1, x_2) = x_1$



$x^* = \{x \mid x_2 \geq 1, x_1 = 0\}$

$f^* = 0$

d)



$2x_1 + x_2 = 1$

$x_1 = x_2$

$3x_1 = 1 \qquad x_1 = x_2 = 1/3$

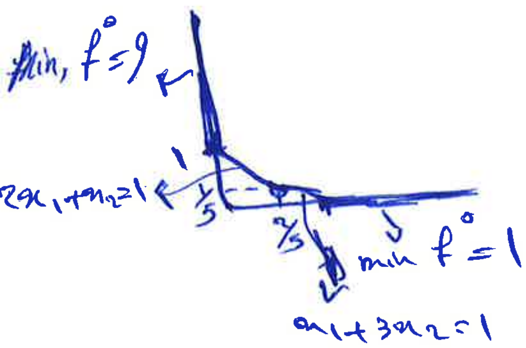
$x^* = (1/3, 1/3) \qquad f^* = 1/3$

e) $f_0(x_1, x_2) = x_1^2 + 9x_2^2$ $\qquad \nabla f_0(x) = \begin{bmatrix} 2x_1 \\ 18x_2 \end{bmatrix}$ monotonically increasing in C

the minum in R_+ is $(0, 0)$ $\qquad \nabla^2 f_0(x) = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} \succeq 0$ convex

where $f_0 = 0$

f_0 is always increasing so the minume f_0 in C is located on boundary of C



On $2x_1 + x_2 = 1$, $x_2 = 1 - 2x_1$

$f_0(x) = x_1^2 + 9x_2^2 = 37x_1^2 - 36x_1 + 9$

$f_0'(x) = 74x_1 - 36 < 0$ for $0 \leq x_1 \leq \frac{2}{5}$

$\min_{0 \leq x_1 \leq \frac{2}{5}} f_0(x) = f_0\left(\frac{2}{5}, \frac{1}{5}\right) = \frac{13}{25}$

e) (4.1) cont.

on $x_1+3x_2=1$ $\quad x_1=1-3x_2$

$0\leq x_2\leq \frac{1}{5}$

$f_0(x)=18x_2^2-6x_2+1$

$f_0'(x)=36x_2-6=0 \quad x_2=\frac{1}{6} \Rightarrow x_1=\frac{1}{2}$

$f_0(\frac{1}{2},\frac{1}{6})=\frac{1}{4}+\frac{9}{36}=\frac{18}{36}=0.5$ the least value of f_0 on boundary

$f^*=0.5$

$x^*=(\frac{1}{2},\frac{1}{6})$

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4) a) minimize $\|Ax-b\|_\infty$

solution is the ~~largest entry~~ minimum absolute value of largest entry of $Ax-b$

~~minimize~~

LP form: minimize t

$Ax-b\preceq t\mathbf{1}$

$Ax-b\succeq -t\mathbf{1}$

$t\geq 0$

solution is $t\geq 0$ where absolute value of $Ax-b$ is (componentwise smaller than $t\mathbf{1}$

so the minimum t is the minimum $\|Ax-b\|_\infty$

b) 4.11 minimize $\|Ax - b\|_1$

optimum value is ~~minimum~~ where sum of ~~o~~absolute value, of entries of $Ax-b$ is minimized

LP form minimize $I^T y$

$$Ax - b \preceq y$$
$$Ax - b \succeq -y$$
$$y \succeq 0$$

$I^T y$ is sum of entries of y

optimum value of $I^T y$ is where y has the smallest entries that are larger than absolute values of entries of $Ax-b$

c) minimize $\|Ax - b\|_1$
subject to $\|x\|_\infty \leq 1$

LP form minimize $\mathbf{1}^T y$

subject to $Ax - b \preceq y$, $Ax - b \succeq -y$ } satisfies the objective

$x \preceq \mathbf{1}$, $x \succeq -\mathbf{1}$ } satisfies the constraints

$y \succeq 0$

1) 4.11 d) minimize $\|x\|_1$

subject to $\|Ax - b\|_\infty \leq 1$

LP for

minimize $\mathbb{1}^T y$

subject to, $x \preceq y$

$x \succeq -y$

$Ax - b \preceq \mathbb{1}$

$Ax - b \succeq -\mathbb{1}$

e) minimize $\|Ax - b\|_1 + \|x\|_\infty$

LP form:

minimize $\mathbb{1}^T y + t$

$Ax - b \preceq y$

$Ax - b \succeq y$

$x \preceq t$

$x \succeq -t$

$t \succeq 0$, $y \succeq 0$