Microeconomics 3: Game Theory
Spring 2021

Problem Set 4

1. Two players compete for a good in a first price auction (ties are broken uniformly). Players have common value ν for the good, $\Pr(\nu = 0) = \Pr(\nu = 1) = 1/2$. Suppose player 1 knows the value of the good and player 2 does not. In this question we'll construct an equilibrium of this game.

   (a) Any equilibrium of this game will be in mixed strategies. Describe each player's payoffs, fixing the other player's bid distribution.

   (b) Show that if $\nu = 0$, then player 1 bids 0.

   (c) Let $\mathcal{B}_2$ be the support of player 2's equilibrium bidding strategy. Show that $\inf \mathcal{B}_2 = 0$ (i.e. the lowest bid 2 makes is 0).

   (d) Observe that if $b \in \mathcal{B}_2$, player 2 must be indifferent between bidding b and 0. Using this, solve for player 1's equilibrium bid distribution when $\nu = 1$.

   (e) Finally, player 1 must be indifferent between all bids in the support of the distribution you found in (d). Solve for player 2's bid distribution.

2. There are two firms, each with marginal cost 0, that compete by choosing quantities ala cournot. Inverse Demand is given by $P = \max(0, \theta - q_1 - q_2)$ where θ is uncertain. With probability α, $\theta = 3$ and with complementary probability $\theta = 4$. Suppose firm 1 knows the value of θ and firm 2 does not.

   (a) Suppose firms choose quantities simultaneously. Characterize the Bayes Nash equilibrium of this game.

   (b) Now suppose firm 1 moves first and player 2 observes firm 1's choice of quantity. Does there exist a PBE where firm 1's chooses a different quantity if $\theta = 3$ and if $\theta = 4$? Does there exist a PBE where firm 1 chooses the same quantity in both states?

3. In the game of Figure 1, Nature chooses L with probability $\frac{3}{4}$.

   (a) What are the PBE of this game?

   (b) Show that in any sequential equilibrium, player 2's beliefs at their information set must put probability $3/4$ on the left node.

   (c) What are the Sequential Equilibria?

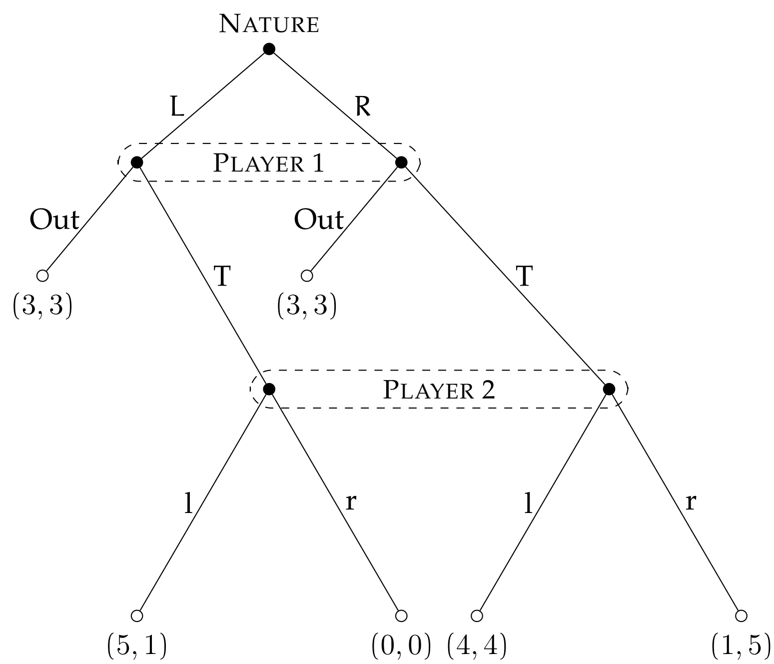

Figure 1: Problem 4

4. Fix a social welfare function $f : \mathcal{R}^n \to \mathcal{R}$ that satisfies the following property: For any $x, y \in X$ and vectors of preferences R, R', such that $xR_iy \Rightarrow xR'_iy$ then

$$xf(R)y \Rightarrow xf(R')y$$
$$xf_p(R)y \Rightarrow xf_p(R')y$$

where f_p denotes the strict part of the relation.

Consider the induced social choice function $\xi(R) = \{x : x f(R) y \, \forall y \in X\}$, and assume $\xi(R)$ is a singleton for all $R \in \mathcal{R}^n$. Show that $\xi(R)$ is strategyproof.