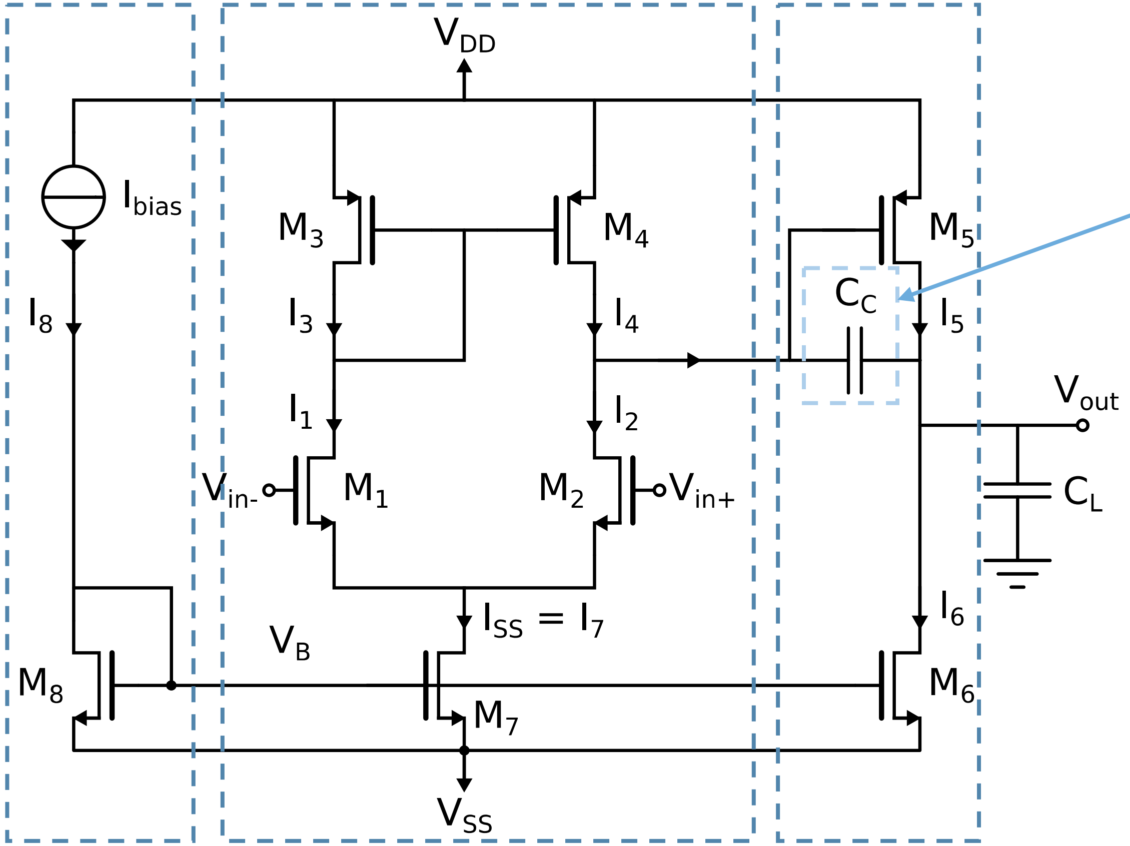
ELEC-E3510 Basics of IC Design

Lecture 5:

Two stage amplifiers

Operational amplifier



Current biasing

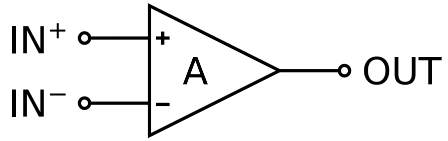
Differential amplifier (1st stage)

(2nd stage)

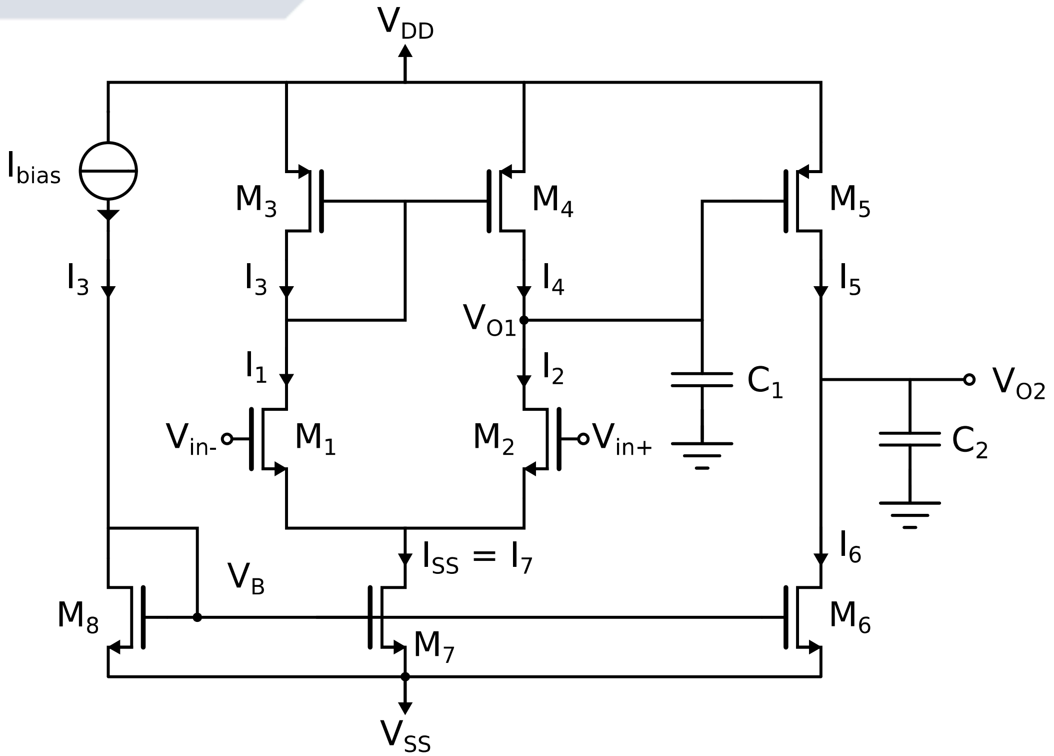
Compensation

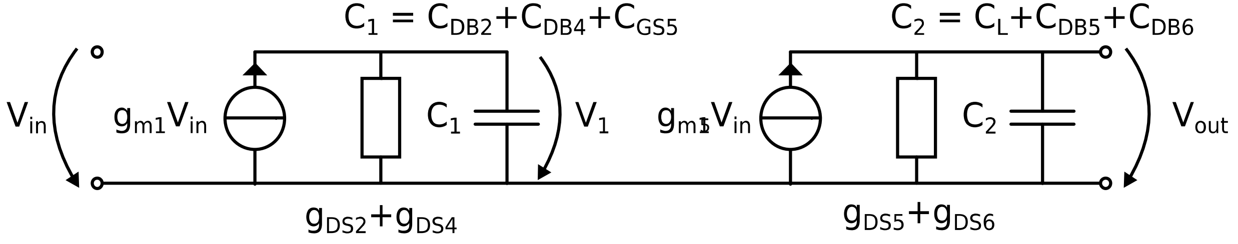
- 2 gain stages (80…120dB)
- Differential input stage
- Miller compensation
- Current biasing

Symbol:



Two stage amplifier





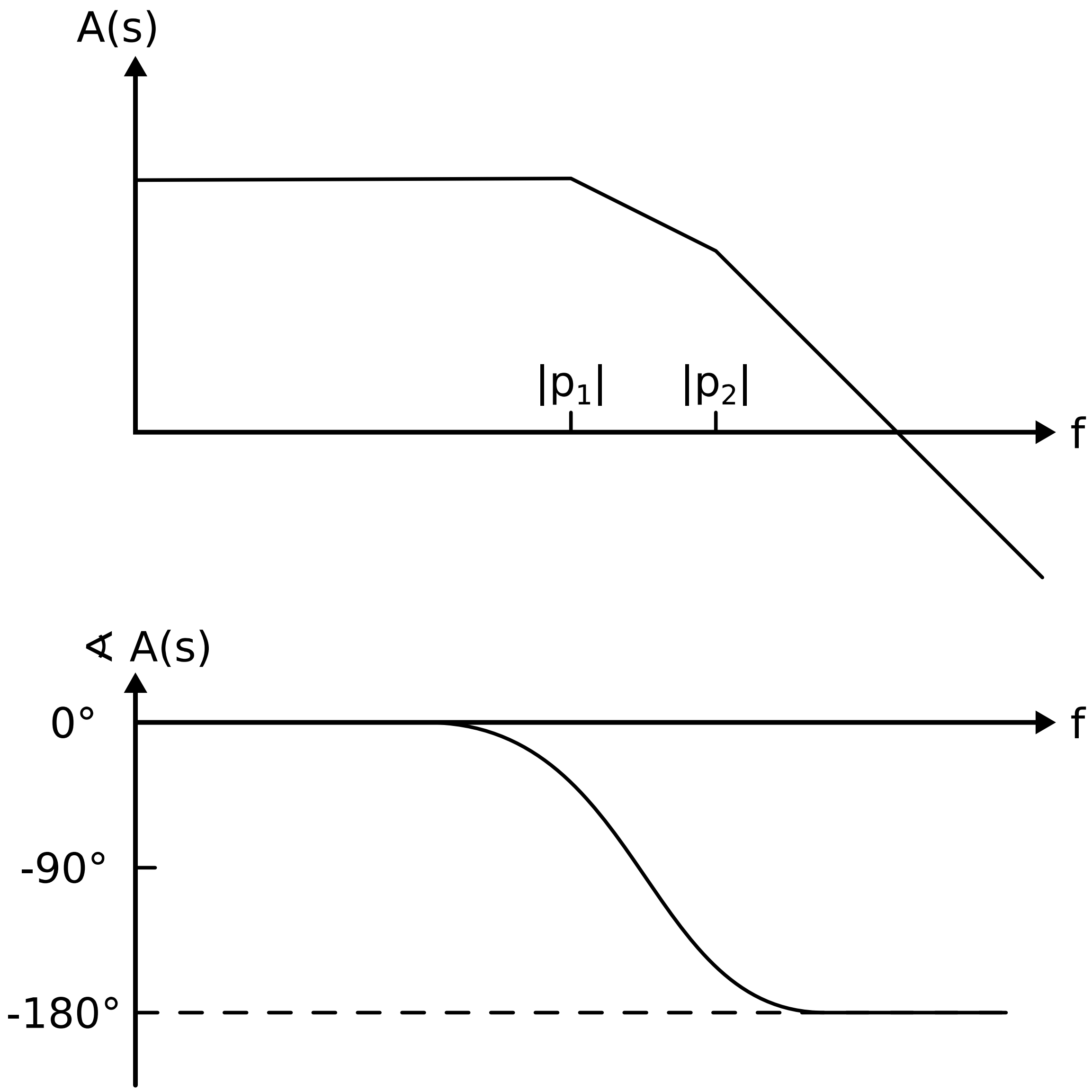
Assume single pole approximation for the gain stages:

$$A_1(s) = \frac{V_1}{V_{in}} = \frac{A}{1+\frac{s}{s_{p1}}}; \quad \begin{cases} A_1 = \dfrac{g_{m1}}{g_{DS2}+g_{DS4}} \\ s_{p1} = \dfrac{g_{DS2}+g_{DS4}}{C_1} \end{cases}$$

$$A_2(s) = \frac{V_{out}}{V_1} = \frac{A_2}{1+\frac{s}{s_{p2}}}; \quad \begin{cases} A_2 = \dfrac{g_{m5}}{g_{DS5}+g_{DS6}} \\ s_{p2} = \dfrac{g_{DS5}+g_{DS6}}{C_2} \end{cases}$$

$$A(s) = A_1(s) \cdot A_2(s) = \frac{A_1 A_2}{\left(1+\frac{s}{s_{p1}}\right)\left(1+\frac{s}{s_{p2}}\right)}$$

$$A_1 A_2 = \frac{g_{m1} g_{m5}}{(g_{DS2}+g_{DS4})(g_{DS5}+g_{DS6})}$$



Two poles are:

$$s_{p1} = \frac{g_{DS2} + g_{DS4}}{C_1}$$

$$s_{p2} = \frac{g_{DS5} + g_{DS6}}{C_2}$$

Typically

$$g_{DS2} \approx g_{DS4} \approx g_{DS5} \approx g_{DS6}$$
$$C_2 = (5 \ldots 10) \cdot C_1$$

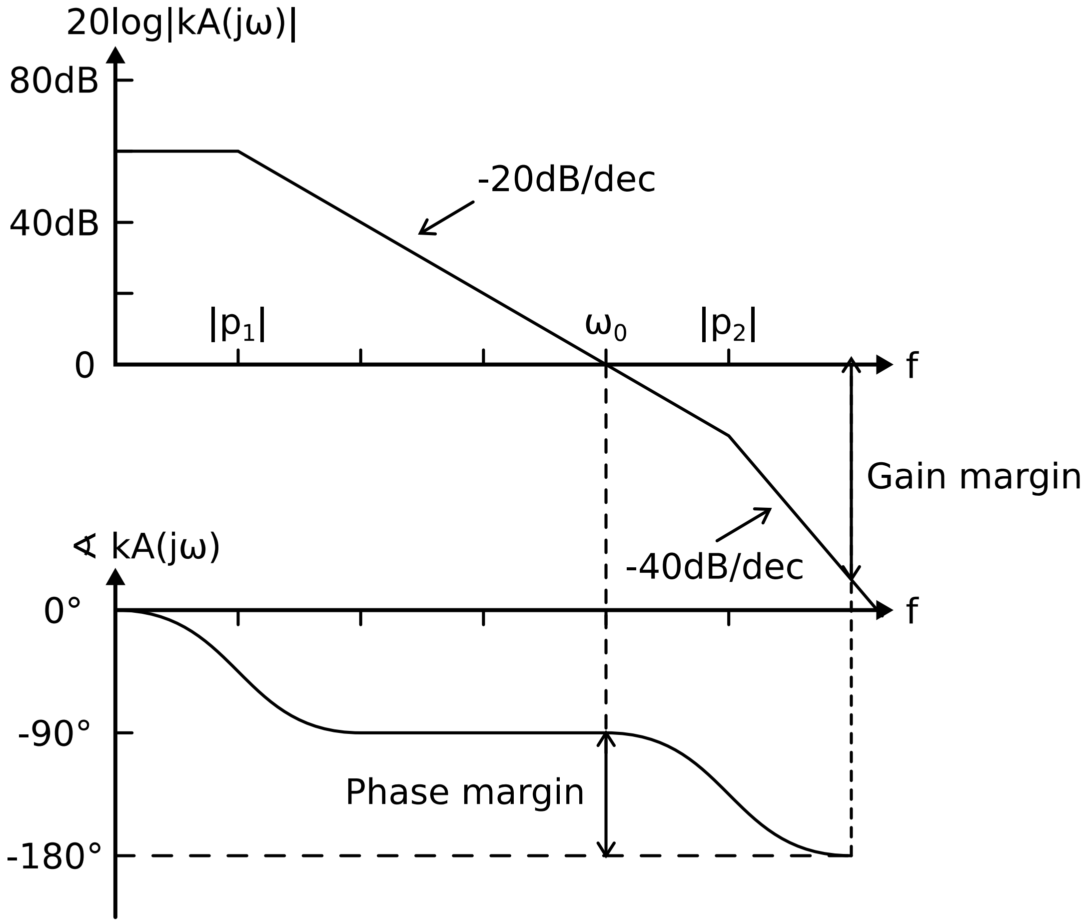
p_1 and p_2 are close to each other

$\Rightarrow$ poor phase margin

An additional pole p_3 will make the system unstable

$\Rightarrow$ frequency compensation needed!

Stability of cascaded amplifiers



Stability:

$|kA(s)| < 1$, when phase shift is -180°

Define:

$|kA(j\omega_0)| = 1$ @ $\omega = \omega_0$

Check for the stability:

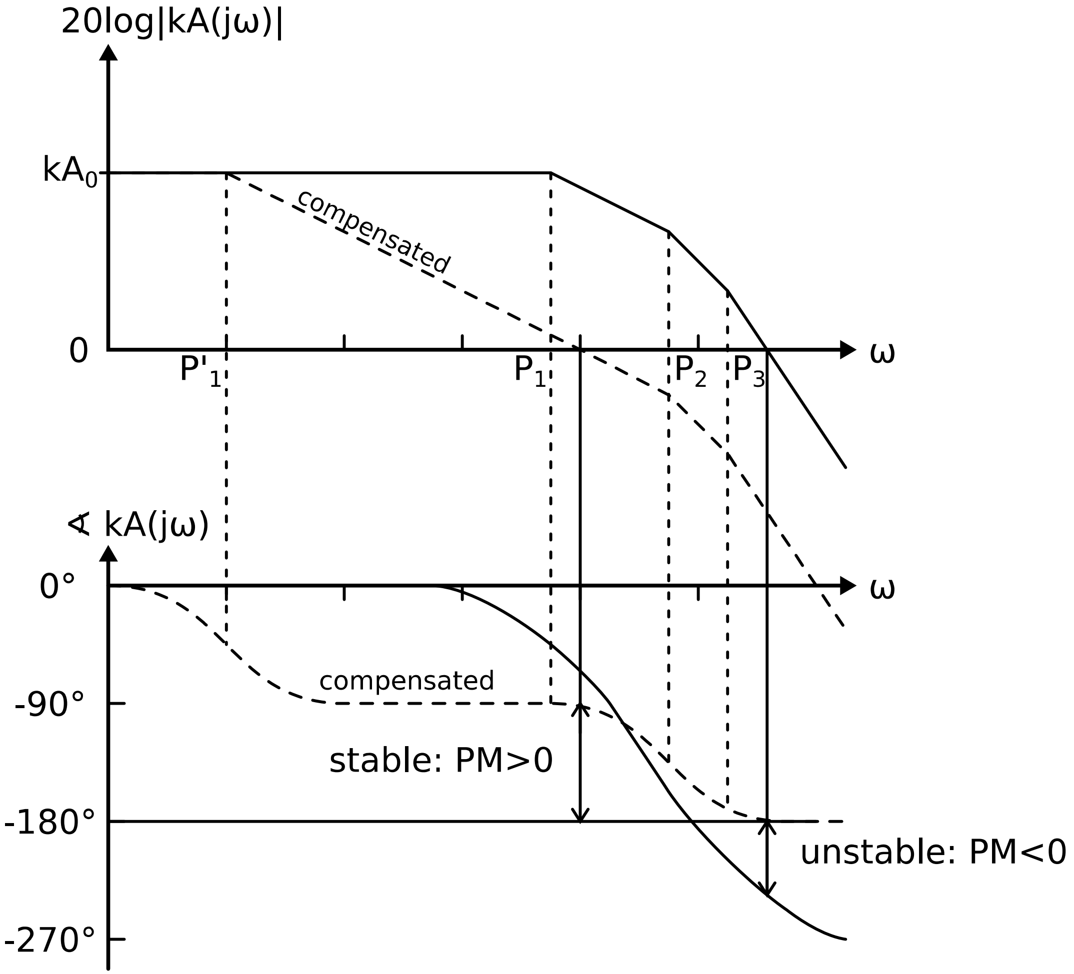
Phase margin: PM = phase$(kA_v(j\omega_0))$+180°

Gain margin: GM[dB] = $20\log|kA_v(j\omega_{180°})|$

For stability: PM > 45°

GM < -12dB

Compensation



Uncompensated

$p_1 \lesssim p_2 \lesssim p_3$

Compensated

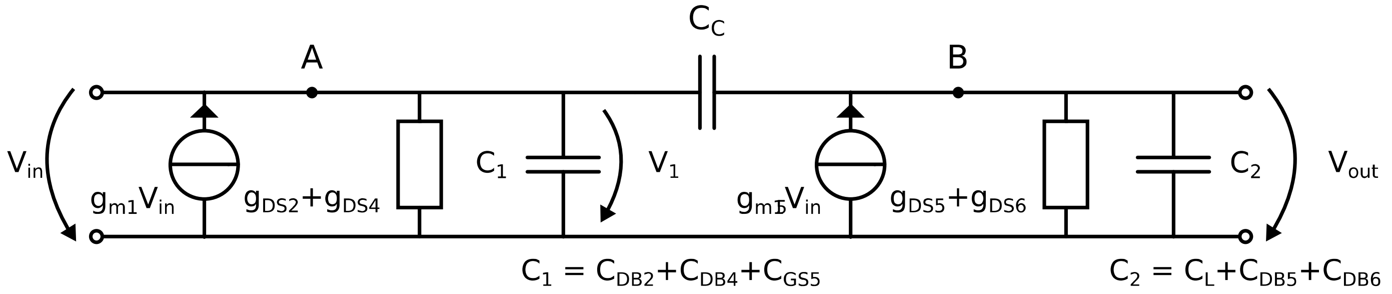
p_1‘<< p_2, p_3

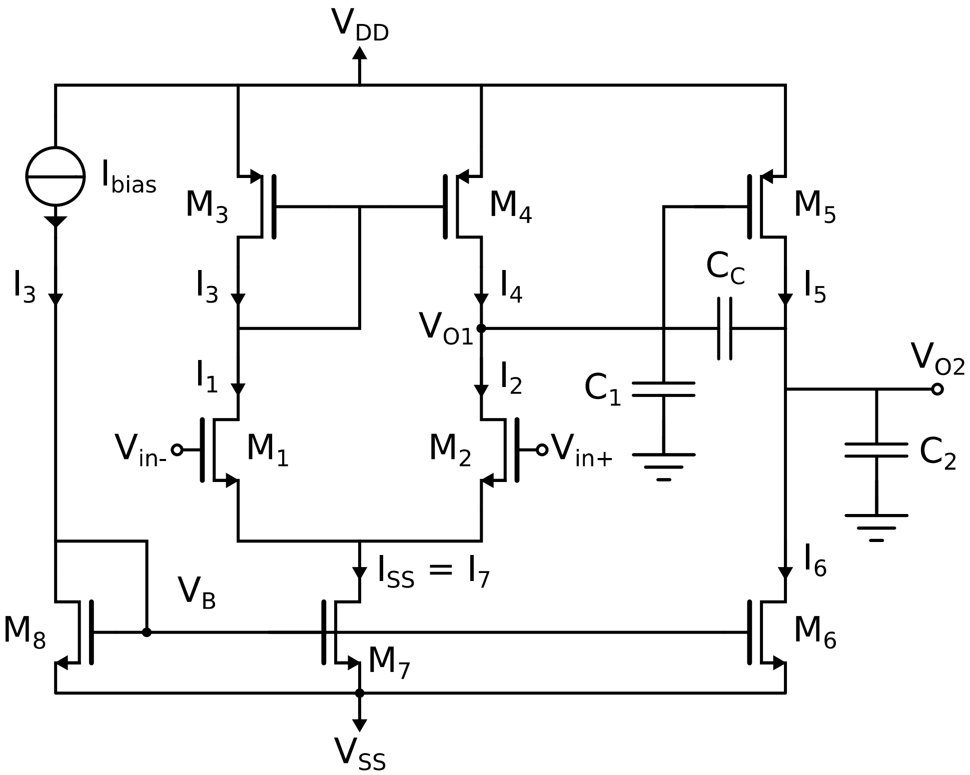
p_1‘ becomes the dominant pole

Miller compensation!

$P_1' << p_2$

Miller compensation





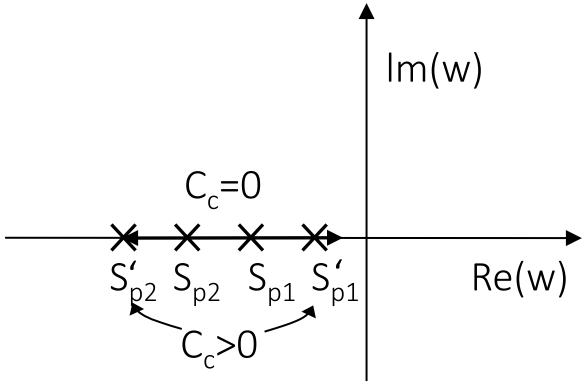
Current equations at nodes A and B

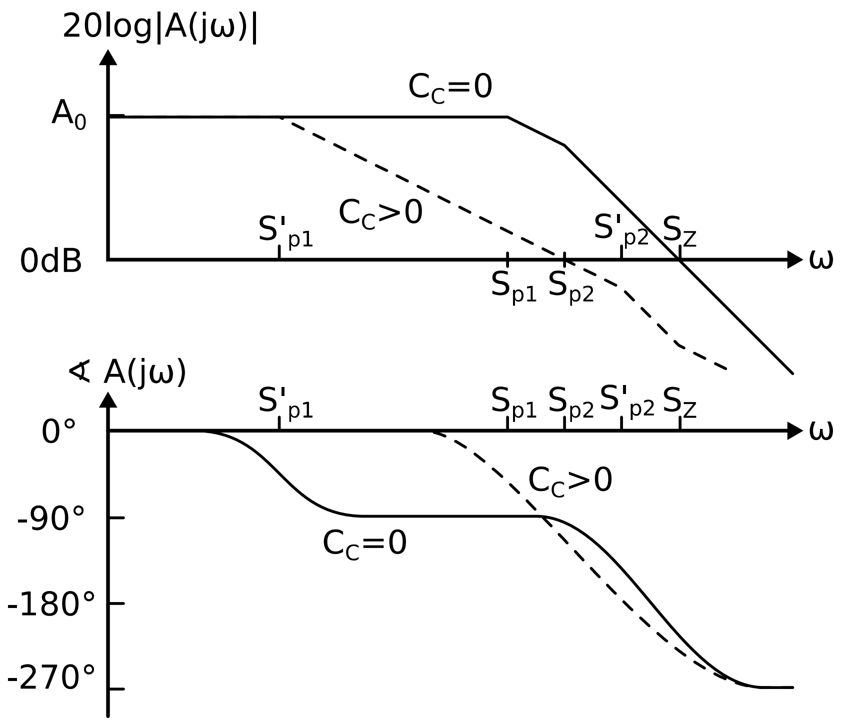
$$\left.\begin{array}{ll} \text{A} & -g_{m1}V_{in} + \left(g_{DS2} + g_{DS4} + sC_1\right)V_1 + sC_c\left(V_1 - V_{out}\right) = 0 \\ \text{B} & sC_c\left(V_{out} - V_1\right) - g_{m5}V_1 + \left(g_{DS5} + g_{DS6} + sC_2\right)V_{out} = 0 \end{array}\right| : V_{in}$$

Eliminate $\frac{V_1}{V_{in}}$, and solve $A(s) = \frac{V_{out}}{V_{in}}$

$$A(s) = \frac{V_{out}}{V_{in}} = \frac{g_{m1} \cdot \left(\frac{g_{m5} - sC_c}{C_1C_2 + C_1C_c + C_2C_c}\right)}{s^2 + s\left(\frac{(C_2 + C_c)(g_{DS2} + g_{DS4}) + (C_1 + C_c)(g_{DS5} + g_{DS6}) + g_{m5}C_c}{C_2C_1 + C_cC_2 + C_cC_1}\right) + \frac{(g_{DS2} + g_{DS4})(g_{DS5} + g_{DS6})}{C_1C_2 + C_1C_c + C_2C_c}}$$

Pole splitting





DC-gain $A_0 = A(s{=}0)$:

$$\boxed{A_0 = \frac{g_{m1}\cdot g_{m5}}{(g_{DS2}+g_{DS4})(g_{DS5}+g_{DS6})}}$$

Miller zero:

$$\boxed{S_z = \frac{g_{m5}}{C_c}}$$

Assume dominant pole approximation, i.e. $|s_{p2}| >> |s_{p1}|$

s_{p1} and s_{p2} are solved from the denominator of A(s):

$$\Rightarrow (S+S_{p1})(S+S_{p2}) = S^2 + s(S_{p1}+S_{p2}) + S_{p1}\cdot S_{p2} \approx S^2 + S\cdot S_{p2} + S_{p1}\cdot S_{p2}$$

$$\Rightarrow S_{p1} \cong \frac{-(g_{DS2}+g_{DS4})(g_{DS5}+g_{DS6})}{(C_2+C_c)(g_{DS2}+g_{DS4})+(C_1+C_c)(g_{DS5}+g_{DS6})+g_{m5}\cdot C_c}$$

$$\approx -\frac{(g_{DS2}+g_{DS4})(g_{DS5}+g_{DS6})}{g_{m5}\cdot C_c}$$

$$= -\frac{g_{m1}}{A_0 C_c}$$

; assume: $g_m >> g_{DS}$

$$\boxed{S_{p2} \cong -\frac{g_{m5}\cdot C_c}{C_1C_2+C_1C_c+C_2C_c} \approx -\frac{g_{m5}}{C_2}}$$

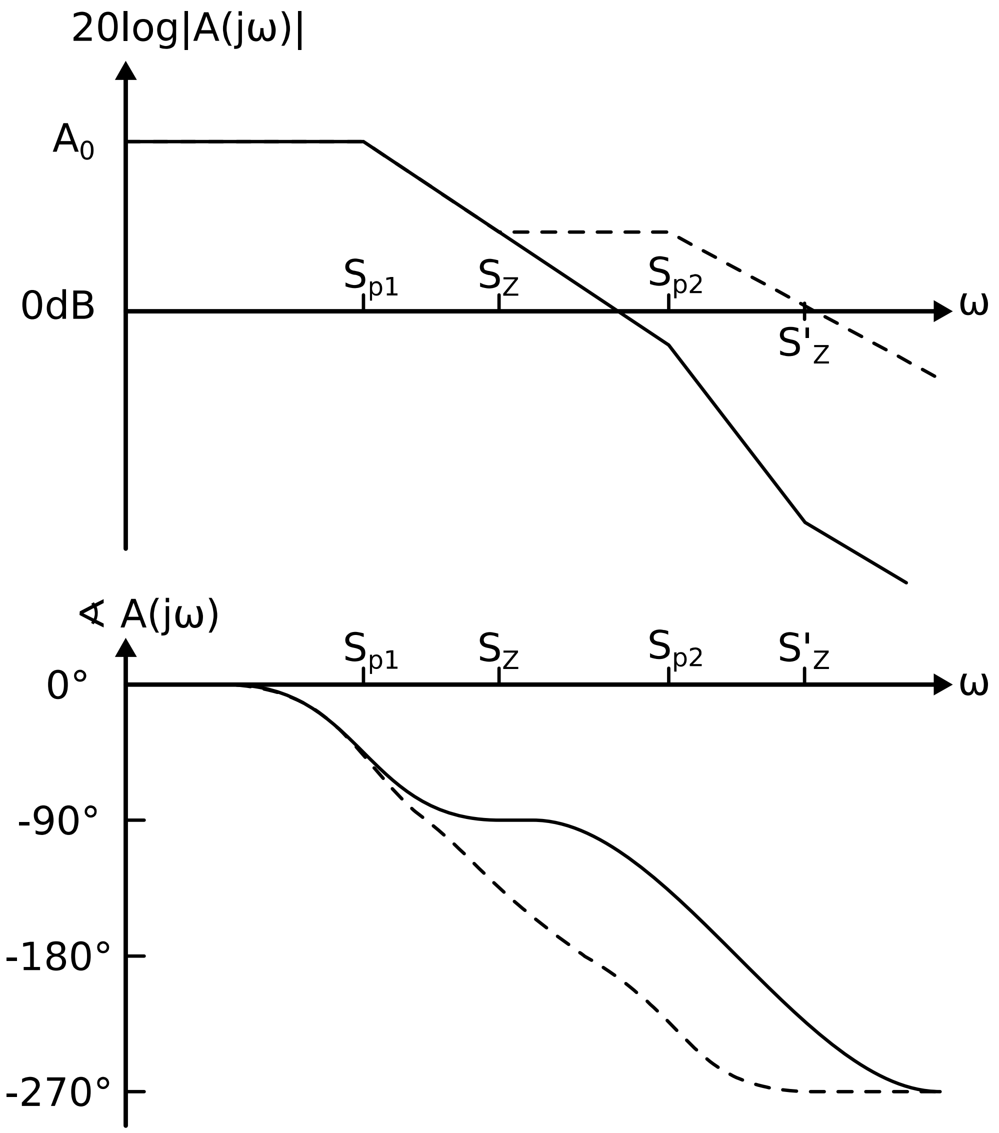
$C_2, C_c >> C_1$

Poles are separated by ~A_0

i.e. DC gain of the amplifier

$$\boxed{GBW = A_0|S_{p1}| = \frac{g_{m1}}{C_c}}$$

Miller zero may cause a stability problem



Miller zero is in the right-hand s-plane causing an additional – 90° phase shift.

For stability we have to design:

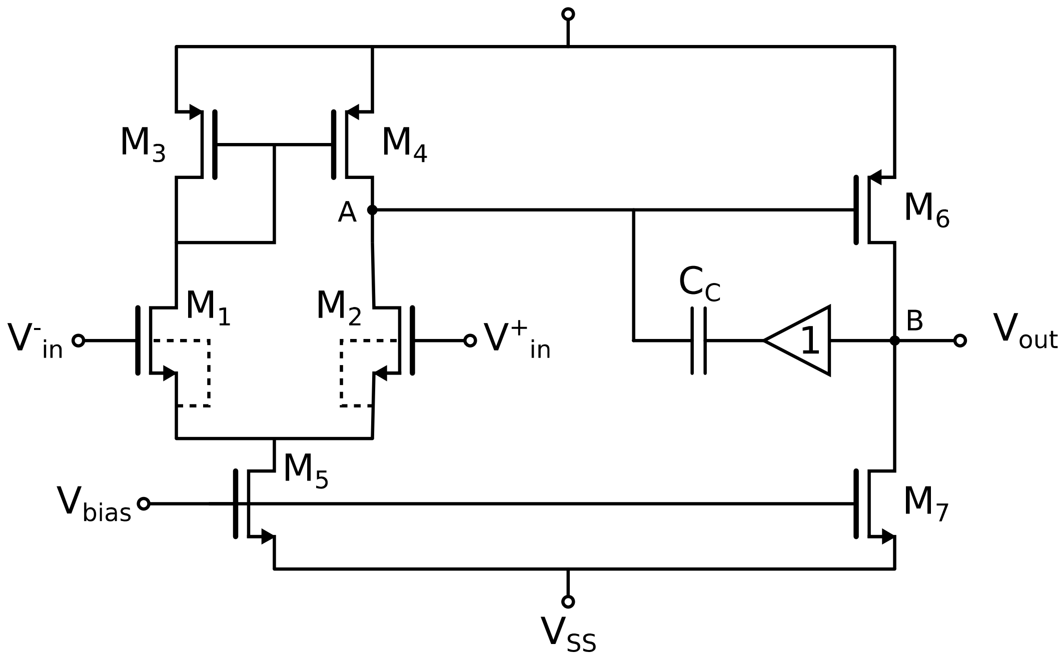
$$\Rightarrow \left|S_z\right| > \left|S_{p2}\right|$$

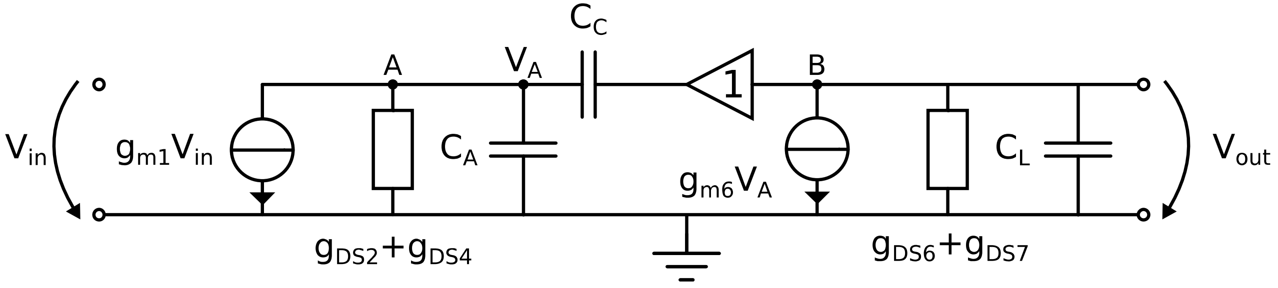
$$\Rightarrow \frac{1}{C_C} > \frac{C_C}{C_A C_2 + \left(C_A + C_2\right) C_C} \approx \frac{1}{C_2}$$

⇒ Block the "feedforward" path through C_C!

If $|s_z|$ < GBW, the amplifier is unstable

Blocking feedforward through C_C





Current equations at nodes A and B:

A: $$g_{m1}V_{in} + \left(g_{DS2} + g_{DS4} + sC_A\right)V_A + sC_C\left(V_A - V_{OUT}\right) = 0$$

B: $$g_{m6}V_A + \left(g_{DS6} + g_{DS7} + sC_L\right)V_{OUT} = 0$$

Eliminate $V_A \Rightarrow$

$$A = \frac{V_{in}}{V_{OUT}} = \frac{-g_{m1} \cdot g_{m6}}{s^2C_L\left(C_A + C_C\right) + s\left[\left(C_A + C_C\right)\left(g_{DS6} + g_{DS7}\right) + C_L\left(g_{DS2} + g_{DS4}\right) + C_C g_{m6}\right] + \left(g_{DS2} + g_{DS4}\right)\left(g_{DS6} + g_{DS7}\right)}$$

$$A = \frac{A_0}{\left(s + s_{p1}\right)\left(s + s_{p2}\right)}$$

$$A_0 = \frac{-g_{m1} \cdot g_{m6}}{\left(g_{DS2} + g_{DS4}\right)\left(g_{DS6} + g_{DS7}\right)}$$

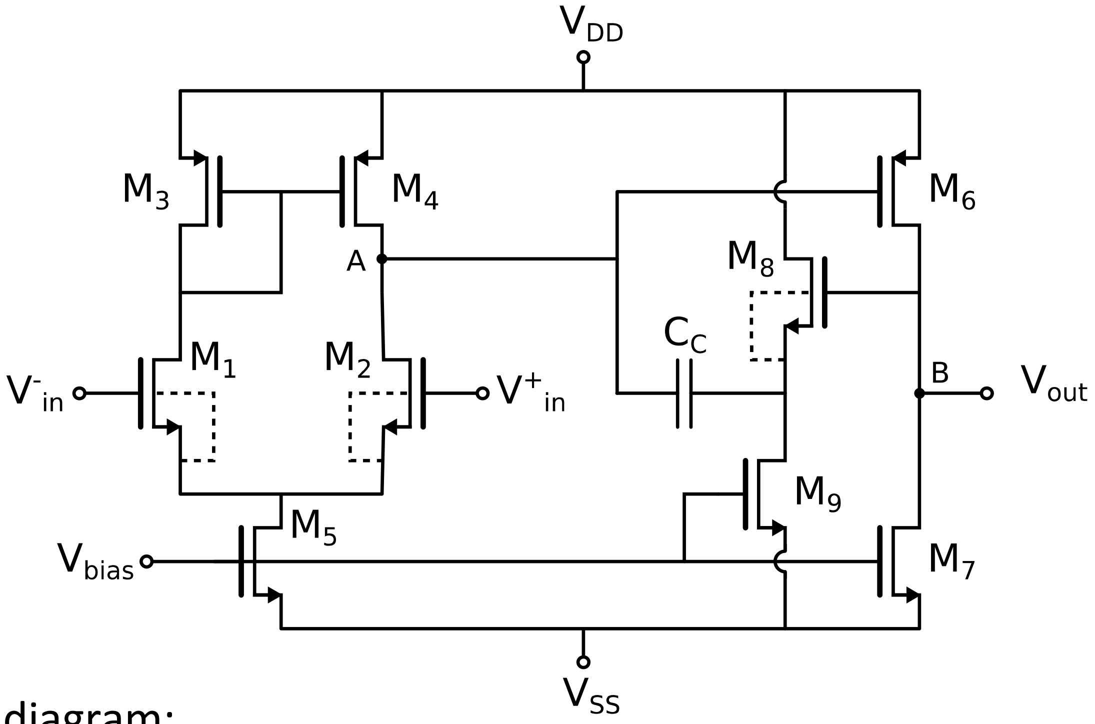
Poles: $$s_{P1} = -\frac{g_{m1}}{A_0 C_C}$$

$$s_{p2} = -\frac{g_{m6}C_C}{C_A C_L + \left(C_A + C_L\right)C_C} \approx -\frac{g_{m6}}{C_L}$$

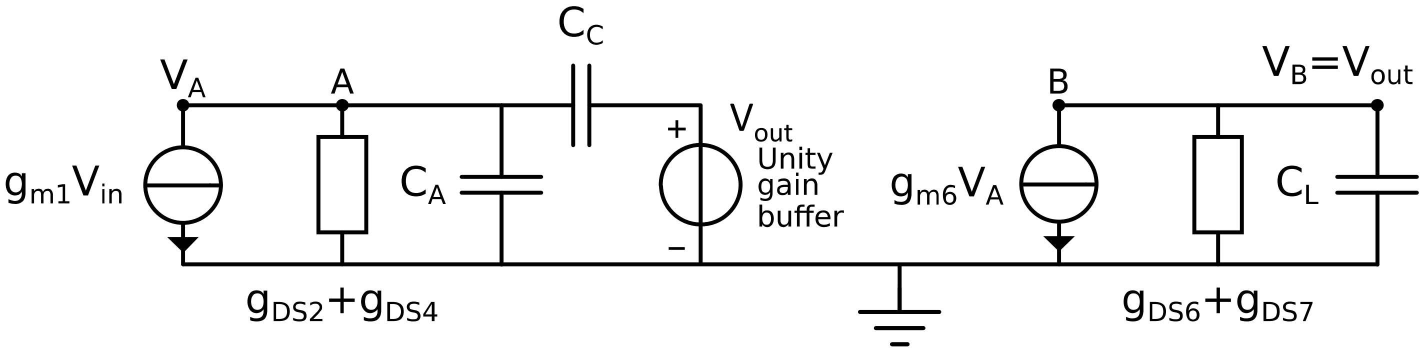
$$GBW = \frac{g_{m1}}{C_C}$$

Source follower: $$A(s) = \frac{1 + s/s_z}{1 + s/s_p} \qquad \left|s_z\right| \approx \left|s_p\right| = \frac{g_{m8}}{C_{GS8}} >> s_{p2}\text{, GBW}$$

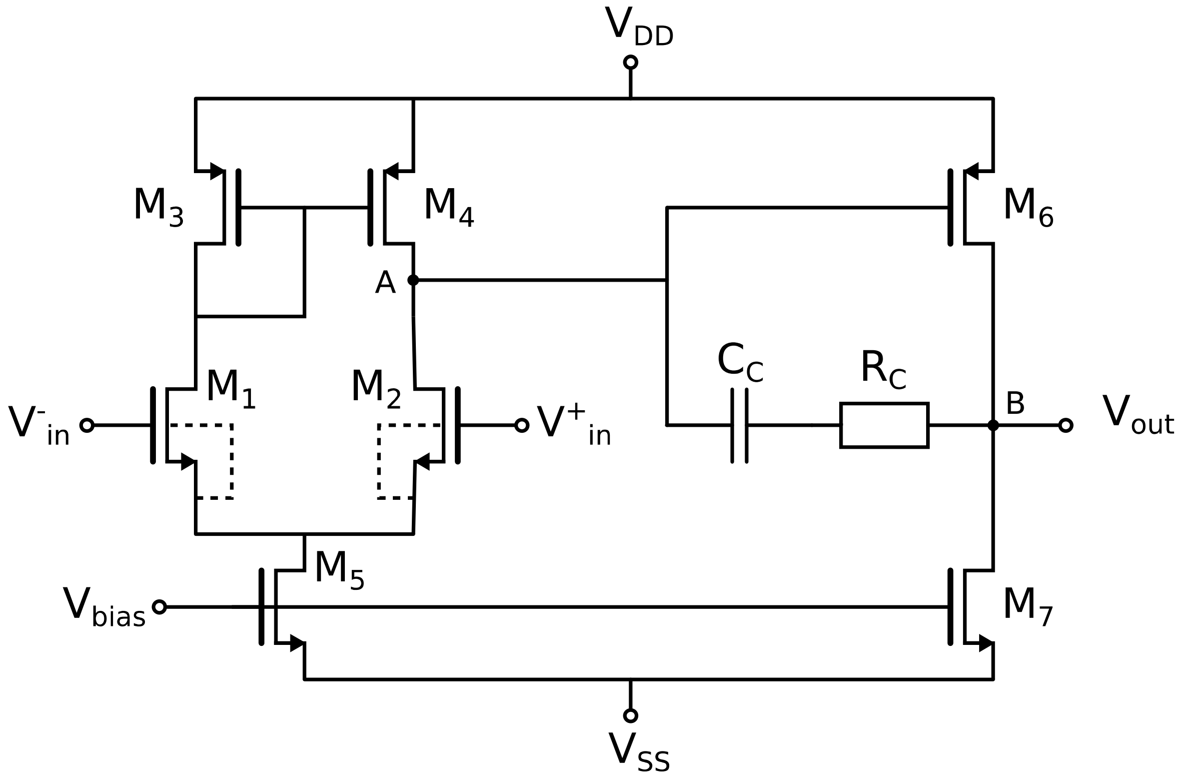
Implementation with the source-follower



Small-signal circuit diagram:

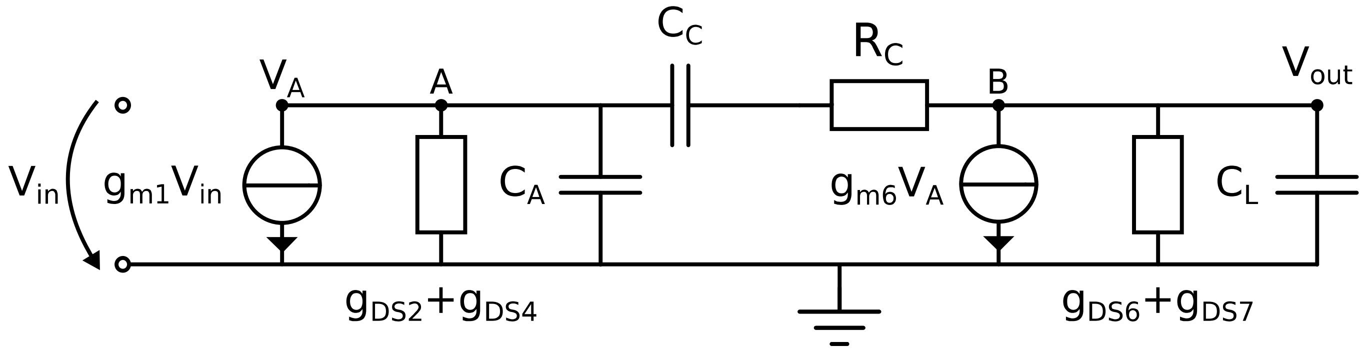


Lead compensation



Phase margin is improved by moving Miller-zero to left-hand s-plane with a series compensation resistor R_C i.e. lead compensation.

Resistor R_C is implemented with a linear region MOS-transistor or a CMOS transmission gate in which transistor gates are connected to V_{DD} and V_{SS}.



In Miller compensated amplifier replace C_C with Y_C

Gain transfer-function:

$$A_v(s) = -\frac{A_0\left(1 - \frac{s}{s_z}\right)}{\left(1+\frac{s}{s_{p1}}\right)\left(1+\frac{s}{s_{p2}}\right)\left(1+\frac{s}{s_{p3}}\right)}$$

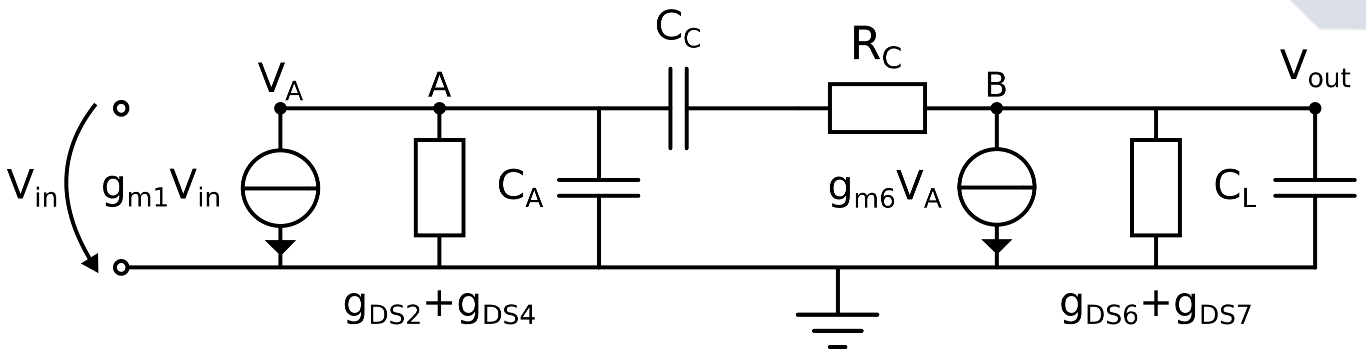
where

$$A_0 = \frac{g_{m1}\cdot g_{m6}}{(g_{DS2}+g_{DS4})(g_{DS6}+g_{DS7})}$$

and poles are

$$s_{p1} = -\frac{g_{m1}}{A_0 C_C}$$

$$s_{p2} = -\frac{g_{m6}C_C}{C_A C_L + (C_A + C_L)C_C}$$



New pole due to compensation:

$$s_{p3} = -\frac{1}{R_C}\left(\frac{1}{C_C}+\frac{1}{C_A}+\frac{1}{C_L}\right)$$

The zero is given by

$$s_z = -\frac{1}{\left(R_C - \frac{1}{g_{m6}}\right)C_C}$$

Compensation resistor R_C is used to eliminate Miller-zero or to move the zero to left-hand s-plane:

Zero elimination:

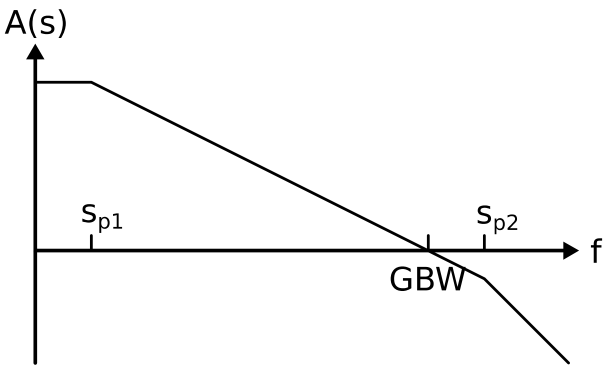
$$s_z \rightarrow \infty \Rightarrow R_C = \frac{1}{g_{m6}}$$

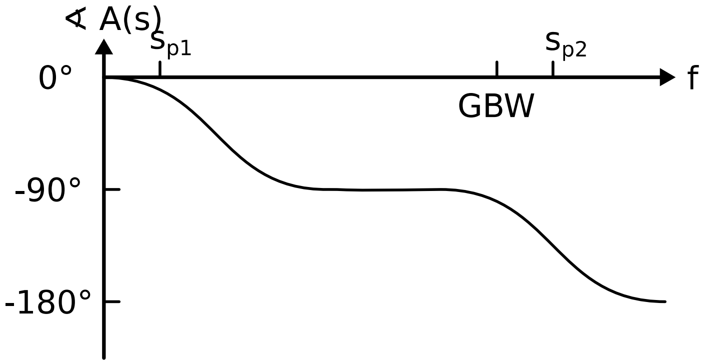
assume $s_{p3} >> s_{p1}, s_{p2} \Rightarrow$

$$PM > 45° \Rightarrow \left| s_{p2} \right| > A_0 \left| s_{p1} \right| = GBW$$

Design principle:

$$\Rightarrow \frac{g_{m6}}{C_L} > \frac{g_{m1}}{C_C}$$

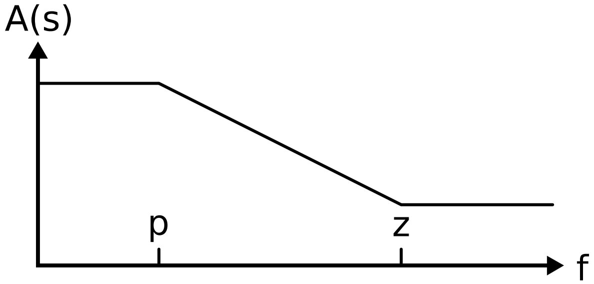


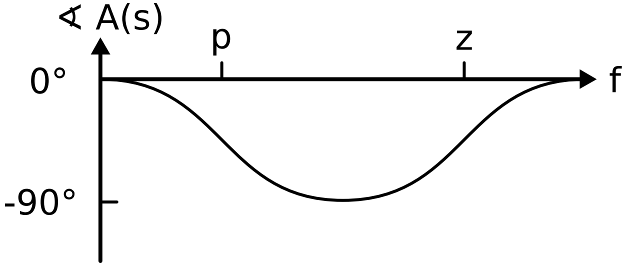


Problems with process varations:
impossible to keep Miller-zero at infinity, it may move to right-hand s-plane causing stability problems.
Move zero to left-hand s-plane:

Zero located in the left half plane, if

$$R_C > \frac{1}{g_{m6}}$$





phase shift +90°!
Improves phase margin!

Compensation strategies

- Move the zero to left-hand s-plane either to compensate s_{p2} or s_{p3}.

Lead compensation:

- Phase shift of the zero in the left-hand s-plane is +90°

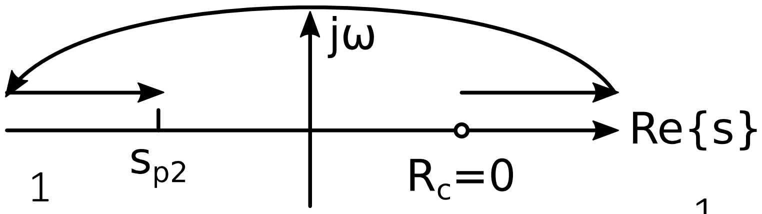
$\Longrightarrow$ Imroves PM

$\Longrightarrow$ Higher GBW is possible

Compensation strategies

at infinity:

$$R_C = \frac{1}{g_{m6}}$$



Left-hand s-plane: $R_C > \frac{1}{g_{m6}}$

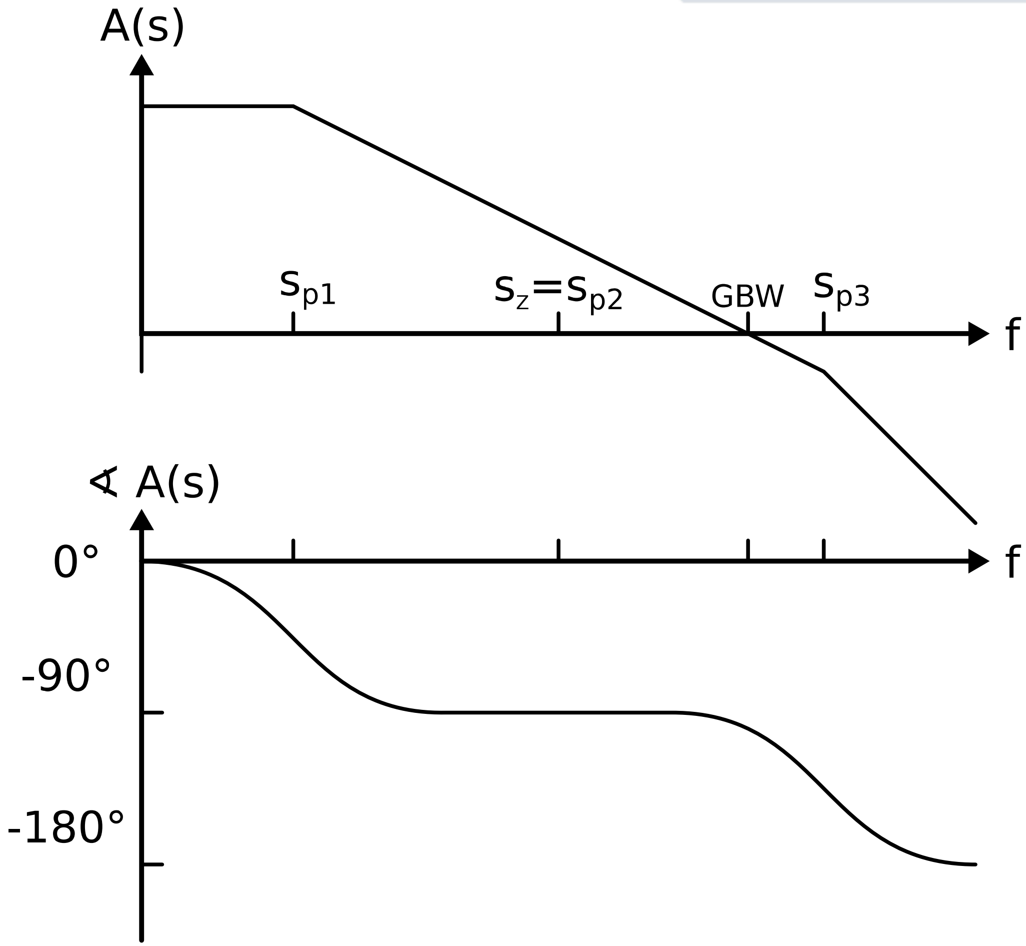
Right-hand s-plane: $R_C < \frac{1}{g_{m6}}$

Pole-zero cancellation:

$$\Rightarrow s_z = s_{p2}$$

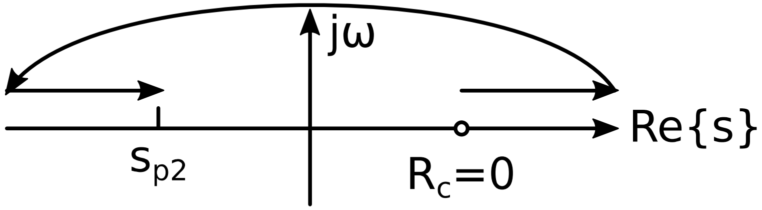
$$\Rightarrow R_C = \frac{C_C + C_A + C_L}{C_C\left(g_{m6}\right)}$$

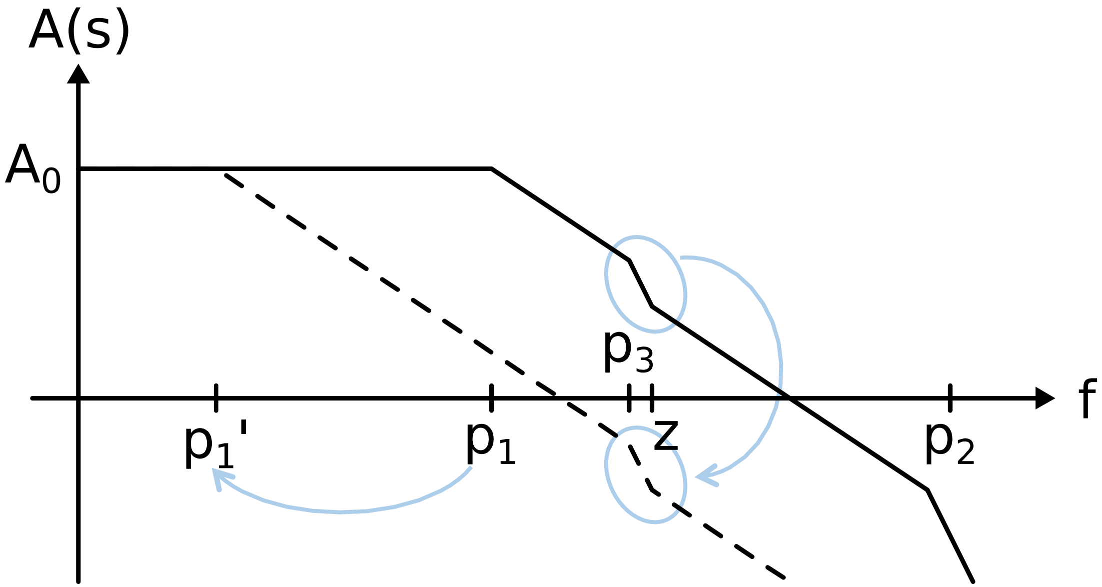
Increased bandwidth!



$$\Rightarrow \left|s_{p3}\right| > GBW = A_0\left|s_{p1}\right| \Rightarrow PM > 45°$$

Problem with pole-zero cancellation





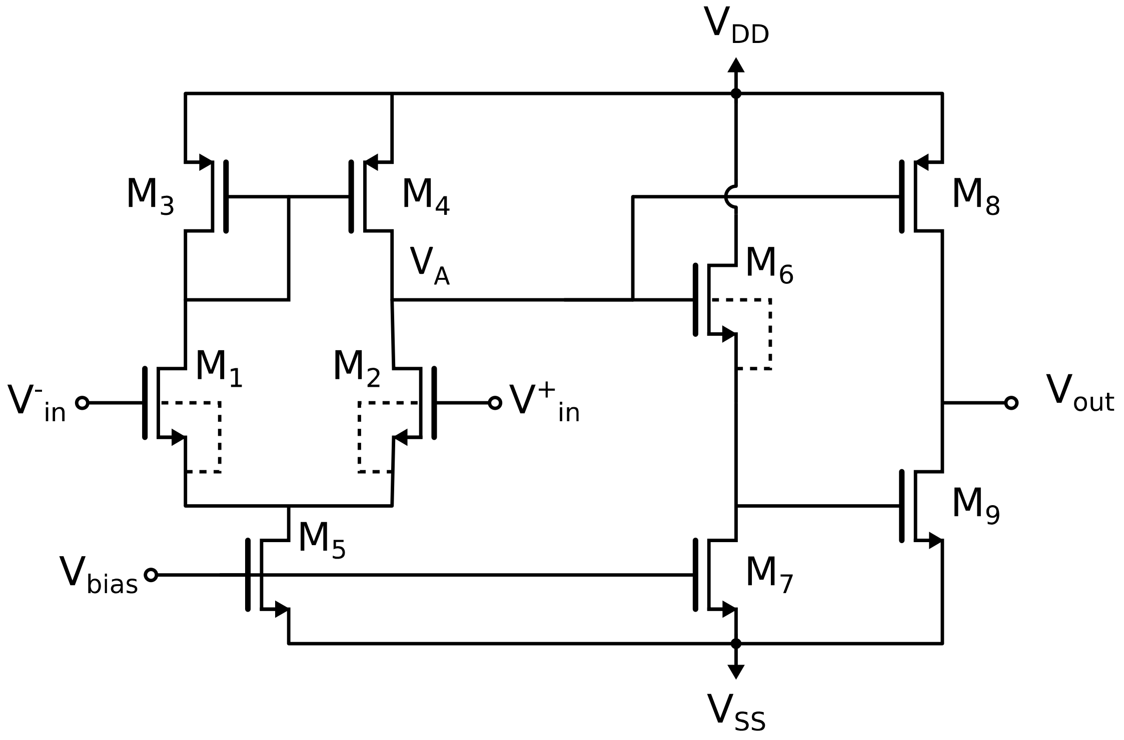
Process variations causes mismatch in pole-zero compensation, which slows down settling of the transients:

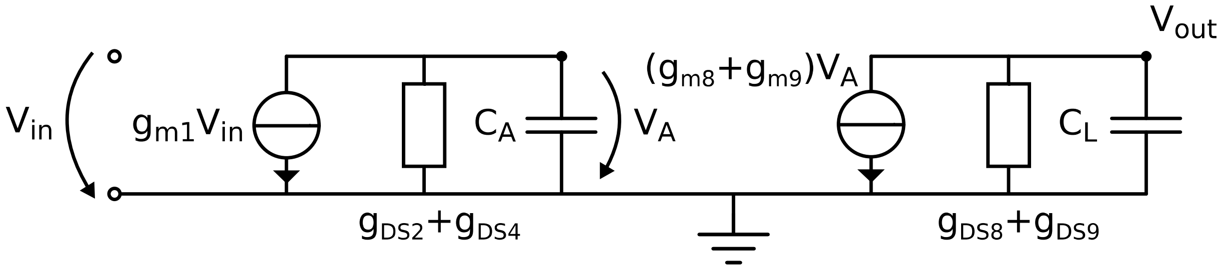
$$H(t) = k_1 e^{-GBW \cdot t} + k_2 e^{-s_{p3} \cdot t}$$

$$k_2 = f(z - p_3)$$

In this case compensate so that GBW $2s_{p3}$

Push-Pull output stage





Assume source follower BW>>GBW!

Source follower:

$$A(s) = \frac{A_0\left(1+\frac{s}{s_z}\right)}{1+\frac{s}{s_p}} \qquad A \approx 1 \qquad s_z \approx s_p = \frac{g_{m6}}{C_{GS6}}$$

$$A(s) = -\frac{A_0}{\left(1+\frac{s}{s_{p1}}\right)\left(1+\frac{s}{s_{p2}}\right)}$$

$$A_0 = \frac{g_{m1}(g_{m8}+g_{m9})}{(g_{DS2}+g_{DS4})(g_{DS8}+g_{DS9})}$$

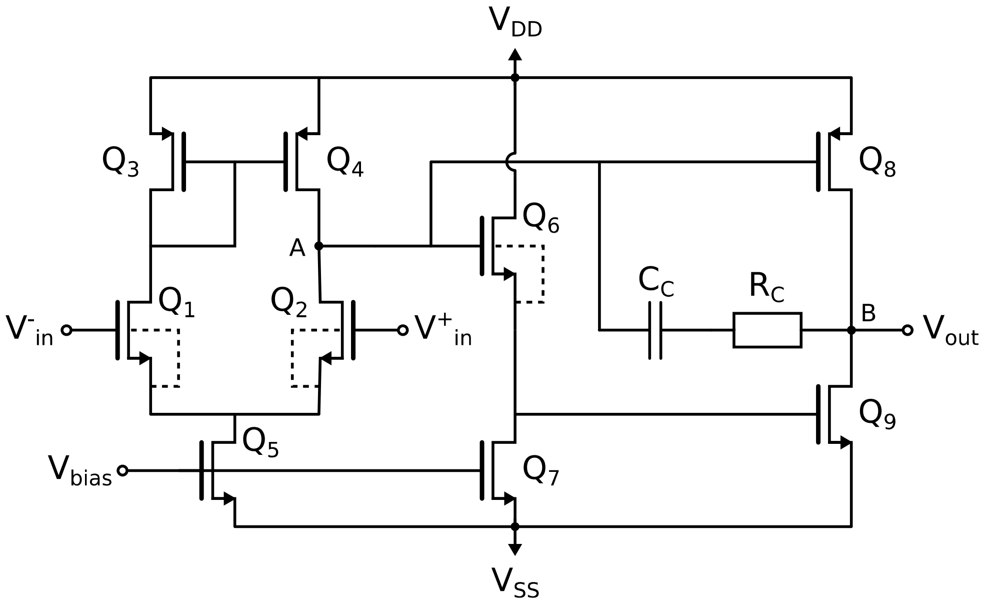
poles:

$$s_{p1} = -\frac{g_{DS2}+g_{DS4}}{C_A}$$

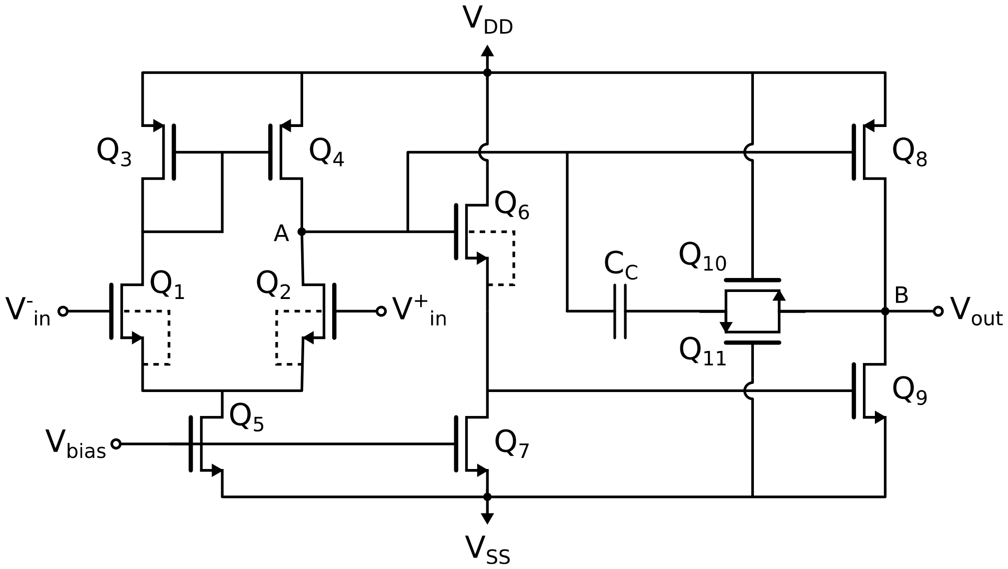
$$s_{p2} = -\frac{g_{DS8}+g_{DS9}}{C_L'}$$

$$\text{assume} \left|s_{pss}\right| = \frac{g_{m6}}{C_{GS6}} >> \left|s_{p2}\right|, \left|s_{p1}\right|$$

Compensation of push-pull output stage:



Lead compensation of push-pull output stage.



Implement R_C with CMOS transmission gate:

$$R_C \approx \frac{1}{g_{m10} + g_{m11}} = \frac{1}{g_{m8} + g_{m9}}$$

In Miller compensated amplifier replace C_C with Y_C

$$A_v(s) = -\frac{A_0\left(1-\frac{s}{s_z}\right)}{\left(1+\frac{s}{s_{p1}}\right)\left(1+\frac{s}{s_{p2}}\right)\left(1+\frac{s}{s_{p3}}\right)}$$

where

$$A_0 = \frac{g_{m1}\cdot(g_{m8}+g_{m9})}{(g_{DS2}+g_{DS4})(g_{DS8}+g_{DS9})}$$

and poles are

$$s_{p1} = -\frac{g_{m1}}{A_0 C_C}$$

$$s_{p2} = -\frac{(g_{m8}+g_{m9})\,C_C}{C_A C_L + (C_A + C_L)C_C}$$

New pole due to compensation:

$$s_{p3} = -\frac{1}{R_C}\left(\frac{1}{C_C}+\frac{1}{C_A}+\frac{1}{C_L}\right)$$

The zero is given by

$$s_z = -\frac{1}{\left(R_C - \frac{1}{g_{m8}+g_{m9}}\right)C_C}$$

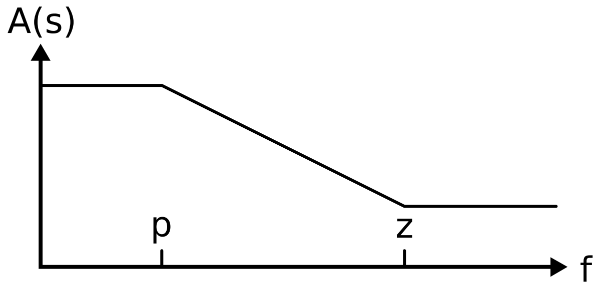
Zero located in the left half plane, if

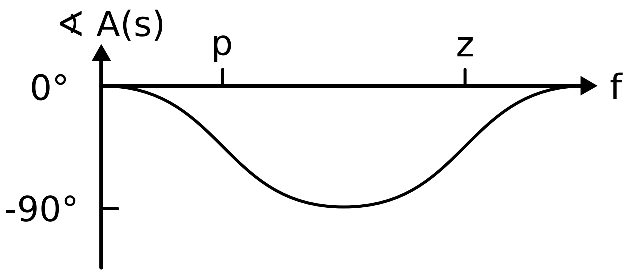
$$R_C > \frac{1}{g_{m8} + g_{m9}}$$

$\longrightarrow$ phase shift +90°!

Zero elimination:

$$s_z \to \infty \Rightarrow R_C = \frac{1}{g_{m8} + g_{m9}}$$

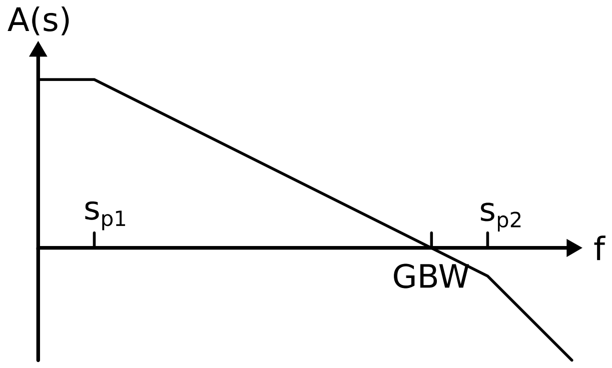


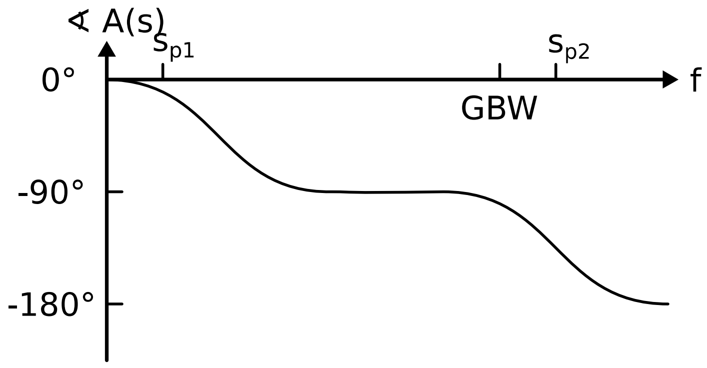


$$\text{assume: } s_{p3} >> s_{p1}, s_{p2} \Rightarrow$$

$$PM > 45° \Rightarrow |s_{p2}| > A_0 |s_{p1}| = GBW$$

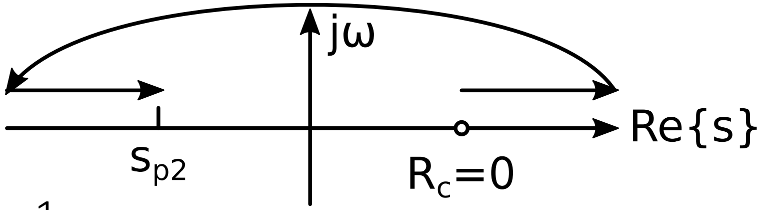
$$\Rightarrow \frac{g_{m8} + g_{m9}}{C_L} > \frac{g_{m1}}{C_C}$$





at infinity:

$$R_C = \frac{1}{g_{m8} + g_{m9}}$$



Left-hand s-plane:

$$R_C > \frac{1}{g_{m8} + g_{m9}}$$

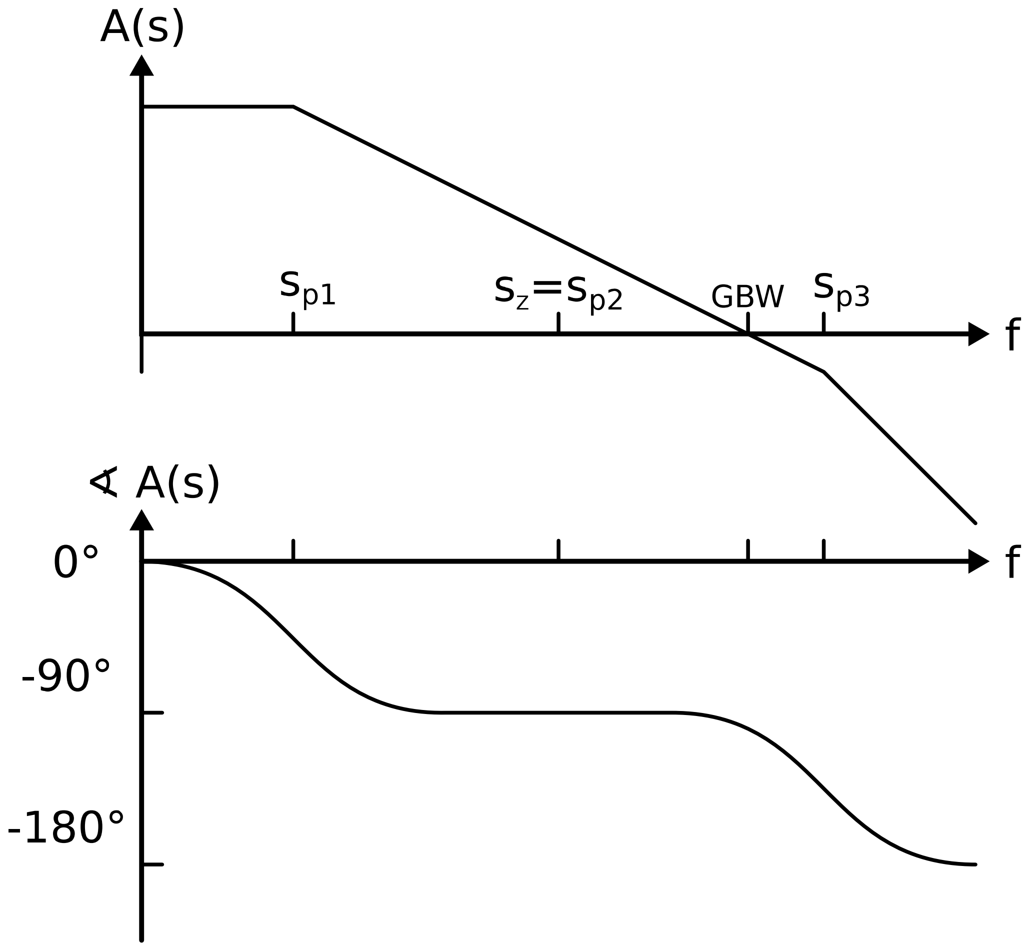
right-hand s-plane:

$$R_C < \frac{1}{g_{m8} + g_{m9}}$$

Pole-zero cancellation:

$$\Rightarrow s_z = s_{p2}$$

$$\Rightarrow R_C = \frac{C_C + C_A + C_L}{C_C\left(g_{m8} + g_{m9}\right)}$$



$$\Rightarrow \left|s_{p3}\right| > GBW = A_0\left|s_{p1}\right| \Rightarrow PM > 45°$$