Exercise problems of Topic 1

In Topic 1, we have four (4) problems.

Write your answers clearly, so that the answer proceeds logically and includes necessary intermediate steps and enough explanations. Your answer should be understandable without oral explanations, too. See further instructions in MyCourses.

The answers are to be returned in a Zoom appointment directly to the teacher. The first chance for the return takes place **on Monday, January 17 at 10-12 in a pre-allocated appointment**. The details will be shown in MyCourses page "Exercise returns".

Be prepared to explain and justify your answer to the teacher. The purpose of this returning method is to enhance your learning through two-way communication and constructive feedback. The teacher will grade your answer in the scale of 0-3 points.

Exercise problem 1.1. A transmission line (see the figure below) has the following unit-per-length equivalent circuit parameters at 5.0 GHz:

- L = series inductance = 0.75 µH/m,
- R = series resistance = 1.0 Ω/m,
- C = shunt capacitor = 300 pF/m,
- G = shunt conductance = 0.0010 S/m.

a. Explain, which physical parts/factors of the transmission line produce/cause the parameters L, R, C and G of the equivalent circuit. For instance, assume a coaxial cable with a low-loss dielectric substrate between the conductors and conductors made of copper.

b. Determine the numeric value of the characteristic impedance Z_0 of the transmission line.

c. Calculate the attenuation per unit length (dB/m). (Note that attenuation > 0 dB.)

d. Recalculate the numeric value of the characteristic impedance Z_0 by neglecting the resistive losses.
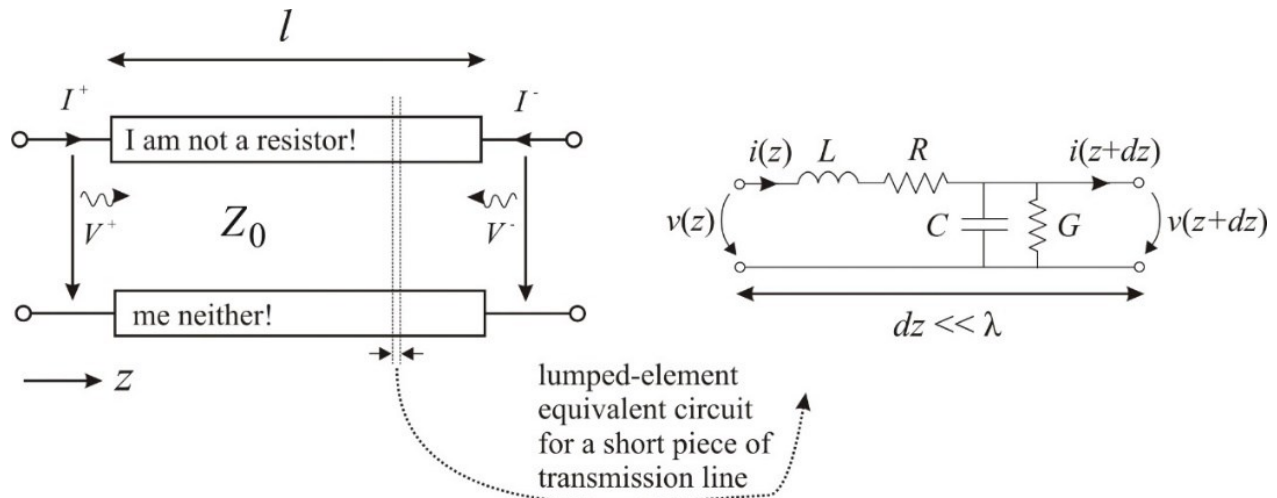
Exercise problem 1.2. FR-4 is a common substrate material for printed circuit boards (PCBs) also at "low" microwave frequencies. For instance, it can have the following parameters:

- relative dielectric constant: ε_r = 4.3,
- substrate thickness: h = 1.5 mm,
- substrate loss tangent: $\tan \delta$ = 0.020,
- a copper layer thickness: t = 35 μm, and the conductivity: σ = 6.0·10^7 S/m
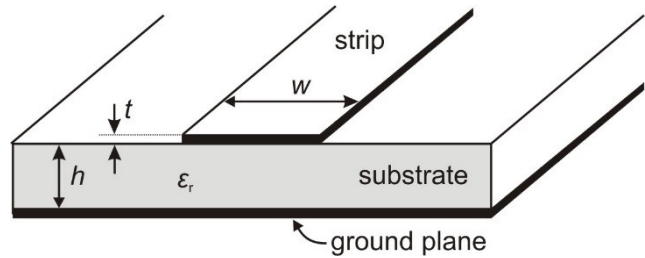
In this problem, we will design a 50-Ω microstrip line. Think first, why microstrip lines are useful and important (you do not need to write an answer). Microstrip lines are discussed in Pozar Chapter 3.8.

a. Sketch a figure of the cross section of a microstrip line on your answer sheet (see an example below).

A wave propagates in the sketch perpendicularly away from the reader. Sketch into your figure the <u>shape</u> and <u>direction</u> of the electric and magnetic field lines of the microstrip line structure when a positive potential is applied to the strip and the ground plane is in the zero potential. Pay attention to the "fringing" fields in the air region and consider the Gauss law for magnetic fields $\nabla \cdot \mathbf{B} = 0$.

b. Based on the a. part, answer with justifications why the effective dielectric constant $\varepsilon_{r,e}$ of a microstrip line satisfies the inequality $1 < \varepsilon_{r,e} < \varepsilon_r$?

c. Calculate the effective dielectric constant $\varepsilon_{r,e}$, the width w of the 50-Ω strip, and the wavelength in the line at 1.0 GHz. For simplicity, you can assume that the thickness of the strip is $t = 0$ μm.
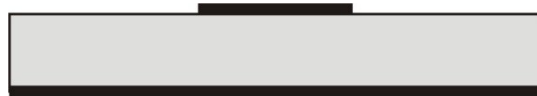

cross section of a microstrip line
⊗ the wave propagates
away from the reader

Exercise problem 1.3. Open the AWR Design Environment circuit simulator (instructions for remote use in MyCourses). Open a transmission line calculator tool: "Tools" → "TXLine...", see figure below. (You do not need any specific AWR skills in this problem.)

a. Check your answers ($\varepsilon_{r,e}$, w and λ at 1 GHz) of part c. of Problem 1.2 with the transmission line calculator. Use the values given in the Problem 1.2 but set the copper layer thickness t = 35 µm. Does the calculator give the same results as you calculated in Problem 1.2? If not, find the reason for difference.

b. Calculate the attenuation (dB/m) of the wave in the same 50-Ω microstrip line at the following frequencies: 0.10, 0.20, 0.50, 1.0, 2.0, 5.0, and 10.0 GHz. Use the same w as in a. part, ε_r = 4.3, h = 1.5 mm, tan δ = 0.020, metal thickness t = 35 µm and its conductivity σ = 6.0·10⁷ S/m (pure copper).

c. Another popular substrate used for microstrip lines at microwave frequencies is RT Duroid 5880, for which ε_r = 2.16, h = 1.575 mm, tan δ = 0.0010, the metal thickness t = 35 µm and its conductivity σ = 6.0·10⁷ S/m (pure copper). Calculate the attenuation (dB/m) of the 50-Ω microstrip line at the same frequencies as in b. part.

d. How does the attenuation generally behave as a function of the frequency (any justification)? Compare the attenuation (dB/m) of FR-4 (part b.) and RT Duroid (part c). Which one seems more suitable PCB material for the implementation of microstrip lines at the microwave frequencies?
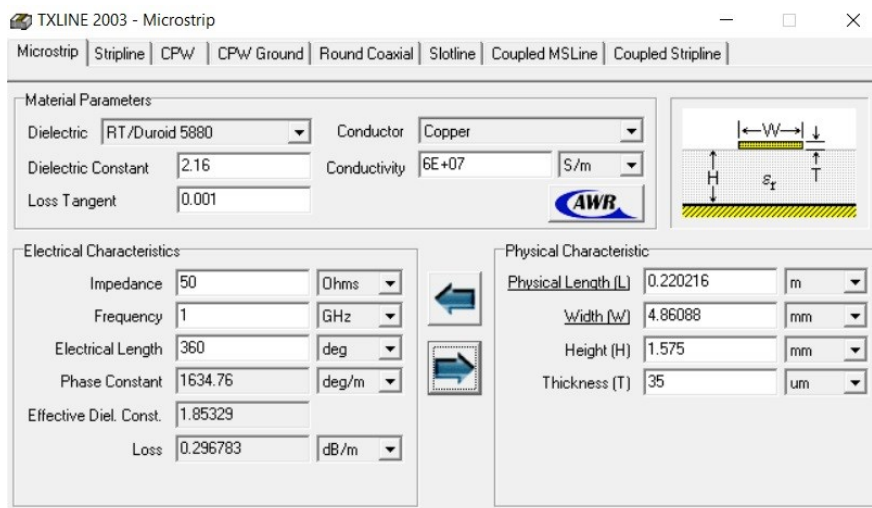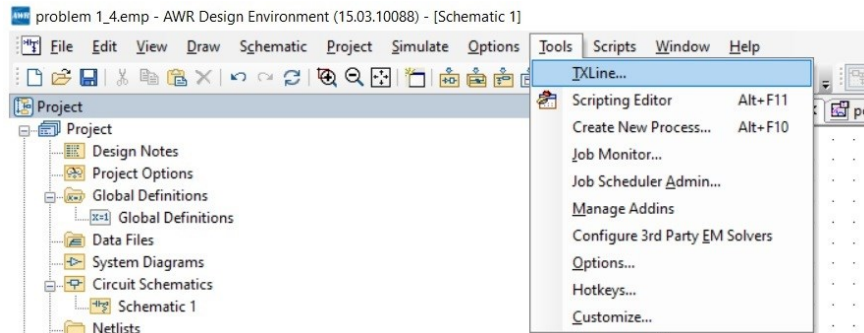
Exercise problem 1.4. A generator is connected to a low-loss (assume lossless for simplicity) transmission line that is terminated with a load impedance Z_L as shown in the figure below. Use the following parameters and initial values:

- $Z_0 = 50\ \Omega$ (transmission line impedance),
- $Z_L = 20 - j\ 50\ \Omega$ (load impedance),
- $Z_g = Z_0$ (generator impedance),
- $l = 1.0\ \lambda$ (unit m),
- $U = 2.0$ V (peak value),
- β = phase constant (unit 1/m), and
- Γ_L = voltage reflection coefficient (unitless) of the load with respect to Z_0 (at z = 0),
- Γ_g = voltage reflection coefficient between the line and the generator impedance (at z = -l).

a. Define the amplitude u^+ of the forward (positive +z direction) travelling wave. Justify.

   Hint: Pozar Chapter 2.6. "Generator and load mismatches" may be helpful.

b. Write an expression for the voltage $u(z)$ and current $i(z)$ functions along the line as a function of z ($-l \leq z \leq 0$).

c. Derive a formula for the average net power flow on the line starting from the formula $P(z) = \frac{1}{2}Re\{u(z)i^*(z)\}$, in which * denotes the complex conjugate and Re is the real part. What is the total power (in W) delivered to the load impedance located in z = 0?

d. Plot (e.g., with Matlab) the magnitudes (envelope curve) of the voltage $u(z)$ and current $i(z)$ standing waves for $-l \leq z \leq 0$ (in wavelengths) in the same figure. For better readability, use different scale for the voltage and the current (e.g., "plotyy" in Matlab). Interpret based on the curve, what is the range (u_{min}, u_{max}) of instantaneous voltage in the load impedance.