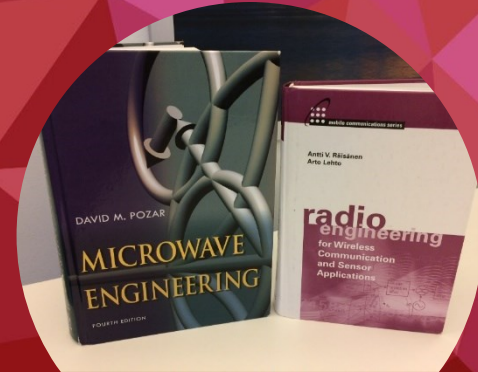# Microwave engineering I (MiWE I)

Interactive lecture 1 of Topic 1
Transmission line theory
January 13, 2022

*The main learning outcome of the course is to create readiness to work in microwave engineering related tasks and projects and enable further studies and continuous learning in microwave engineering.*

# Topic 1: Learning outcomes and content

- The student can
  - **explain** the wave propagation of a radio-frequency signal in transmission lines (such as signal propagation, attenuation, reflection),
  - **calculate** and **simulate** (AWR) related circuit parameters (such as voltage, current, power, characteristic impedance, loss, reflection coefficient) related to transmission lines,
  - **design** transmission lines (such as microstrip lines) with calculations and AWR simulations.

- Transmission line model, wave equations and its solution (Pozar Chapter 2.1)
- Wave propagation along a transmission line and characteristic impedance (2.1, 2.7)  } MAIN TOPICS TODAY
- Connection of the transmission line theory and EM field theory (2.2)
- Microstrip line (3.8)
- Voltage reflection from an impedance discontinuity and standing wave along a transmission line (2.3)
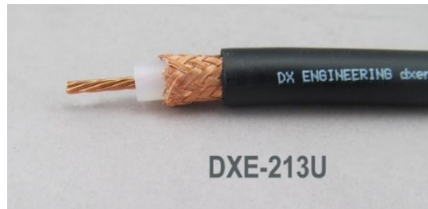
> These lecture slides and notes are not designed for self-study.
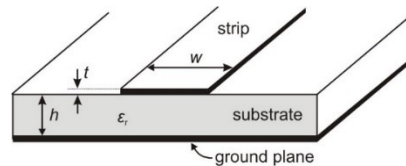> Please, use the course book chapters 2 and 3 for self-study.

# The "world" becomes wireless, why do we still study transmission line theory?

Coaxial cable, e.g., in
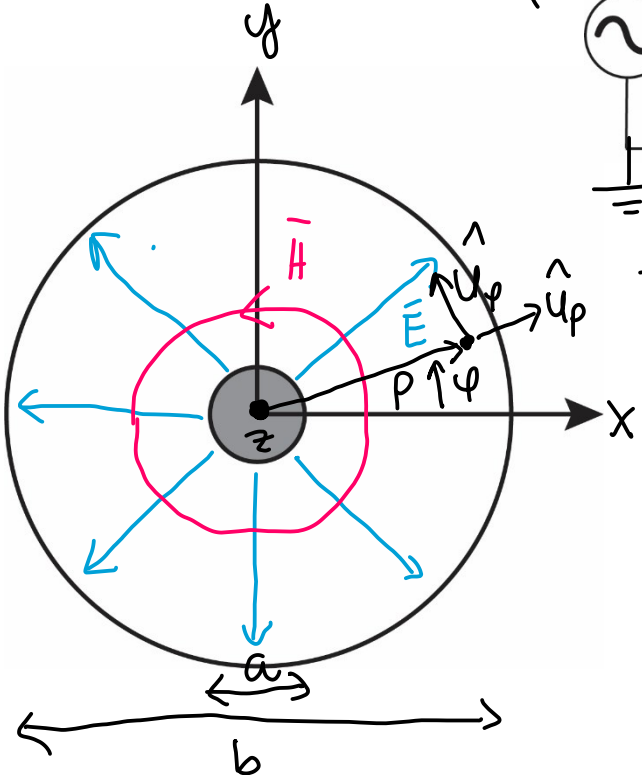- WiFi router
- TV antenna cable
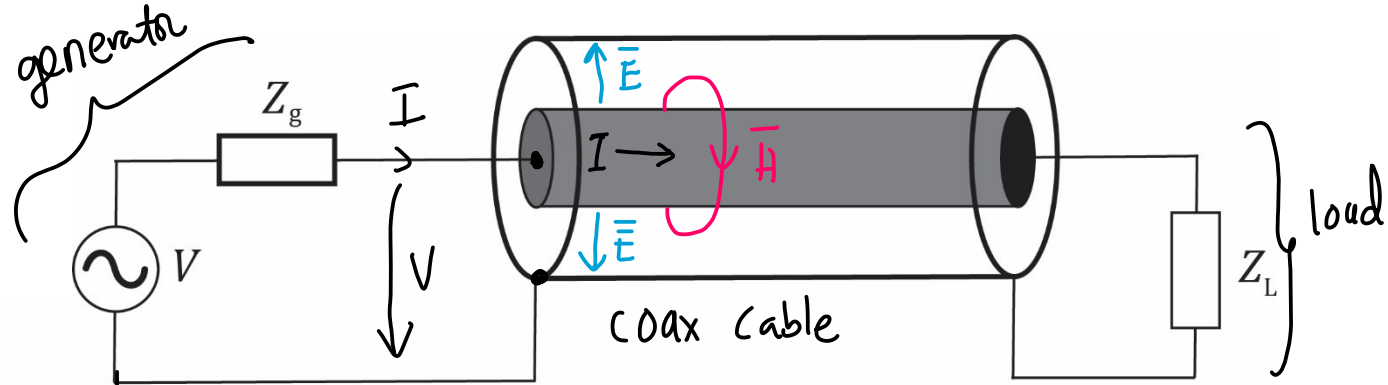- measurement devices,
- etc.

Microwave amplifier integrated on printed circuit board (consists of transistor and microstrip lines)

Mobile phone disassembled – lots of transmission lines are needed for signal transfer

# Transmission line carries a wave with E and H fields

generator

$Z_g$

$I$

$V$

$\bar{E}$

$I \rightarrow$

$\bar{H}$

$\bar{E}$

$Z_L$  load

coax cable

$y$

$\bar{H}$

$\hat{U}_\rho$

$\bar{E}$ $\hat{U}_\rho$

$\rho \uparrow \varphi$

$x$

$z$

$a$

$b$

$$\bar{E}(\rho) = \hat{U}_\rho \underbrace{\frac{V}{\rho \ln\left(\frac{b}{a}\right)}}_{E(\rho)} e^{-jkz} \leftarrow \text{wave behaviour}$$

$$\bar{H}(\rho) = \hat{U}_\rho \underbrace{\frac{I}{2\pi\rho}}_{H(\rho)} e^{-jkz}$$

$$Z_0 = \frac{U}{I} = \frac{E(\rho) \cdot \rho \left(\ln\left(\frac{b}{a}\right)\right) \cdot e^{-jkz}}{\underbrace{H(\rho)}\, 2\pi\rho \quad e^{-jkz}}$$

wave impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$$Z_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right)$$

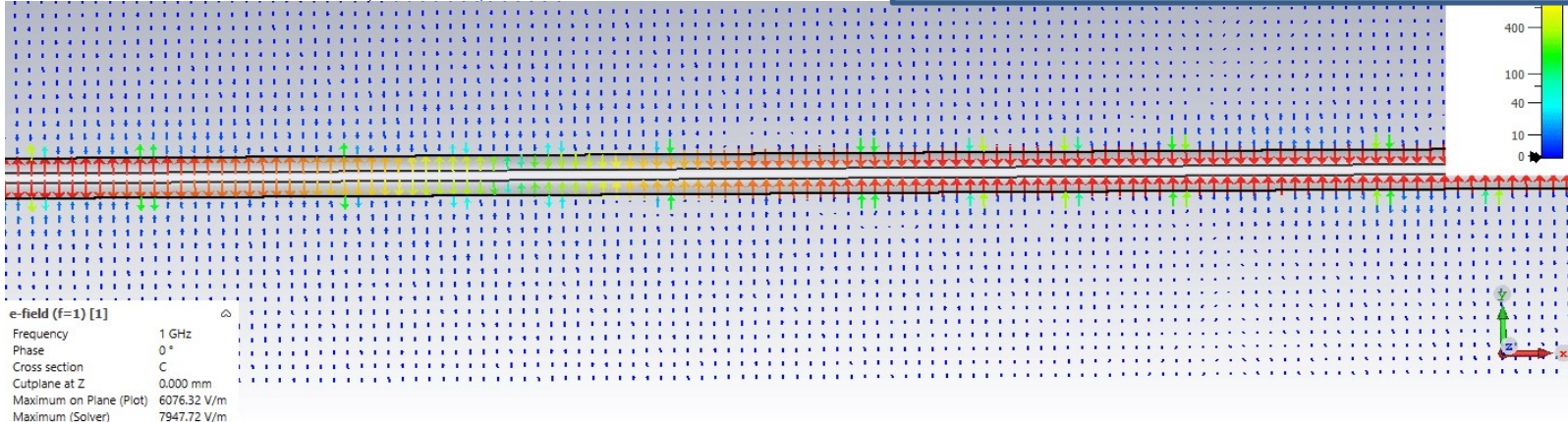# The wave behaviour is visible in the time-domain presentation
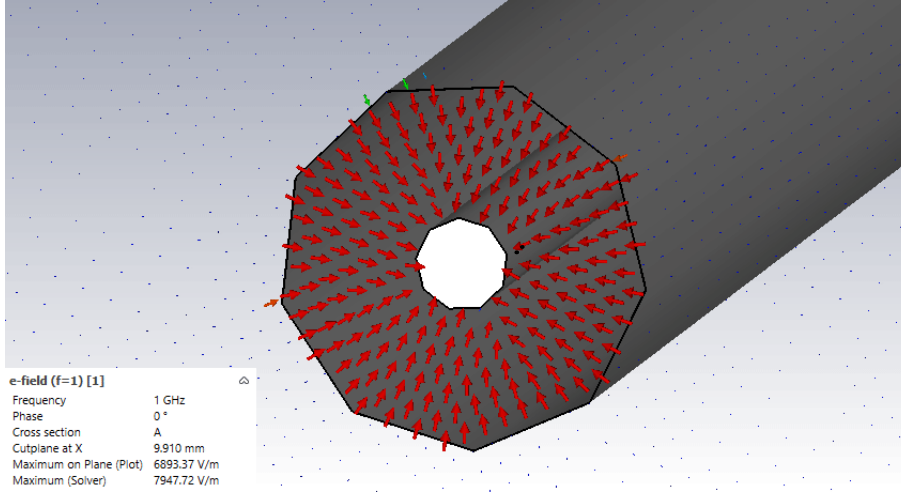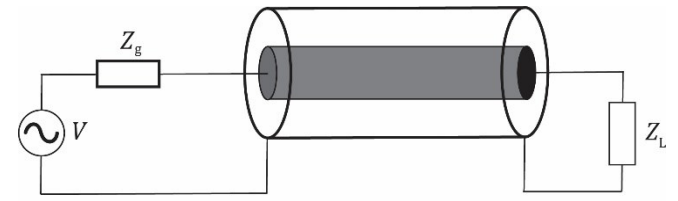


$$\mathbf{E}(z) = \mathbf{u}_\rho \frac{U}{\rho \ln\frac{b}{a}} e^{-\mathrm{j}kz}$$
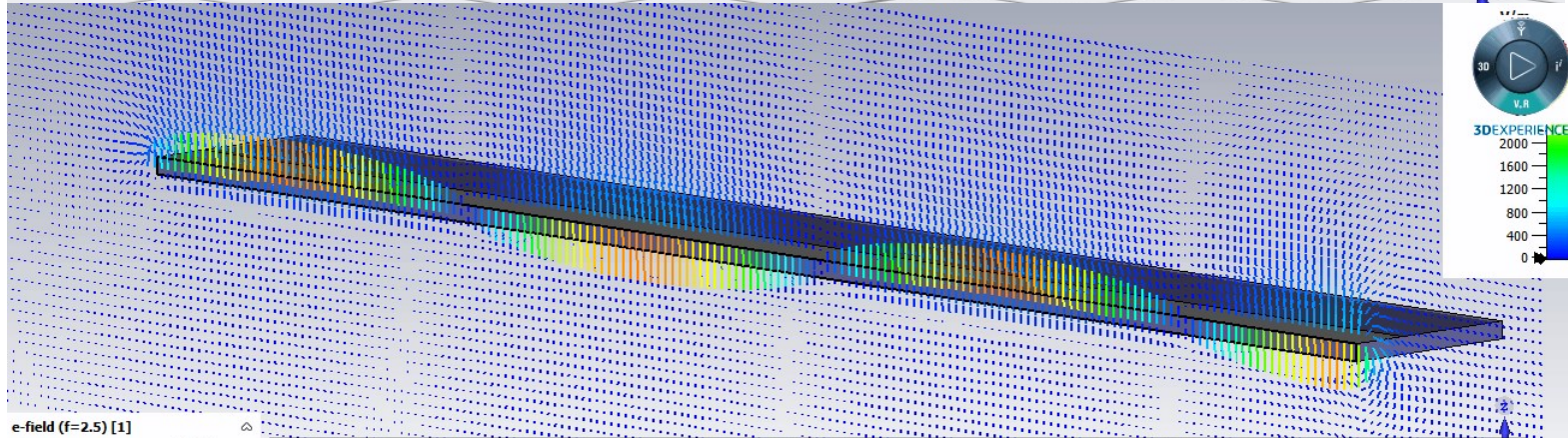
$$\mathbf{E}(z,t) = \mathcal{R}e\{\mathbf{E}(z)e^{\mathrm{j}\omega t}\}$$

$$\mathbf{E}(z,t) = \mathbf{u}_\rho \frac{U}{\rho \ln\frac{b}{a}} \cos(\omega t - kz)$$

$\omega = 2\pi f$

# Microstrip line is one important transmission line in microwave engineering



load impedance

dielectric substrate

strip

ground plane

feed port

e-field (f=2.5) [1]

| | |
|---|---|
| Frequency | 2.5 GHz |
| Phase | 0 ° |
| Cross section | D |
| Cutplane at X | -2.000 mm |
| Maximum on Plane (Plot) | 3623.91 V/m |
| Maximum (Solver) | 3740.13 V/m |

electric field strength vector (red = strong, blue = weak) illustrated at the centre ($x = 0$) of the line

# Microstrip line is one important transmission line in microwave engineering



load impedance

dielectric substrate

strip

ground plane

feed port
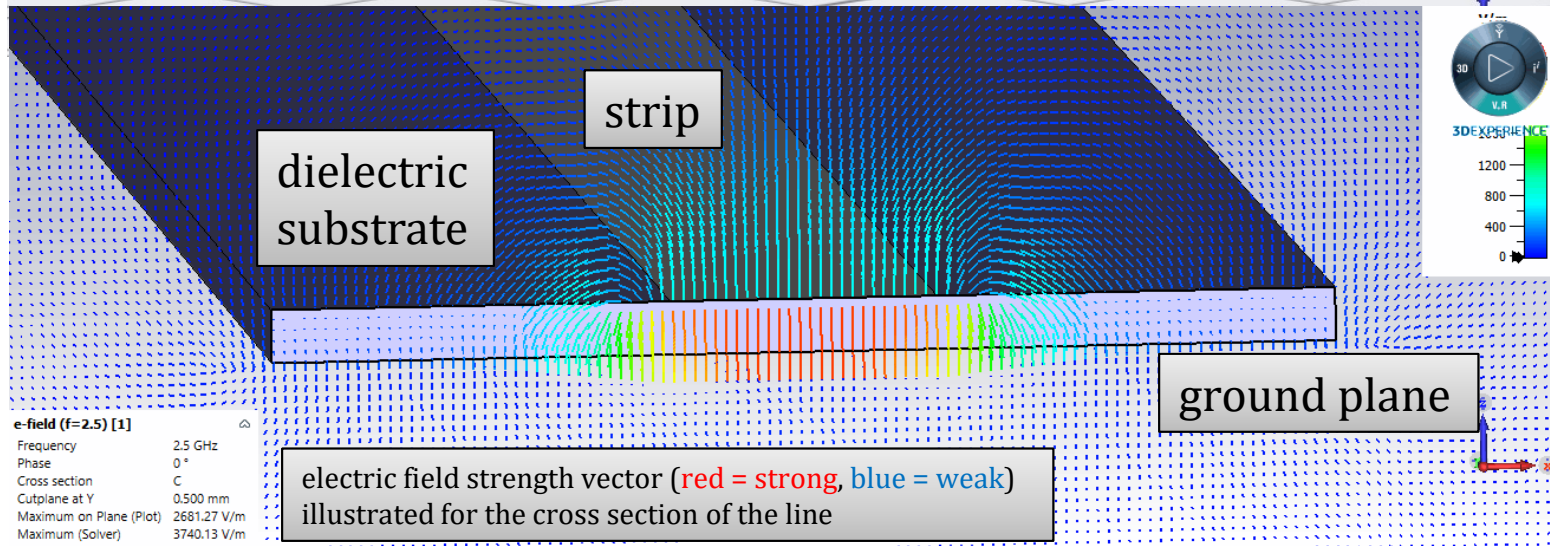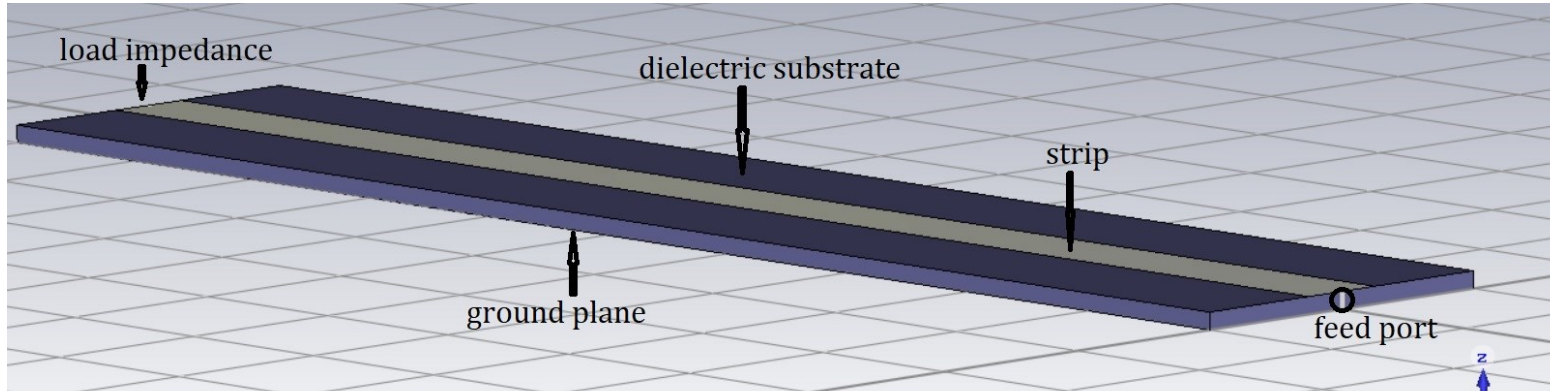
Exercise problem 3 of Topic 1.

The plug-in-type design formulas in Pozar chapter 3.8.

strip

dielectric substrate

ground plane

e-field (f=2.5) [1]

| | |
|---|---|
| Frequency | 2.5 GHz |
| Phase | 0 ° |
| Cross section | C |
| Cutplane at Y | 0.500 mm |
| Maximum on Plane (Plot) | 2681.27 V/m |
| Maximum (Solver) | 3740.13 V/m |

electric field strength vector (red = strong, blue = weak) illustrated for the cross section of the line

# Transmission line theory and wave equations

Components and lines whose physical length is a fraction **(?)** of the wavelength or longer must be analysed using the transmission line theory

$L$ = inductance/m
$R$ = resistance/m
$C$ = capacitance/m
$G$ = conductance/m



$dz \ll$

$z + dz$
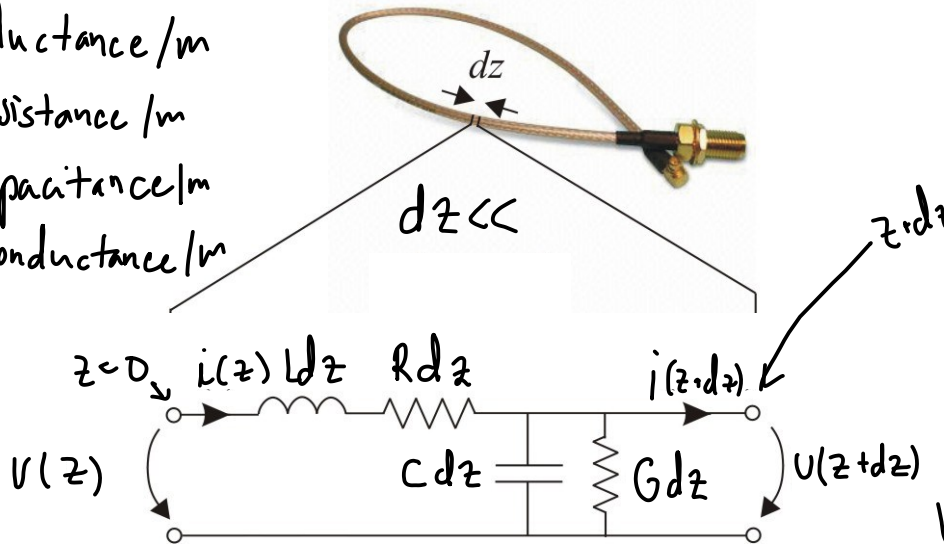
telegrapher's equations

$z = 0$, $i(z)$ $L\,dz$ $R\,dz$     $i(z+dz)$

$v(z)$    $C\,dz$   $G\,dz$   $v(z+dz)$

$dv = v(z) - v(z+dz) = -(R+j\omega L)\,dz \cdot i(z) \Rightarrow \dfrac{dv}{dz} = -(R+j\omega L)\,i(z)$

$di = i(z) - i(z+dz) = -(G+j\omega C)\,dz \cdot v(z) \Rightarrow \dfrac{di}{dz} = -(G+j\omega C)\,v(z)$

**Wave equations:**

$$
\begin{cases}
\dfrac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\[2mm]
\dfrac{d^2 i(z)}{dz^2} = \gamma^2 i(z)
\end{cases}
$$

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$\gamma$ = propagation constant

complex number

# One solution of the wave equations



$$dz \ll \lambda$$

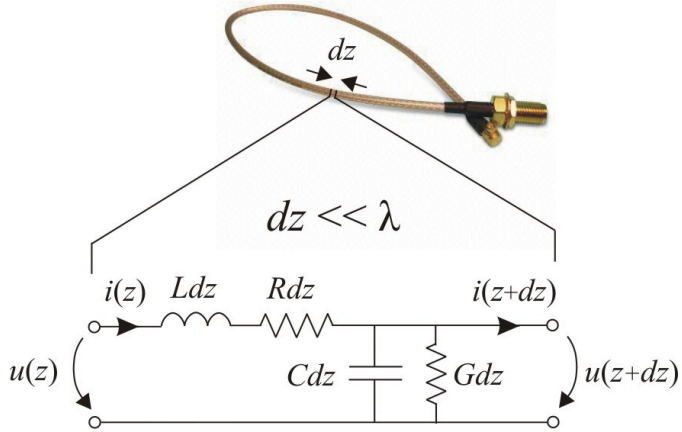$$\begin{cases} \dfrac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \dfrac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

$$\gamma^2 = (\cancel{R} + j\omega L)(\cancel{G} + j\omega C)$$

$$u(z,t) = \mathcal{Re}\{ u(z) \cdot e^{j\omega t} \}$$

phase constant

Lossless case: $R = 0, G = 0$   $\gamma^2 = j\omega L \cdot j\omega C = j^2 \omega^2 LC \implies \gamma = j\underbrace{\omega\sqrt{LC}}_{\beta} = j\beta$

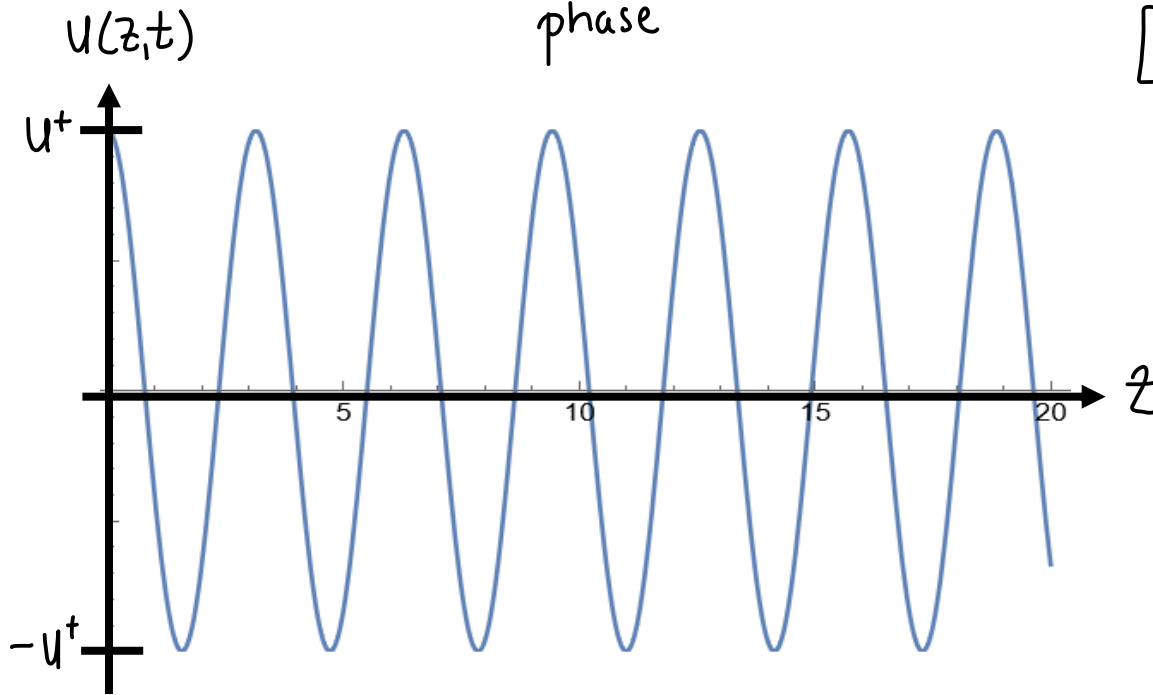One solution:  $U(z) = U^+ e^{-j\beta z}$   $i(z) = i^+ e^{-j\beta z}$   $U^+$ real number

Time-domain presentation:  $U(z,t) = \mathcal{Re}\{ u(z) \cdot e^{j\omega t} \} = U^+ \cos(\omega t - \beta z)$

# This wave propagates to positive z-direction

$$u(z,t) = u^+ \cos(\underbrace{\omega t - \beta z}_{\text{phase}})$$

phase

$u(z,t)$

$u^+$

$-u^+$

$z$

5    10    15    20

The phase $(\omega t - \beta z)$ changes $2\pi$ within one full wavelength:

$$\left[\omega t - \beta z\right] - \left[\omega t - \beta(z + \lambda)\right] = 2\pi$$

$$\beta \lambda = 2\pi$$

$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

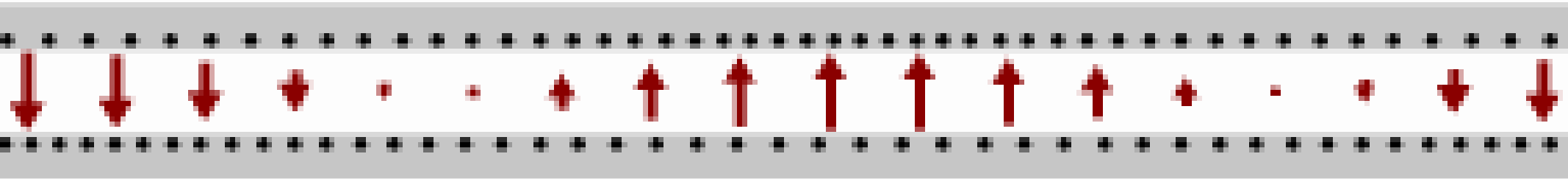$\beta = $ phase constant

$$[\beta] = \frac{1}{m}$$

# Wave travelling in the +z direction in a lossless line
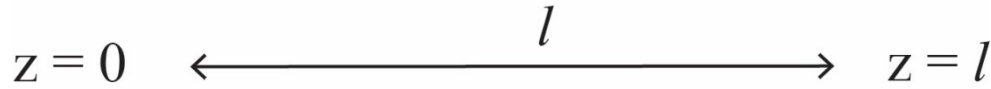
$$u(t, z) = u^+ \cos(\omega t - \beta z)$$

↑ = **E** field vector, represents "voltage" $u(z,t)$     ● = charge

# When to apply the transmission line theory?

$z = 0$ ⟵———————— $l$ ————————⟶ $z = l$

$$u(t, z) = u^+ \cos(\omega t - \beta z)$$

$t = 0$

$u(t=0) = u^+ \cos(-\beta z)$

$u(z=0) = u^+$

$u(z=l)$

$u(z=0, t=0) = u^+ \cos 0 = u^+$

$\llcorner$ phase $= 0$

$u(z=l, t=0) = u^+ \cos(\underbrace{-\beta l})$

$\quad \quad \quad \quad \quad \quad \quad \llcorner$ phase $= -\beta l$

$\dfrac{2\pi}{\lambda}$

$\downarrow$
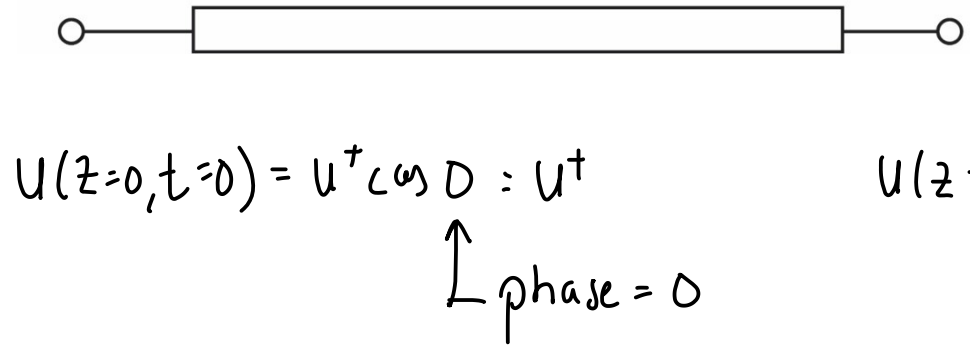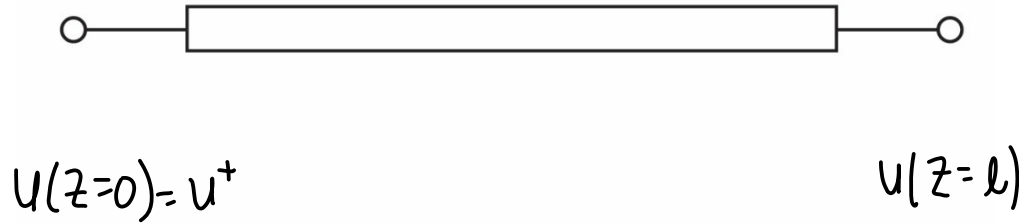
phase difference $= 0 - (-\beta l) = \beta l = \dfrac{2\pi}{\lambda} l = 2\pi \dfrac{l}{\lambda} = 360° \dfrac{l}{\lambda}$

# Q1: What is the <u>maximum length *l*</u> of the line when the phase difference of the voltage in the line is no more than <u>10 degrees</u>?

$z = 0 \longleftarrow \overset{l}{\hspace{6cm}} \longrightarrow z = l$

$u(t = 0, z = 0)$
$= u^+ \cos(0)$

$u(t = 0, z = l)$
$= u^+ \cos(-\beta l)$

phase diff. $\beta l = \frac{2\pi}{\lambda} \cdot l = 360° \frac{l}{\lambda} = 10°$
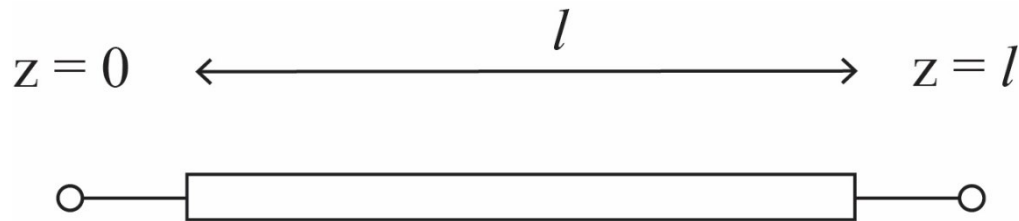
$\frac{1}{360} \lambda$

$l = \frac{10°}{360°} \lambda = \frac{1}{36} \lambda$

Select **one** alternative:

13%. *1.* $l = \frac{1}{5}\lambda = 0.20\lambda$

10%. *2.* $l = \frac{1}{10}\lambda = 0.10\lambda$

6%. *3.* $l = \frac{1}{12}\lambda = 0.083\lambda$

58% ④. $l = \frac{1}{36}\lambda = 0.028\lambda$ %✓

6%. *5.* $l = \frac{1}{72}\lambda = 0.014\lambda$

0%. *6.* $l = \frac{1}{99}\lambda = 0.010\lambda$

IDN 6%.

# Q2: What is the physical interpretation of this solution?

Note! Plus sign!

$$u(z) = ue^{+\gamma z} = ue^{+\alpha z}\, e^{+j\beta z}$$

$$\begin{cases} \dfrac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \dfrac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

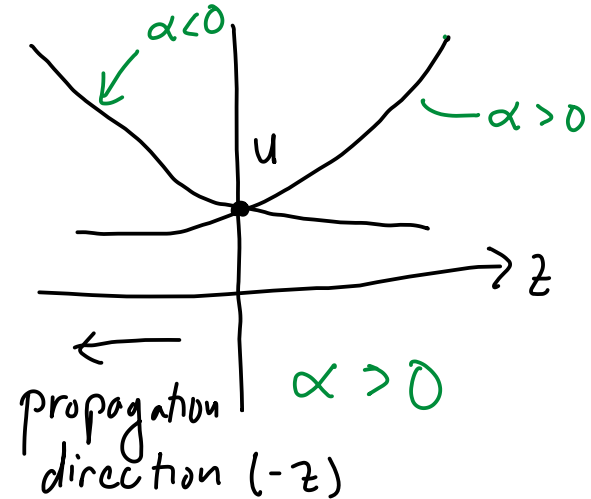$$\boxed{\begin{aligned} \gamma &= \pm\sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \alpha + j\beta,\ \alpha \neq 0,\ \beta > 0 \end{aligned}}$$

$u(z) = ue^{+\alpha z} \cdot e^{j\beta z}$

$u(z,t) = \underbrace{ue^{+\alpha z}}_{} \cdot \cos(\omega t + \beta z)$

Select the best alternative.

This solution is a wave travelling...

19%. 1.    in the +z direction with an exponential damping factor

19%. 2.    in the +z direction with an exponential growth factor

0%. 3.    in the +z direction with constant magnitude

%. 32%. (4.)    in the -z direction with an exponential damping factor

16%. 5.    in the -z direction with an exponential growth factor

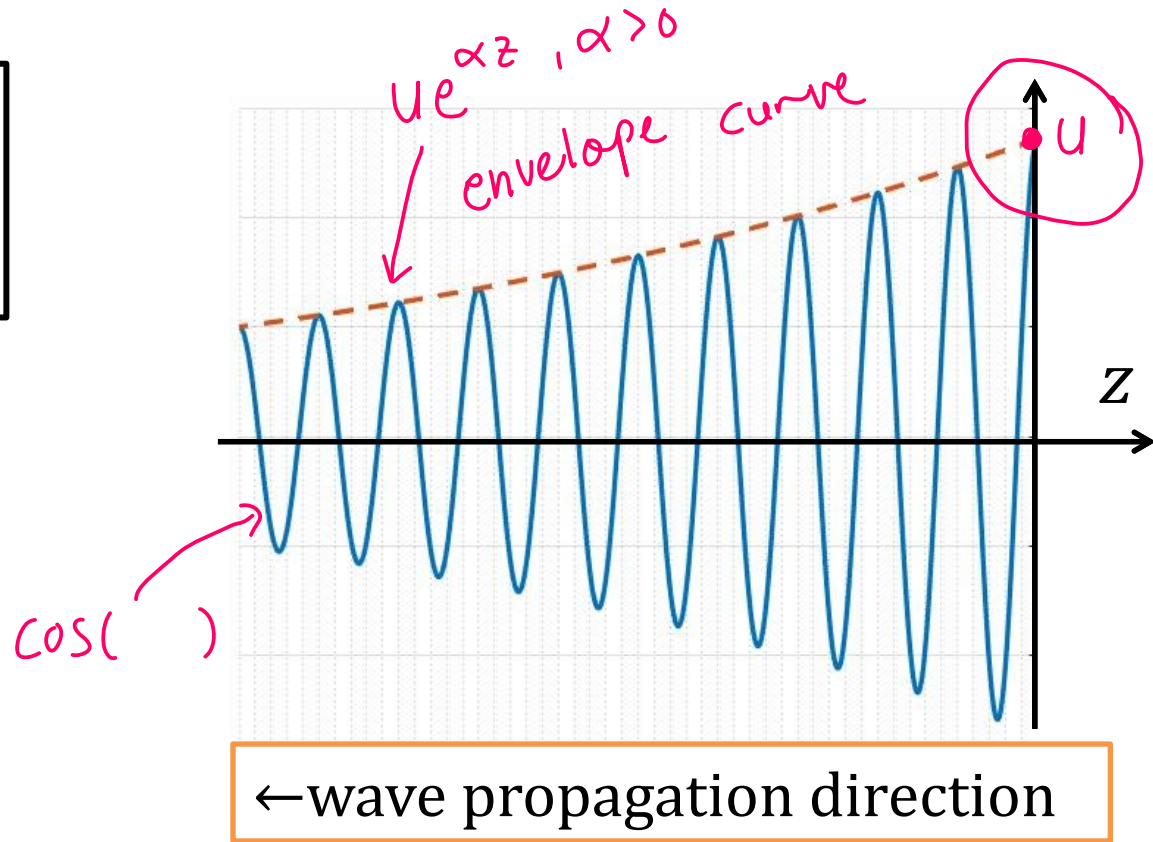6%. 6.    in the -z direction with constant magnitude   IDN 6%.

$\alpha < 0$
$\alpha > 0$
$u$
$\alpha > 0$
$z$

Propagation direction (-z)

# A wave travelling in the +z direction with an exponential damping factor

$$\gamma = \pm\sqrt{(R + j\omega L)(G + j\omega C)}$$

$\quad$ = complex propagation constant

$\quad = \alpha + j\beta \quad$ (unit 1/m)

$$u(z) = u^- e^{+\gamma z}$$

$$u(z,t) = \mathcal{Re}\{\, u(z)e^{j\omega t}\,\}$$

the sign of $\alpha$ selected
so that the solution
is physical

$u e^{\alpha z}, \alpha > 0$

envelope curve

$u$

$z$

$\cos(\quad)$

←wave propagation direction
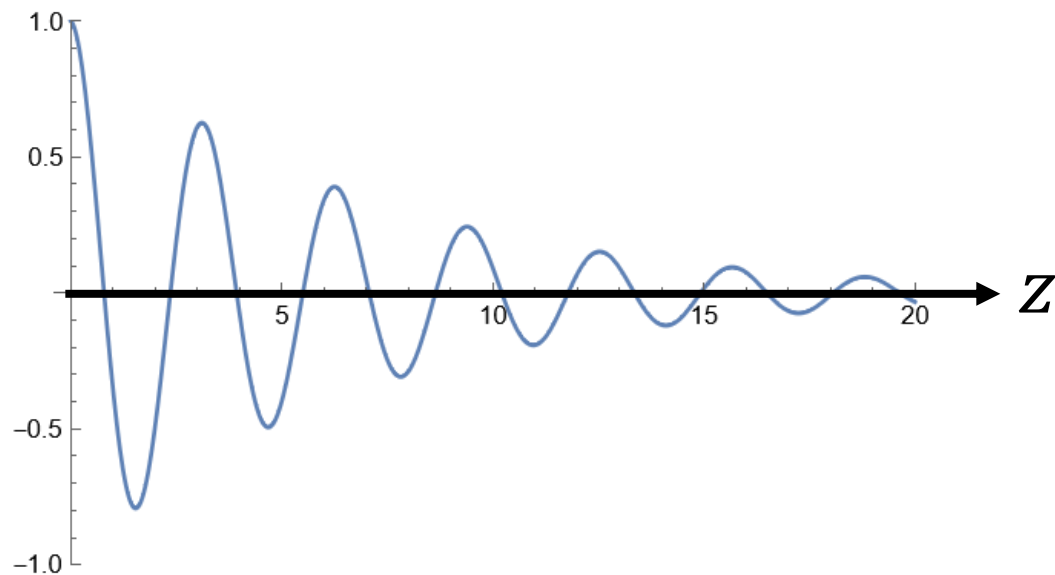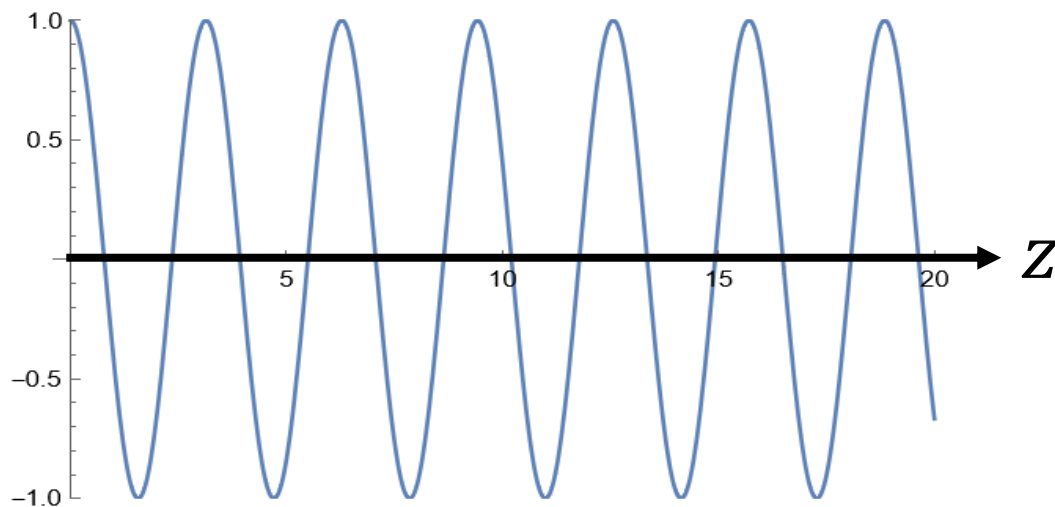
$$u(z,t)$$

lossless line

$$u^+ = 1\,\text{V}; \beta = 2\,\frac{1}{\text{m}}; \alpha = 0$$

$$u(z,t) = u^+ e^{\alpha z}$$
$$\cdot \cos(\omega t - \beta z)$$

lossy line

$$u^+ = 1\,\text{V}; \beta = 2\frac{1}{\text{m}};$$
$$\alpha = -0{,}15\,\frac{1}{\text{m}}$$

# Full solution of the wave equations



$dz$

$dz \ll \lambda$

$i(z)$ $Ldz$ $Rdz$ $i(z+dz)$

$u(z)$ $Cdz$ $Gdz$ $u(z+dz)$
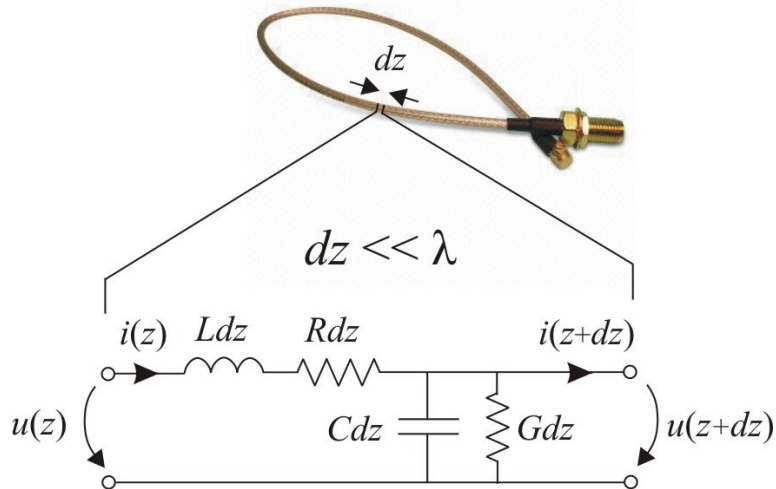
$$\begin{cases} u(z) = u^+ e^{-\gamma z} + u^- e^{+\gamma z} \\ i(z) = i^+ e^{-\gamma z} + i^- e^{+\gamma z} \end{cases}$$

Wave equations:

$$\begin{cases} \dfrac{d^2 u(z)}{dz^2} = \gamma^2 u(z) \\ \dfrac{d^2 i(z)}{dz^2} = \gamma^2 i(z) \end{cases}$$

$$\gamma = \pm\sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \alpha + j\beta$$

# Full solution and characteristic impedance

$$\begin{cases} u(z) = u^+ e^{-\gamma z} + u^- e^{\gamma z} \\ i(z) = i^+ e^{-\gamma z} + i^- e^{\gamma z} \end{cases}$$

positive z-direction

neg. z-direction
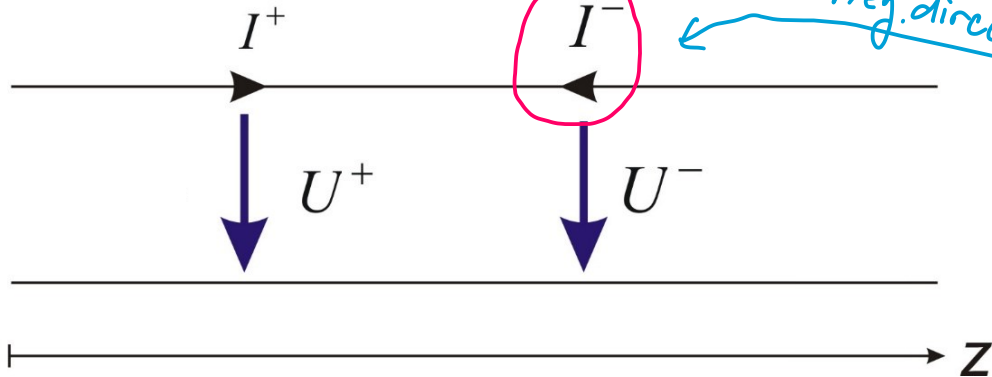
We define the characteristic impedance

$$Z_0 = \frac{\text{voltage}}{\text{current}} = \frac{U^+}{i^+} = \frac{U^-}{-i^-}$$

$$i^+ = \frac{U^+}{Z_0}$$

$$i^- = -\frac{U^-}{Z_0}$$

$$i(z) = \frac{U^+}{Z_0} e^{-\gamma z} \left(-\right) \frac{U^-}{Z_0} e^{+\gamma z}$$

neg. direction

$I^+$  $I^-$

$U^+$  $U^-$

$z$

# Characteristic impedance relates the voltage and current on the line

$$\frac{du}{dz} = -(R + j\omega L) \cdot i(z)$$

(from slide 8)

$$\begin{cases} u(z) = u^+ e^{-\gamma z} + u^- e^{\gamma z} \\ i(z) = i^+ e^{-\gamma z} + i^- e^{\gamma z} \end{cases}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\frac{du}{dz} = -u^+ \gamma \, e^{-\gamma z} + u^- \gamma \, e^{\gamma z} = -(R + j\omega L) \cdot i(z) \quad \Big| : -(R + j\omega L)$$

$$i(z) = \frac{\gamma}{(R + j\omega L)} \left( u^+ e^{-\gamma z} - u^- e^{\gamma z} \right) = i^+ e^{-\gamma z} + i^- e^{\gamma z}$$

compare

$$\frac{\gamma}{(R + j\omega L)} u^+ = i^+ \iff \frac{u^+}{i^+} = Z_0 = \frac{R + j\omega L}{\gamma} = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

# In-class task: average power of a lossy transmission line

Let us assume that the transmission line has some resistive losses – i.e., in the equivalent circuit model $R$ and/or $G \neq 0$.

A wave travels in the positive +z direction only. Let the average power at $z = 0$ be $P_0$ and it depends upon the voltage $u^+$ and the characteristic impedance $Z_0$.

Starting from the definition of the average power, derive a formula for the average power $P(z)$ of a lossy transmission line in a location $z$ (>0).

The teacher is available here to help you! Do not hesitate to ask!

Return your effort in the return box in MyCourses under Topic 1 latest at 12 noon today. It is completely okay if it is not correct or ready at the end, the most important thing is that you try to do it!

$$P(z) = \frac{1}{2}\mathcal{R}e\{u(z) \cdot i^*(z)\}$$

\* is the complex conjugate
$\mathcal{R}e$ denotes the real part

$$u(z) = u^+ e^{-\gamma z}$$
$$i(z) = i^+ e^{-\gamma z}$$

$$\gamma = \alpha + \mathrm{j}\,\beta$$

$$Z_0 = \frac{u^+}{i^+}$$

# Lecture task: average power of a lossy transmission line - solution

$$P(z) = \frac{1}{2}\mathcal{Re}\{u(z) \cdot i^*(z)\} = \frac{1}{2}\mathcal{R}\{u^+ e^{-\gamma z} \cdot (i^+ e^{-\gamma z})^*\}$$

$$P(z) = \frac{1}{2}\mathcal{Re}\left\{u^+ e^{-\gamma z} \cdot \left(\frac{u^+}{Z_0}\right)^* (e^{-\gamma z})^*\right\} \qquad \gamma = \alpha + j\beta$$

$$P(z) = \frac{1}{2}\mathcal{Re}\left\{\underbrace{u^+(u^+)^*}_{|u^+|^2} \frac{1}{Z_0{}^*} \underbrace{e^{-\alpha z} e^{-\alpha z}}_{\text{purely real}} \underbrace{e^{-j\beta z} e^{+j\beta z}}_{1}\right\}$$

$$P(z) = \underbrace{\frac{1}{2}|u^+|^2 \mathcal{Re}\left\{\frac{1}{Z_0{}^*}\right\}}_{P(z=0)=P_0} \cdot e^{-2\alpha z}$$

The average power decays twice as fast as the voltage and the current. This makes sense as the power is the product of the voltage and the current.

If we set the condition of the lossless line; $\alpha = 0$ and $Z_0$ is real, we get a familiar formula:

$$P(z) = \frac{|u^+|^2}{2Z_0}$$

# What's up on Monday

- Find the exercise problems (1-4) of Topic 1 in MyCourses

- The exercise session takes place on Monday morning

- We reserve everyone a 15-minute one-to-one session with one of the course teachers. See further instructions in MyCourses.

- We assume you to return 1-2 of the problems during the pre-booked meeting.