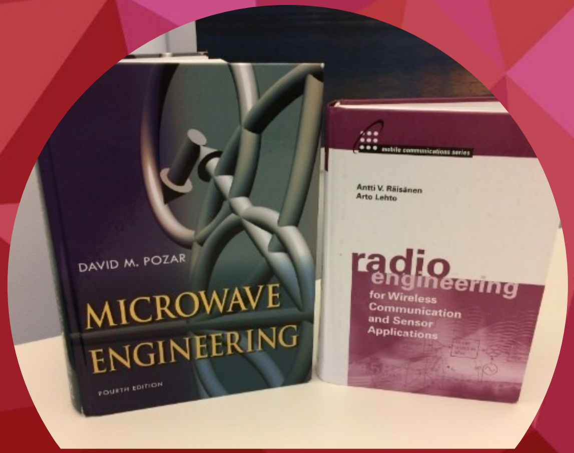
Microwave engineering I (MiWE I)

Interactive lecture 2 of Topic 1
Transmission line theory

January 20, 2022

The main learning outcome of the course is to create readiness to work in microwave engineering related tasks and projects and enable further studies and continuous learning in microwave engineering.



Topic 1: Learning outcomes and content

- The student can
  - **explain** the wave propagation of a radio-frequency signal in transmission lines (such as signal propagation, attenuation, reflection),
  - **calculate** and **simulate** (AWR) related circuit parameters (such as voltage, current, power, characteristic impedance, loss, reflection coefficient) related to transmission lines,
  - **design** transmission lines (such as microstrip lines) with calculations and AWR simulations.

- Transmission line model, wave equations and its solution (Pozar Chapter 2.1)
- Wave propagation along a transmission line and characteristic impedance (2.1, 2.7)

} WEEK AGO

- Connection of the transmission line theory and EM field theory (2.2)
- Microstrip line (3.8)
- Voltage reflection from an impedance discontinuity and standing wave along a transmission line (2.3) TODAY

These lecture slides and notes are not designed for self-study.
Please, use the course book chapters 2 and 3 for self-study.

Recap of the last week session on one slide

Wave equations:

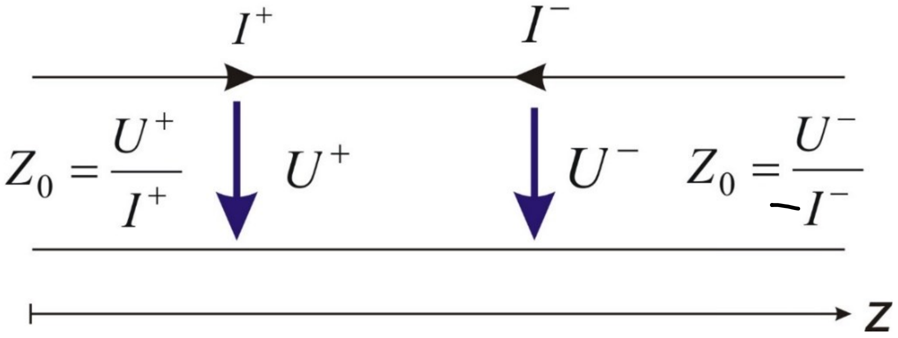
$$\frac{d^2u(z)}{dz^2} = \gamma^2 u(z); \quad \frac{d^2i(z)}{dz^2} = \gamma^2 i(z)$$

$$\gamma = \pm\sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \alpha + j\,\beta$$

$$\begin{cases} u(z) = u^+ e^{-\gamma z} + u^- e^{\gamma z} \\ i(z) = i^+ e^{-\gamma z} + i^- e^{\gamma z} \end{cases}$$

forward wave

reverse wave



Characteristic impedance (Ω):

$$Z_0 = \frac{\text{voltage}}{\text{current}} = \frac{u^+}{i^+} = -\frac{u^-}{i^-}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\left.\begin{matrix} R = 0 \\ G = 0 \end{matrix}\right\} Z_0 = \sqrt{\frac{L}{C}}$$

$\frac{dB}{m}$

$$i(z) = \frac{u^+}{Z_0} e^{-\gamma z} - \frac{u^-}{Z_0} e^{\gamma z}$$

neg. z direction

In-class task final answer:

$$P(z) = \underbrace{\frac{1}{2} |u^+|^2 \mathcal{Re}\left\{\frac{1}{Z_0{}^*}\right\}}_{P(z=0)=P_0} \cdot e^{-2\alpha z}$$

Previous in-class task: the power decays as $e^{-2\alpha z}$ along the line

$$P(z) = \frac{1}{2}\mathcal{R}e\{u(z)\cdot i^{\boxed{*}}(z)\} = \frac{1}{2}\mathcal{R}\{u^{+}e^{-\gamma z}\cdot(i^{+}e^{-\gamma z})^{*}\}$$

$$P(z) = \frac{1}{2}\mathcal{R}e\left\{u^{+}e^{-\gamma z}\cdot\left(\frac{u^{+}}{Z_0}\right)^{\boxed{*}}(e^{-\gamma z})^{\boxed{*}}\right\}$$

$$a\cdot a^{*} = |a|^{2}$$

$$P(z) = \frac{1}{2}\mathcal{R}e\left\{\underbrace{u^{+}(u^{+})^{*}}_{|u^{+}|^{2}}\frac{1}{Z_0{}^{*}}\underbrace{e^{-\alpha z}e^{-\alpha z}}_{\text{Purely real}}\underbrace{e^{-j\beta z}e^{+j\beta z}}_{1}\right\}$$

$$\mathcal{R}e\left\{\frac{1}{Z_0^{*}}\right\} \neq \frac{1}{\mathcal{R}e\{Z_0^{*}\}}$$

$$P(z) = \underbrace{\frac{1}{2}|u^{+}|^{2}\mathcal{R}e\left\{\frac{1}{Z_0{}^{*}}\right\}}_{P_0 = P(z=0)}\cdot e^{-2\alpha z} = \frac{1}{2}|u^{+}|^{2}\,\frac{\mathcal{R}e\{Z_0\}}{|Z_0|^{2}}\cdot e^{-2\alpha z}$$

1 Np = 8.6859 dB

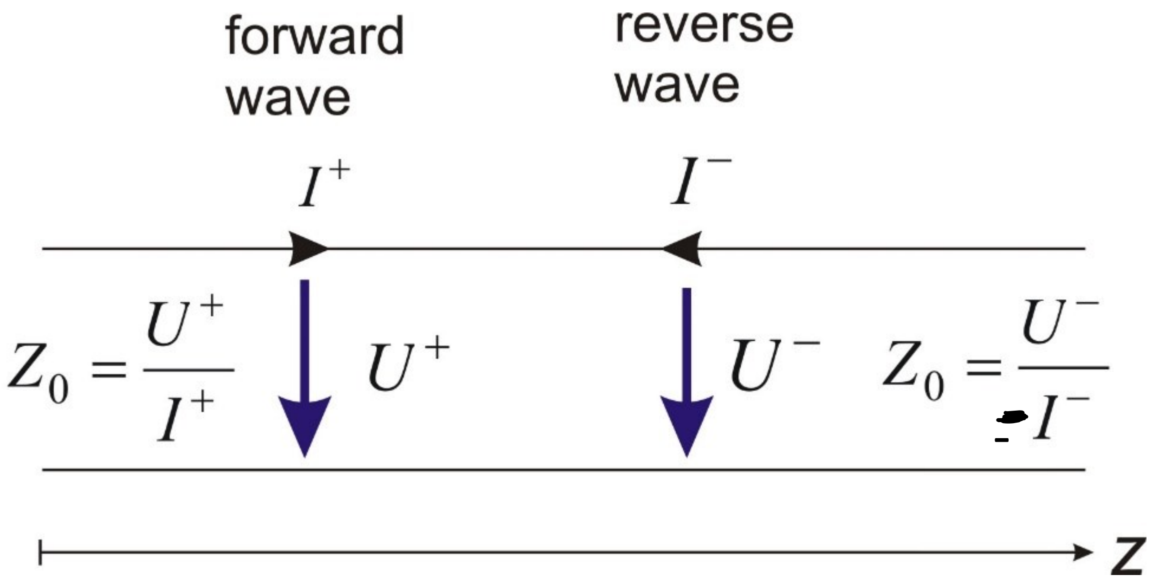
$$P(z=1m) = \underset{\underset{P(z=0)}{\uparrow}}{P_0}\, e^{-2\alpha\cdot 1m}$$

$$L(dB) = -10\log_{10}\frac{P(z=1\,m)}{P_0} = -10\log e^{-2\alpha\cdot 1m}$$

$$= \alpha\cdot 10\cdot 2\cdot 1m\,\log_{10} e = \alpha\cdot\underbrace{20\,m\cdot\log e}_{1\,Np = 8,6859\ \cdot dB}$$

$$L(dB) = \alpha\left[\frac{1}{m}\right]\cdot 8,6859 \quad dB \quad \left(\frac{dB}{m}\right)$$

Q1: What does the given power *P* mean physically?



forward

$$\boxed{P = \frac{|u^{\oplus}|^2}{2Z_0}}$$

Z_0 assumed real number.

U^+ = peak voltage

$U_{rms} = \frac{U^+}{\sqrt{2}}$

0% 1. Total loss power due to the resistive losses on the line.

7% 2. Peak power of the wave propagating in the positive +*z* direction.

67% [✓ 3.] Average power flow of a wave propagating in the positive +*z* direction. %

13% 4. Instantaneous power propagating in the positive +*z* direction when *z* = 0.

10% 5. Net power flow (= forward power – reverse power) on the line.

0% 6. Reflected power from the mismatched load impedance ($Z_L \neq Z_0$).

IDN 3%

$p(t) = u(t) \cdot i(t)$

↑

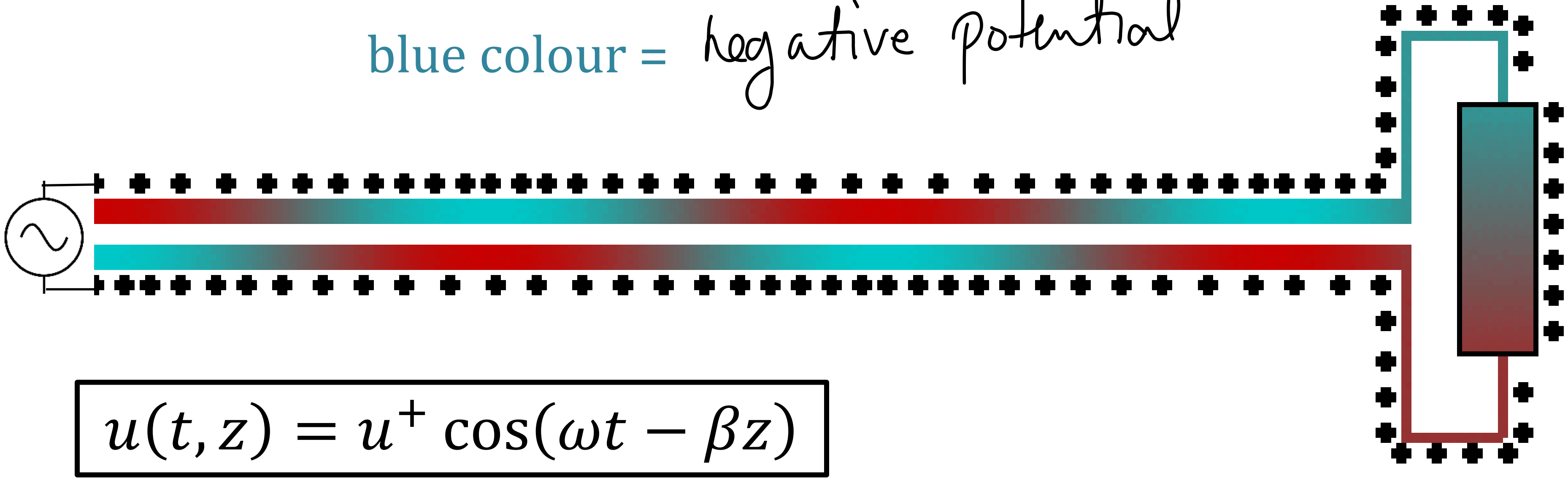
instantaneous power

Wave travelling in the +z direction with constant magnitude

● = charge (electron)

red colour = positive potential

blue colour = negative potential

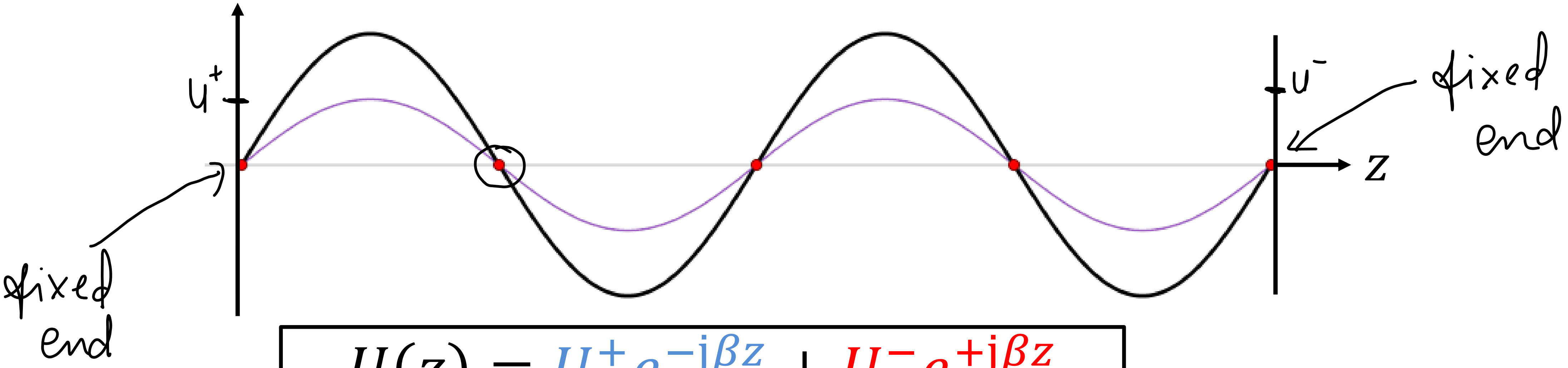


impedance-matched load
- no reflection!

$$\boxed{u(t,z) = u^{+} \cos(\omega t - \beta z)}$$

Animation Source: https://physics.stackexchange.com

Standing wave is a superposition (interference) of two opposite-travelling waves



$$\boxed{U(z) = U^+ e^{-j\beta z} + U^- e^{+j\beta z}}$$

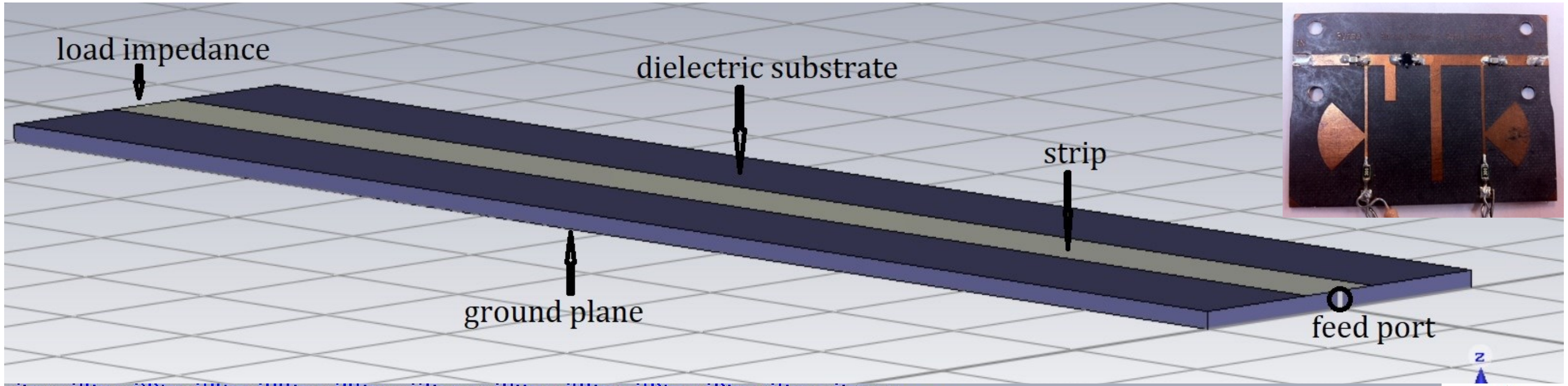
- Blue curve = forward-propagating wave
- Red curve = reverse- — || — -||-
- Black curve = sum of waves
- Red dots ● = nodes
- The "dome" between the red dots = antinode "extrema" / supremum

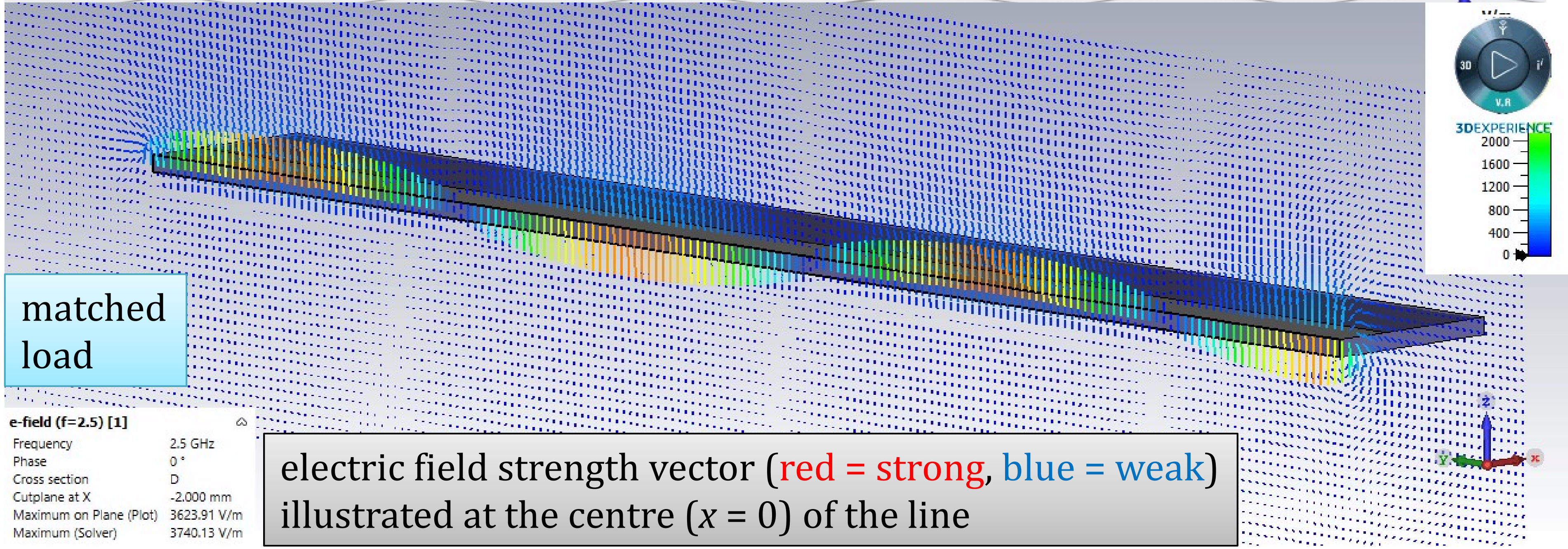
$$|U_{max}| = u^+ + u^-$$

$$|U_{min}| = 0$$

$$VSWR = \frac{|U_{max}|}{|U_{min}|} = \frac{u^+ + u^-}{0} = \text{"}\infty\text{"}$$

Forward propagating wave in a 50-Ω microstrip line – matched load impedance





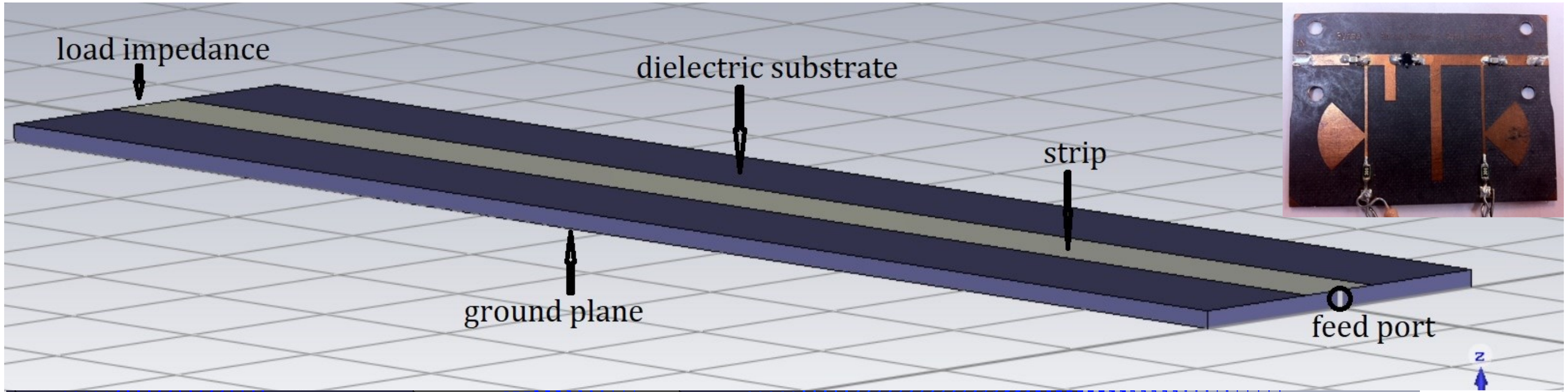
matched load

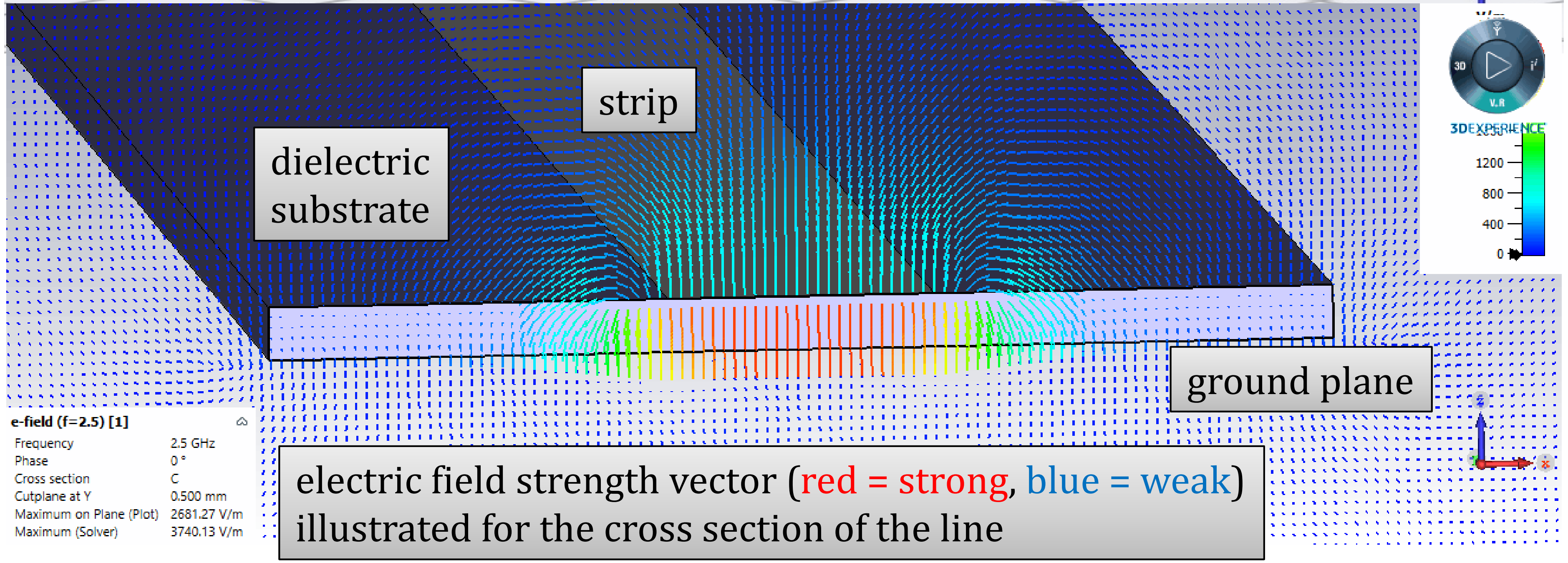
e-field (f=2.5) [1]

Frequency	2.5 GHz
Phase	0 °
Cross section	D
Cutplane at X	-2.000 mm
Maximum on Plane (Plot)	3623.91 V/m
Maximum (Solver)	3740.13 V/m

electric field strength vector (red = strong, blue = weak) illustrated at the centre (x = 0) of the line

Forward propagating wave in a 50-Ω microstrip line – matched load impedance

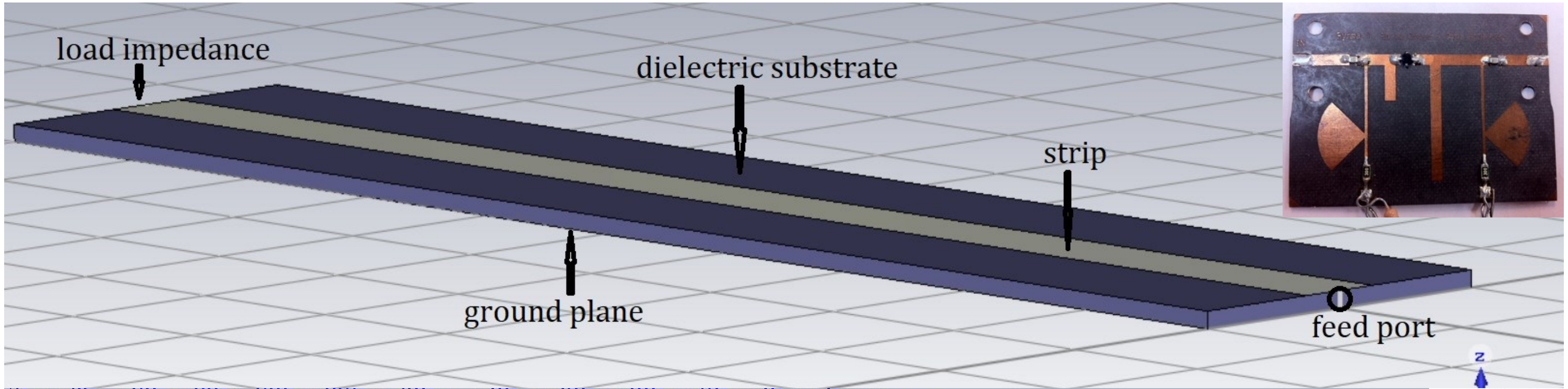


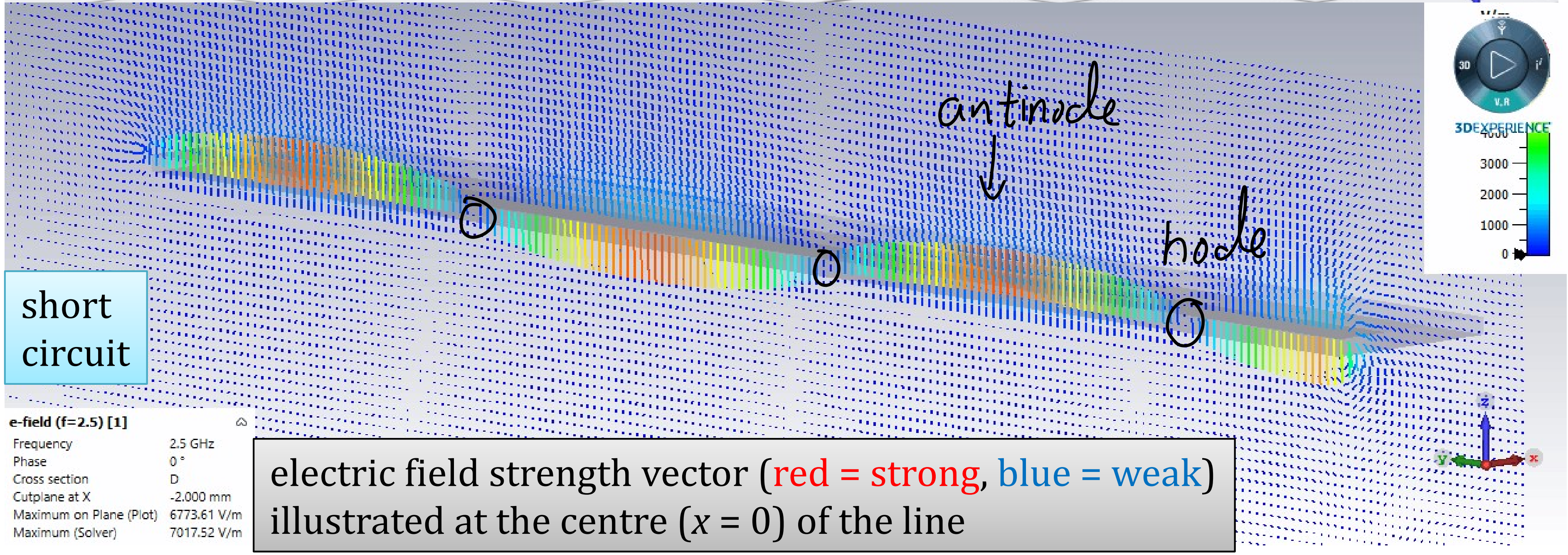


```
e-field (f=2.5) [1]
 Frequency               2.5 GHz
 Phase                   0 °
 Cross section           C
 Cutplane at Y           0.500 mm
 Maximum on Plane (Plot) 2681.27 V/m
 Maximum (Solver)        3740.13 V/m
```

electric field strength vector (red = strong, blue = weak)
illustrated for the cross section of the line

Stationary standing wave in a 50-Ω microstrip line – short circuit as the load impedance





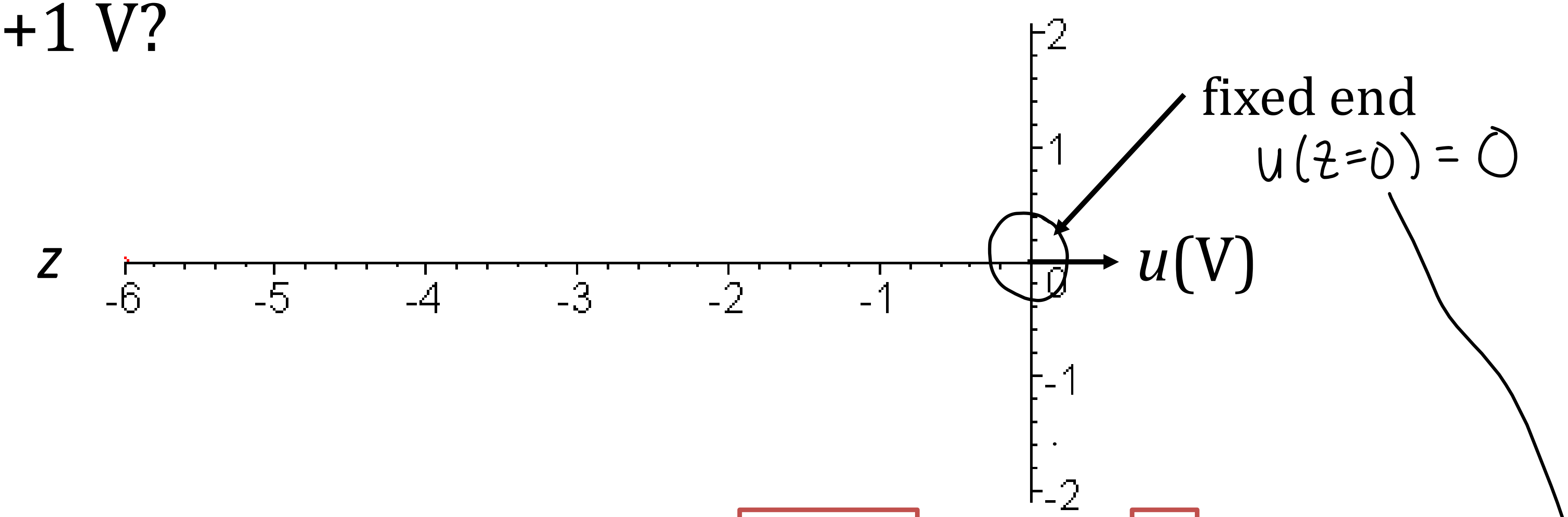
short circuit

e-field (f=2.5) [1]

Frequency	2.5 GHz
Phase	0 °
Cross section	D
Cutplane at X	-2.000 mm
Maximum on Plane (Plot)	6773.61 V/m
Maximum (Solver)	7017.52 V/m

electric field strength vector (red = strong, blue = weak) illustrated at the centre (x = 0) of the line

Q2. What is the value u^- of the **reverse**-propagating voltage wave when $u^+ = +1$ V?



2. vote

7%. 1. $u^- = +2$ V

6%. 47%. 2. $u^- = +1$ V

3%. 3. $u^- = 0$ V

94%. 40%. [✓ 4.] $u^- = -1$ V

3%. 5. $u^- = -2$ V

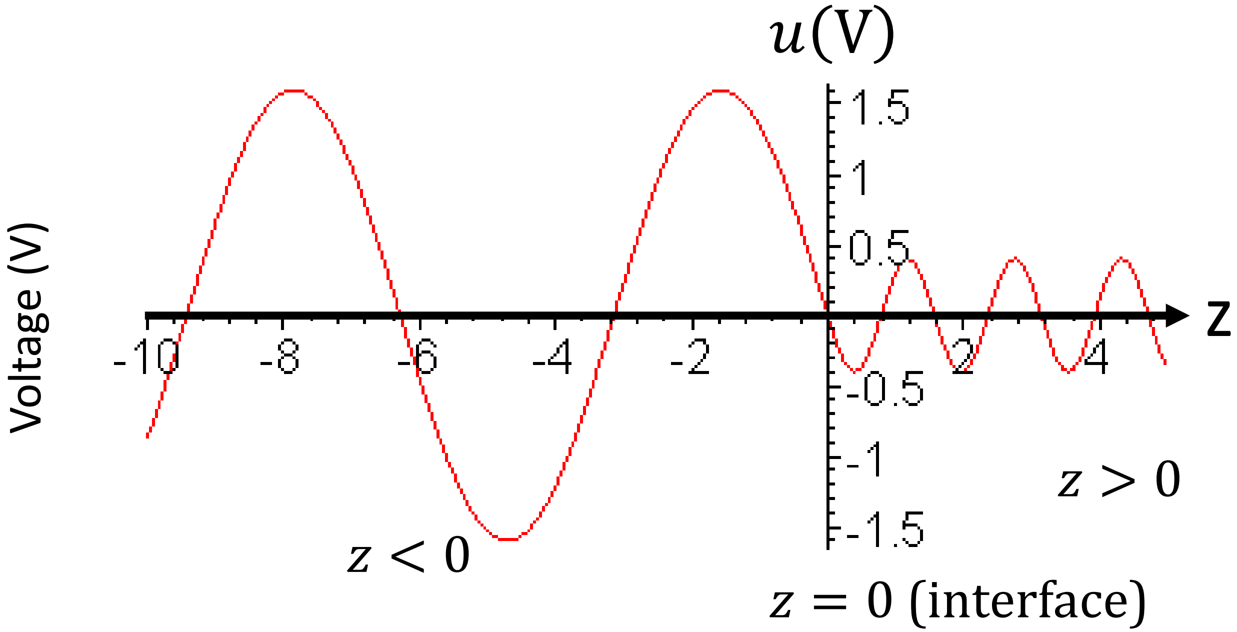
6. I don't know

$$u(z) = \overset{\boxed{+1\text{ V}}}{u^+} e^{-j\beta z} + \overset{\boxed{?}}{u^-} e^{+j\beta z}$$

$$u(z=0) = u^+ e^{-j0} + u^- e^{j0} = u^+ + u^- = 0\text{ V}$$

$$u^- = -u^+ = -1\text{ V}$$

Q3. Interface of two **unequal** characteristic impedances at $z = 0$. What kind of wave takes place in the **region $z < 0$**?



$$u(z \le 0) = u^{+}e^{-\mathrm{j}\beta z} + u^{-}e^{+\mathrm{j}\beta z}$$

transmitted voltage

$$u(z > 0) = \boxed{u^{T}}e^{-\mathrm{j}\beta z}$$

13%. 1. A pure forward (to positive z) propagating wave – i.e., $u^{-} = 0$ V .

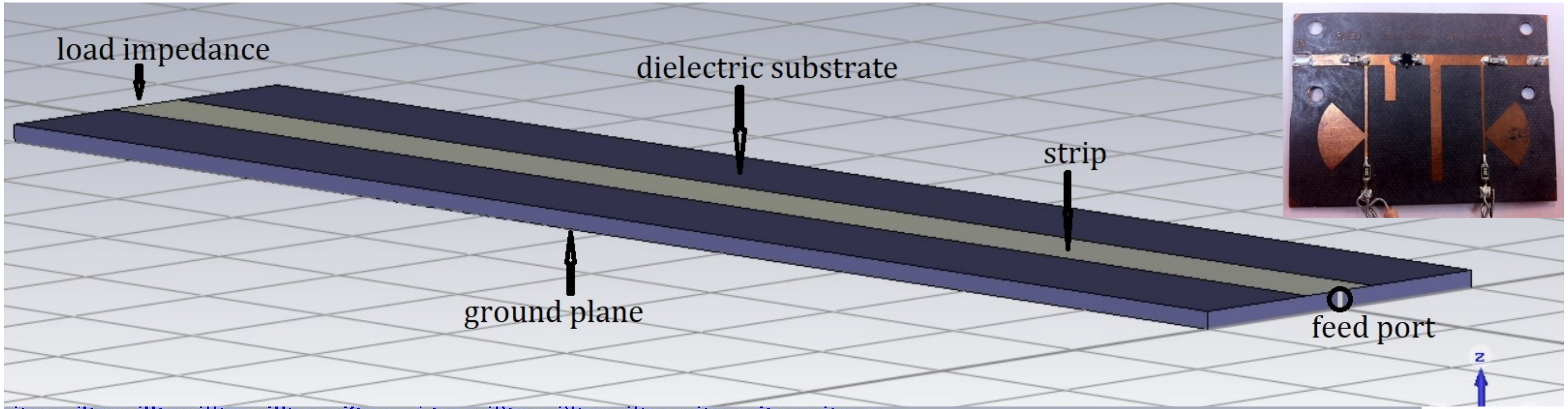
0%. 2. A pure reverse (to negative z) propagating wave – i.e., $u^{+} = 0$ V.

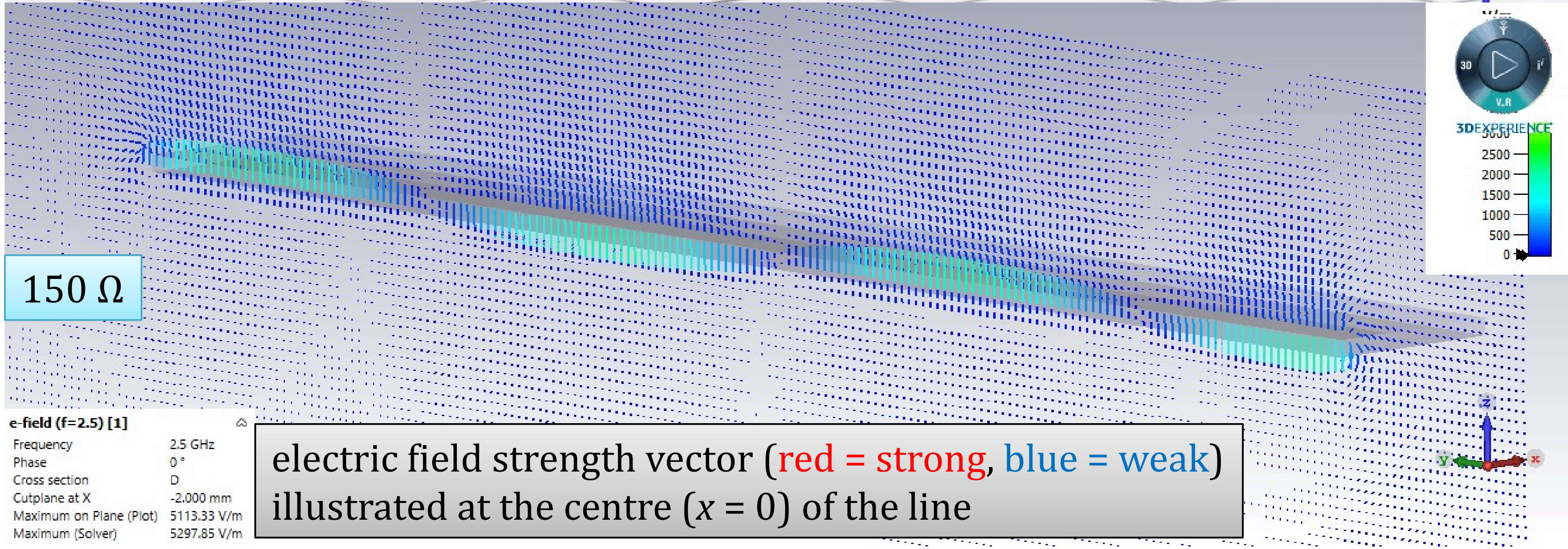
13%. 3. A stationary standing wave – i.e., the net power flow is zero, $|u^{+}| = |u^{-}|$

71%. [✓ 4.] A partial standing wave – i.e., net power flow forward (to positive z), $|u^{+}| > |u^{-}|$.

3%. 5. I don't know

Partial standing wave in a 50-Ω microstrip line – 150 Ω as the load impedance



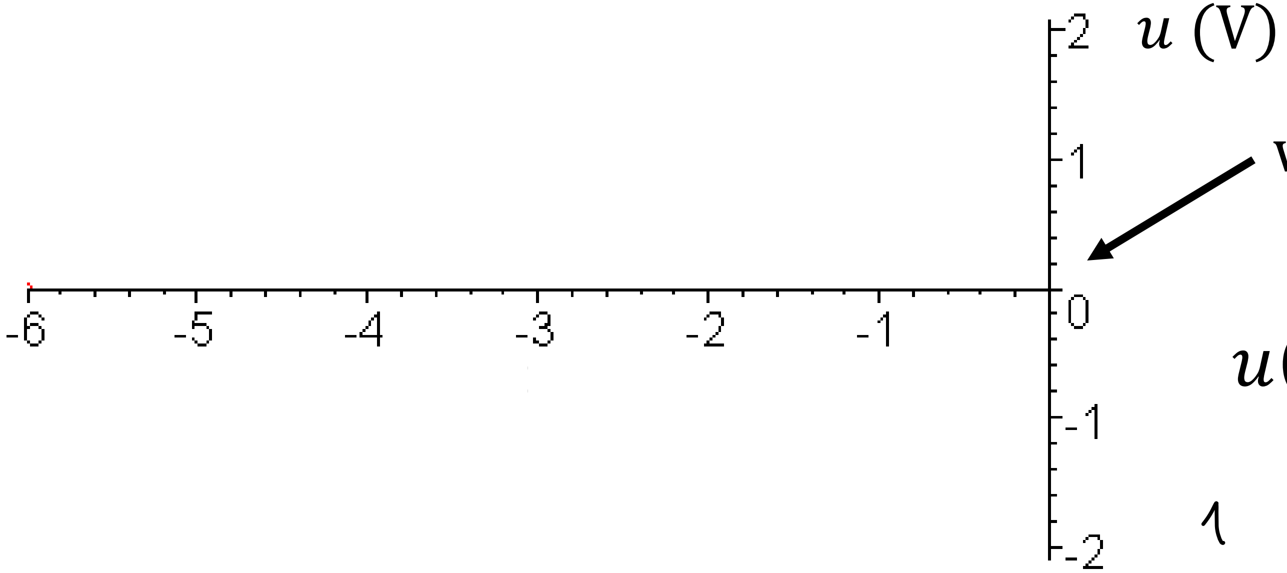


150 Ω

```
e-field (f=2.5) [1]
Frequency                2.5 GHz
Phase                    0 °
Cross section            D
Cutplane at X            -2.000 mm
Maximum on Plane (Plot)  5113.33 V/m
Maximum (Solver)         5297.85 V/m
```

electric field strength vector (red = strong, blue = weak)
illustrated at the centre ($x = 0$) of the line

Voltage reflection coefficient Γ is defined as the ratio of the reflected and incident waves



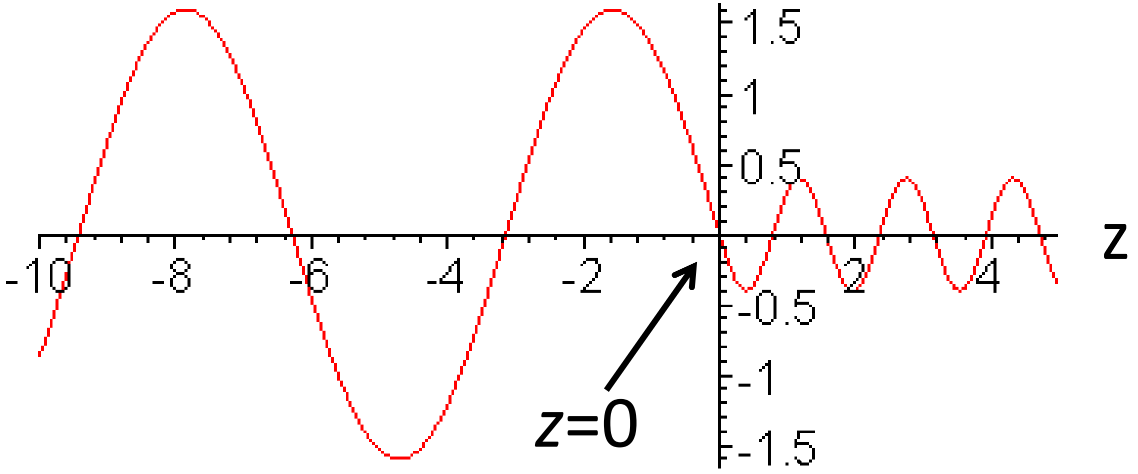
what is Γ here at $z = 0$? $-1\,\mathrm{V}$

$$u(z \leq 0) = \underbrace{\overset{+1\,\mathrm{V}}{u^+} e^{-\mathrm{j}\beta z}}_{\substack{\text{forward} \\ \text{incident}}} + \underbrace{u^- e^{+\mathrm{j}\beta z}}_{\substack{\text{reverse} \\ \text{reflected}}}$$

$$\Gamma = \frac{\text{reflected voltage (at } z = 0)}{\text{incident voltage (at } z = 0)} = \frac{u^- \overbrace{e^{+j\beta\cdot 0}}^{1}}{V^+ \underbrace{e^{-j\beta\cdot 0}}_{1}} = \frac{u^-}{u^+} = \frac{-1\,V}{+1\,V} = -1$$

$\Gamma = -1$ means short circuit

Q4. $u^+ = +1$ V. What is the voltage reflection coefficient Γ at $z = 0$?



$$\boxed{+1\text{ V}}$$

$$u(z \le 0) = u^+ e^{-j\beta z} + u^- e^{+j\beta z}$$

$$u(z > 0) = u^T e^{-j\beta z}$$

$$\boxed{+0.4\text{ V}}$$

$$\boxed{\Gamma = \frac{u^-}{u^+}}$$

3% 1. $\Gamma = 0$

2.vote 9% 2. $\Gamma = -1$

↓ 0% 3. $\Gamma = 1$

21% 35% 4. $0 < \Gamma < 1$

79% 50% [✓ 5.] $-1 < \Gamma < 0$

3% 6. I don't know

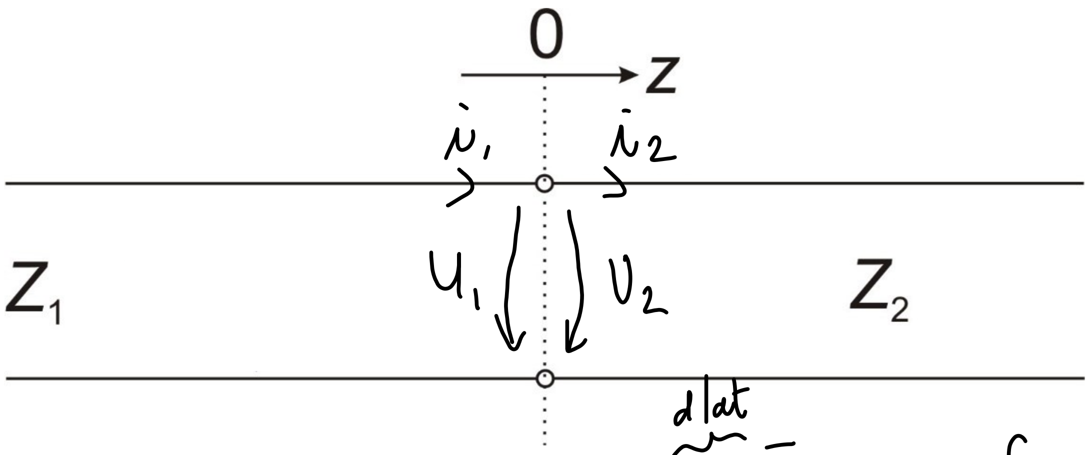
$$U(z=0) = U^+ e^{-j\beta\cdot 0} + U^- e^{+j\beta 0} = U^+ + U^- = 0.4\text{ V}$$

$$U^- = 0{,}4\text{ V} - 1\text{ V}$$

$$U^- = -0{,}6\text{ V}$$

$$\Gamma = \frac{U^-}{U^+} = \frac{-0{,}6\text{ V}}{+1\text{ V}} = -0{,}6 \;(<0)$$

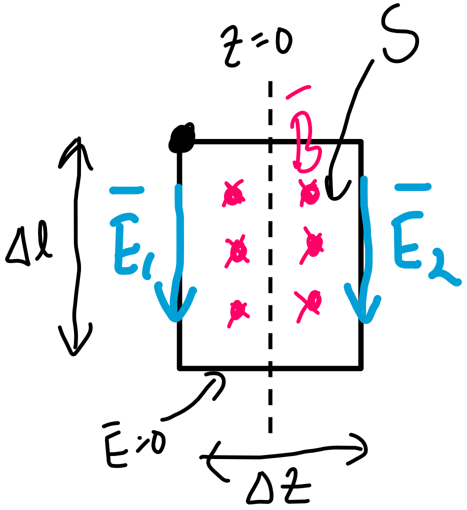
Voltage and current are continuous in the interface (z = 0)



voltage is continuous : $u_1 = u_2$

current — '' — : $\underbrace{i_1 = i_2}_{\text{Kirchhoff's (I) current law}}$

Faraday's law : $\nabla \times \bar{E} = -\overbrace{j\omega}^{d/dt} \bar{B} \Leftrightarrow \oint \bar{E}\cdot d\bar{l} = -j\omega \iint \bar{B}\cdot d\bar{S}$ ↖ select integration path/area freely



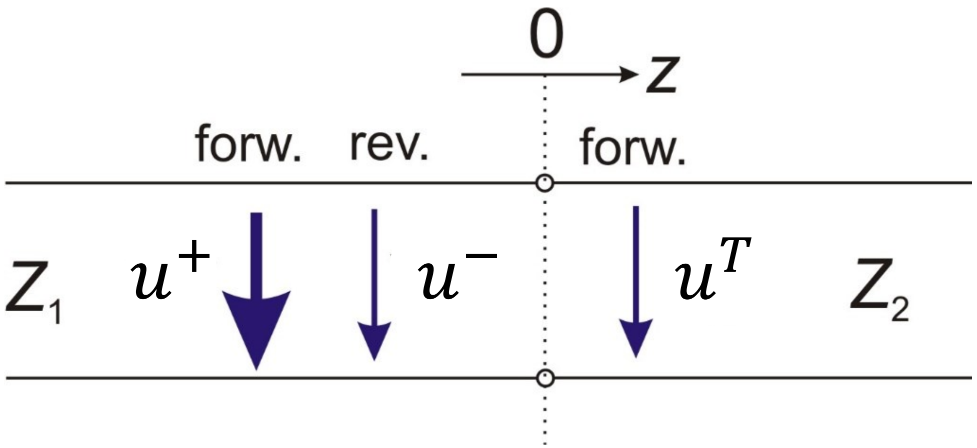
$$\oint \bar{E} d\bar{l} = \qquad -j\omega$$

$$E_1\Delta l - E_2\Delta l \xrightarrow{\Delta z \to 0} \overbrace{E_1\Delta l}^{U_1} - \overbrace{E_2\Delta l}^{U_2} = 0 \Leftarrow \boxed{U_1 = U_2}$$

$$||$$

$$\iint \bar{B} d\bar{S} = B\cdot\Delta z\cdot\Delta l \xrightarrow[S \to 0]{\Delta z \to 0} 0$$

Voltage and current are continuous in the interface (*z* = 0)



$$u(z = 0) = u^+ e^{-j\beta 0} + u^- e^{+j\beta 0} = u^T e^{-j\beta 0}$$

$$I(z = 0) = \frac{u^+}{Z_1} e^{-j\beta 0} \ominus \frac{u^-}{Z_1} e^{+j\beta 0} = \frac{u^T}{Z_2} e^{-j\beta 0}$$

neg. current direction

voltage continuity

$$\begin{cases} u(z=0) = u^+ + u^- = u^T \leftarrow \\ i(z=0) = \frac{u^+}{Z_1} - \frac{u^-}{Z_1} = \frac{u^T}{Z_2} \end{cases}$$

$$\frac{u^-}{u^+} = \Gamma = \text{voltage refl. coeff.}$$

$$\boxed{\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$

$$\boxed{\Gamma = \frac{u^-}{u^+}}$$

Transmission coefficient:

$$T = \frac{u^T}{u^+}$$

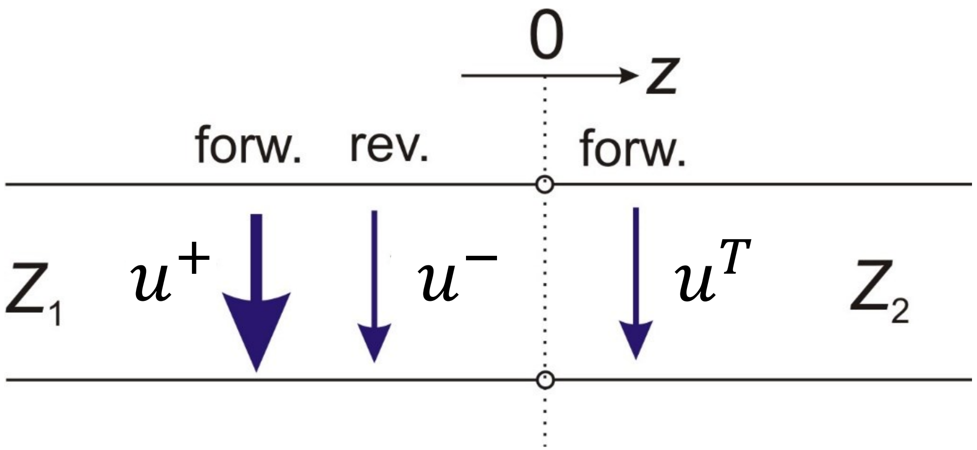
$$T = \frac{U^T}{U^+} = \frac{\text{transmitted voltage}}{\text{incident voltage}} = \frac{U^+ + U^-}{U^+}$$

$$= \frac{U^+}{U^+} + \frac{U^-}{U^+} = \boxed{1 + \Gamma = T}$$

Γ is complex number / phasor

Q5: **Γ = -0.6** and ***V^+* = 1** V. What is the **maximum** instantaneous voltage of the standing wave in **$z \le 0$**?

2 vote ↓



$$u(z \le 0) = u^+ e^{-j\beta z} + u^- e^{+j\beta z}$$
$$u(z > 0) = u^T e^{-j\beta z}$$

$$u^+ = 1\ \mathrm{V}, \Gamma = -0.6, u^T = 0.4\ \mathrm{V}$$

$$\Gamma = \frac{u^-}{u^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$T = \frac{u^T}{u^+} = 1 + \Gamma$$

13% 22% 1. 0.4 V

3% 2. 0.6 V

6% 3. 1 V

9% 4. 1.4 V

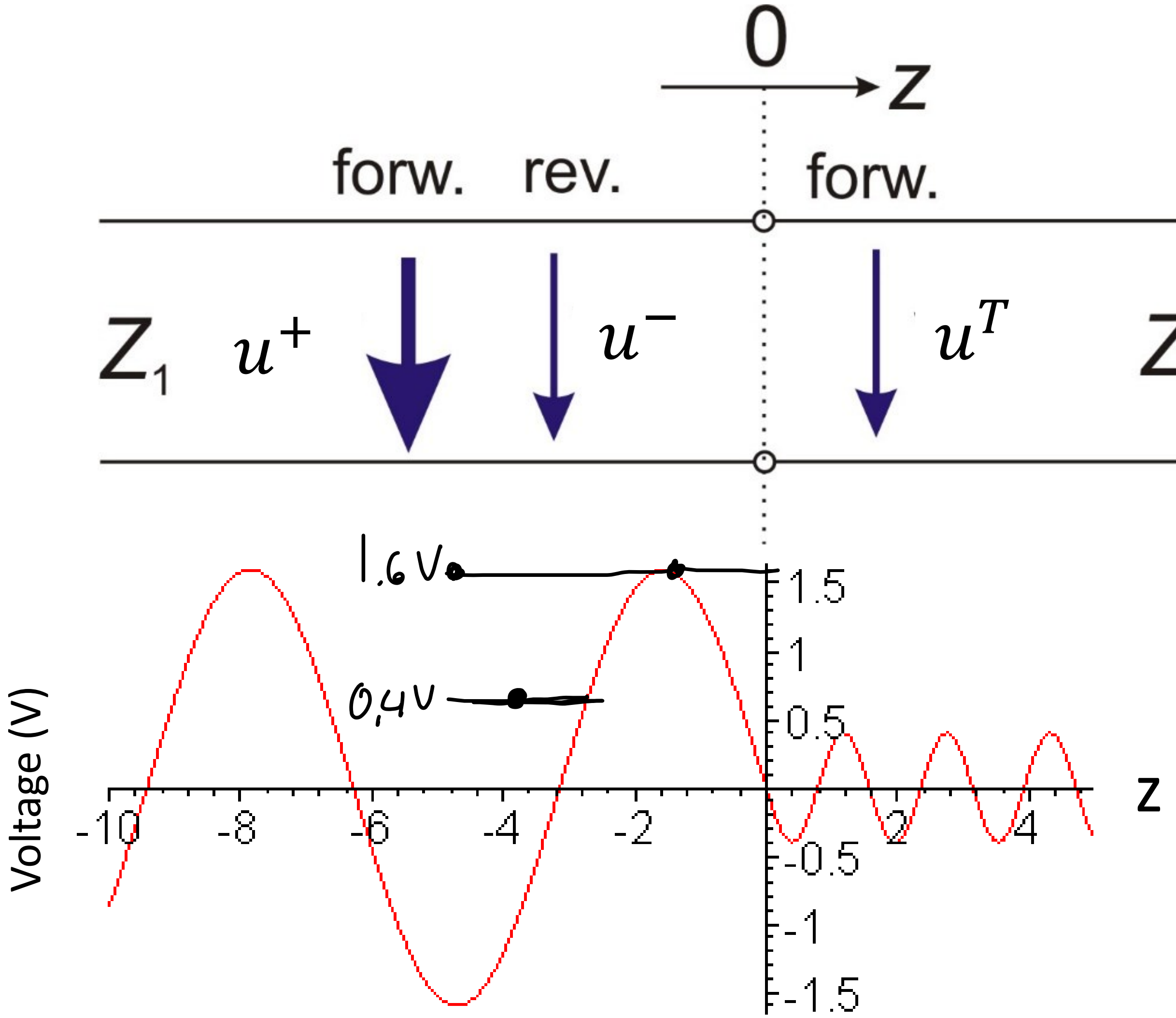
88% 56% [✓ 5.] 1.6 V

3% 6. I don’t know

$$U(z=0) = U^+ + U^- = U^T = 0.4V \Leftrightarrow U^- = -0.6V$$

$$z \le 0 \quad U(z) = 1V \cdot \underbrace{e^{-j\beta z}}_{+1} - 0.6V \underbrace{e^{+j\beta z}}_{-1}$$

$$|U_{max}| = 1V \cdot 1 - 0.6V(-1) = 1V + 0.6V = 1.6V$$



$$u^+ = 1\ \mathrm{V}, u^- = -0.6\ \mathrm{V}$$
$$u^T = 0.4\ \mathrm{V}$$

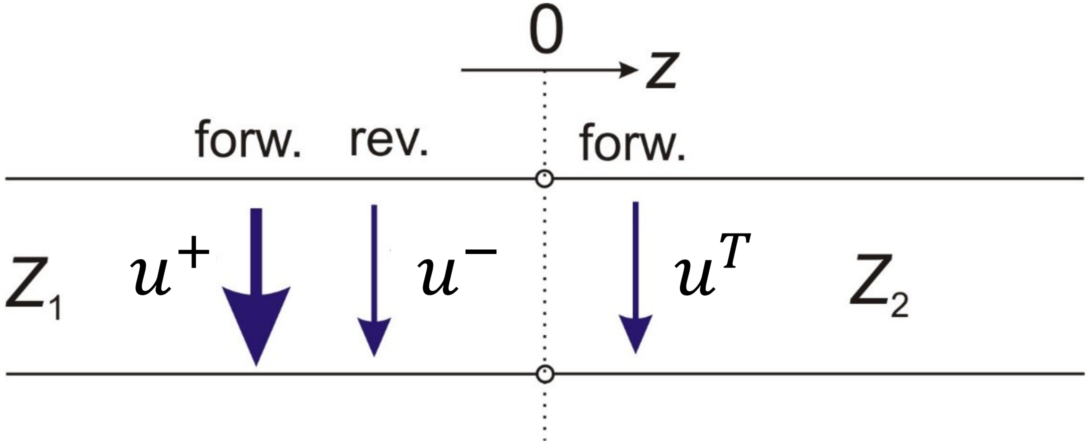
$$\Gamma = \frac{V^-}{V^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$T = \frac{V^T}{V^+} = 1 + \Gamma$$

$$u(z \leq 0) = 1\ \mathrm{V} \cdot e^{-\mathrm{j}\beta z} - 0.6\ \mathrm{V} \cdot e^{+\mathrm{j}\beta z}$$

$$u(z > 0) = 0.4\ \mathrm{V} \cdot e^{-\mathrm{j}\beta z}$$

Envelope of the standing wave has a λ/2 periodicity



$$\boxed{\Gamma = \frac{u^-}{u^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$

$$\boxed{T = \frac{u^T}{u^+} = 1 + \Gamma}$$

$$u(z) = u^+ e^{-j\beta\cdot z} + u^- e^{+j\beta\cdot z} = u^+ e^{-j\beta z} + \Gamma \cdot u^+ e^{-j\beta z} = u^+ e^{-j\beta z}\left(1 + \Gamma e^{j2\beta z}\right)$$

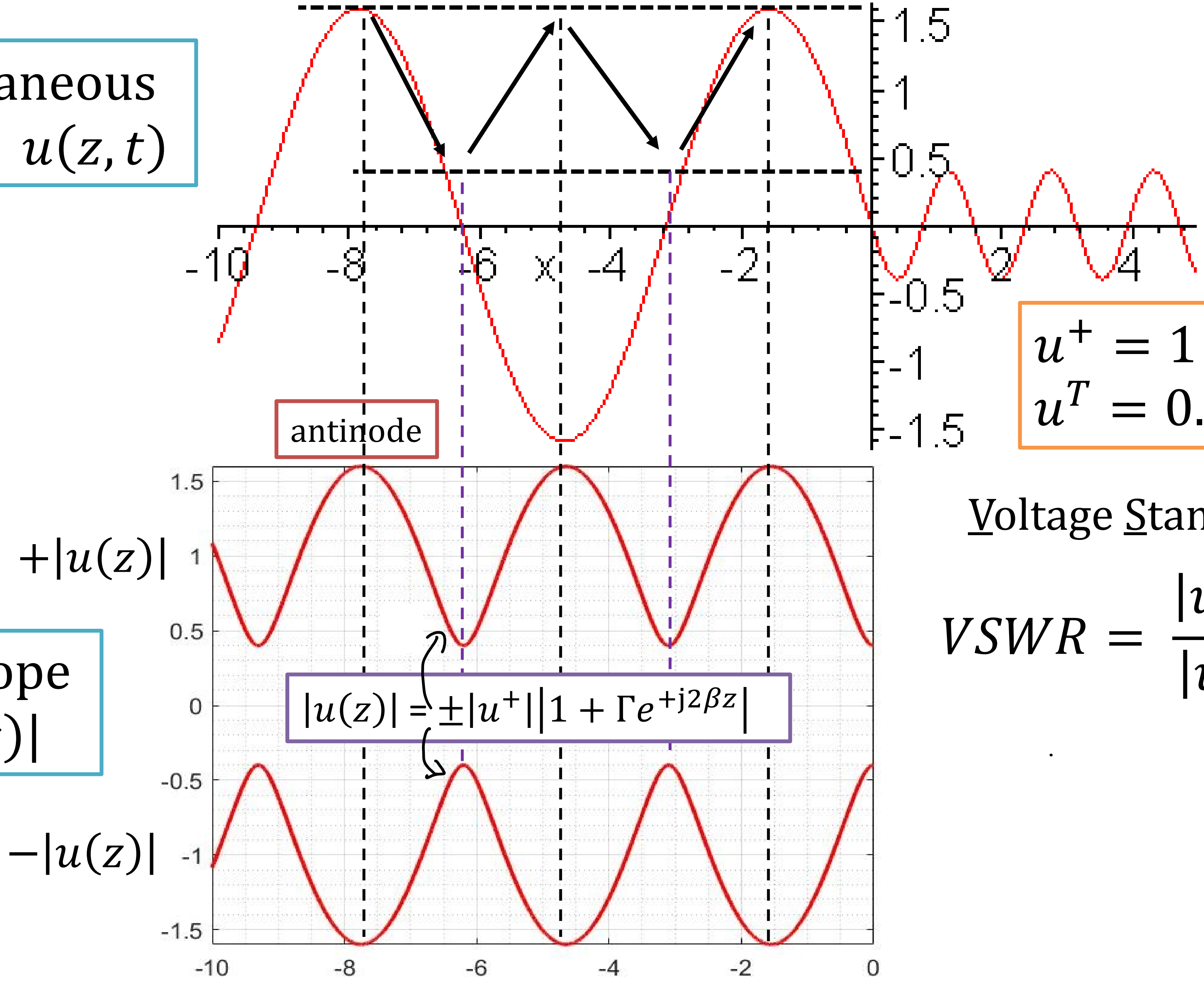
$$|u(z)| = \left|u^+ e^{-j\beta z}\right| \left|1 + \Gamma e^{j2\beta z}\right| = \pm \left|u^+\right| \left|1 + \Gamma e^{j2\beta z}\right|$$

$$i(z) = \frac{u^+}{Z_1} e^{-j\beta\cdot z} - \frac{u^-}{Z_1} e^{+j\beta\cdot z} = \frac{u^+}{Z_1} e^{-j\beta z} - \frac{\Gamma u^+}{Z_1} e^{+j\beta z} = \frac{u^+}{Z_1} e^{-j\beta z}\left(1 - \Gamma e^{j2\beta z}\right)$$

$$|i(z)| = \pm \left|\frac{u^+}{Z_1}\right| \left|1 - \Gamma e^{j2\beta z}\right|$$

instantaneous voltage $u(z,t)$

envelope $\pm|u(z)|$



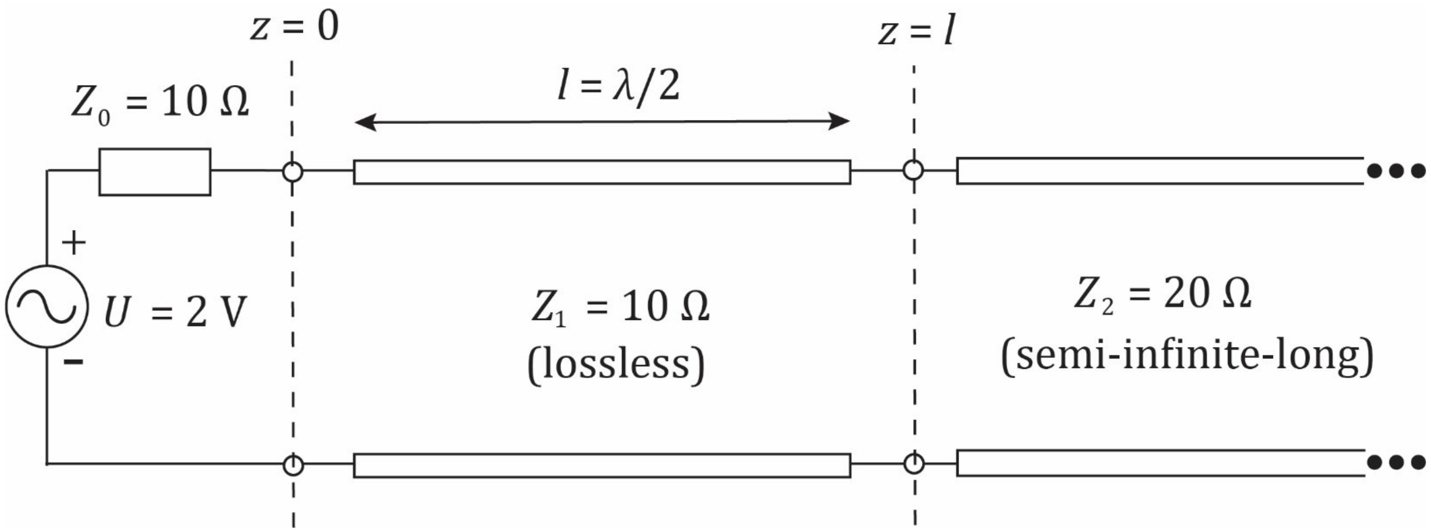
$$|u(z)| = \pm|u^+|\left|1+\Gamma e^{+j2\beta z}\right|$$

$u^+ = 1\text{ V}, u^- = -0.6\text{ V}$
$u^T = 0.4\text{ V}$

Voltage Standing Wave Ratio:

$$VSWR = \frac{|u_{max}|}{|u_{min}|} = \frac{1.6V}{0.4V} = 4$$

In-class task



$$\boxed{u(z) = u^{+}e^{-\mathrm{j}\beta\cdot z} + u^{-}e^{+\mathrm{j}\beta\cdot z}}$$

$$\boxed{I(z) = \frac{u^{+}}{Z_1}e^{-\mathrm{j}\beta\cdot z} - \frac{u^{-}}{Z_1}e^{+\mathrm{j}\beta\cdot z}}$$

$$\boxed{\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$

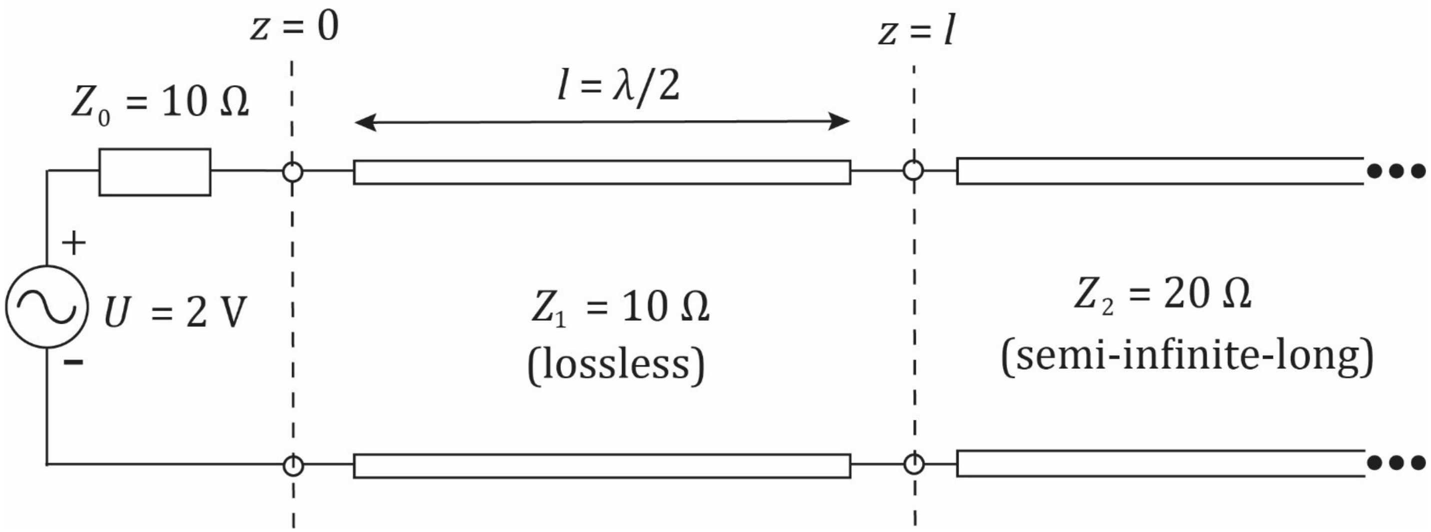
Determine the numerical values of the

1) voltage of the forward-propagating wave u^{+},
2) transmitted voltage amplitude $u^{T} = u(z = l)$ in region $z \geq l$,
3) total voltage $u(z = 0)$ and total current $i(z = 0)$,
4) input impedance $Z(z = 0) = \frac{u(z=0)}{i(z=0)}$ looking into +z direction.
5) Answer the question: why the input impedance $Z(z = 0) \neq Z_1$?

The teachers will be circleing in the breakout rooms for help.

Return your effort in the return box in MyCourses at the end of the session, latest at 12:30 pm.

In-class task



Determine the numerical values of the

1) voltage of the forward-propagating wave u^+,
2) transmitted voltage amplitude $u^T = u(z = l)$ in region $z \geq l$,
3) total voltage $u(z = 0)$ and total current $i(z = 0)$,
4) input impedance $Z(z = 0) = \frac{u(z=0)}{i(z=0)}$ looking into +z direction.
5) Answer the question: why the input impedance $Z(z = 0) \neq Z_1$?

1) At $z = 0$, the forward-propagating wave sees only the impedance Z_1 of the first transmission line. Hence, we have a voltage division of U between Z_1 and Z_0. See further details in Chapter 2.8 ("Transients of transmission lines"). $u^+ = U\frac{Z_1}{Z_0+Z_1} = 1$ V

2) First, we determine the value of the voltage reflection coefficient at $z = l$: $\Gamma = \frac{u^- e^{+j\beta \cdot l}}{u^+ e^{-j\beta \cdot l}} = \frac{u^- e^{+j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}}{u^+ e^{-j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}} = \frac{u^- e^{+j\pi}}{u^+ e^{-j\pi}} = \frac{u^-(-1)}{u^+(-1)} = \frac{u^-}{u^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{1}{3}$.

   Then we can calculate the voltage $u^T = u\left(z = \frac{\lambda}{2}\right) = u^+ e^{-j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}} + u^- e^{+j\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}} = u^+ e^{-j\pi} + \Gamma u^+ e^{+j\pi} = u^+\left(-1 - \frac{1}{3}\cdot 1\right) = -\frac{4}{3}u^+ = -\frac{4}{3}$ V

3) Total voltage at z = 0, $u(z = 0) = u^+ e^{-j0} + u^- e^{+j0} = u^+ + \Gamma u^+ = u^+(1 + \Gamma) = \frac{4}{3}$ V

   Total current at z = 0, $i(z = 0) = \frac{u^+}{Z_1} e^{-j0} - \frac{u^-}{Z_1} e^{+j0} = \frac{u^+}{Z_1} - \Gamma\frac{u^+}{Z_1} = \frac{u^+}{Z_1}(1 - \Gamma) = \frac{1}{15}$ A

4) The input impedance looking into +z direction $Z(z = 0) = \frac{u(z=0)}{i(z=0)} = \frac{\frac{4}{3}\text{ V}}{\frac{1}{15}\text{ A}} = 20\ \Omega$.

5) The total input impedance is affected by the reflected wave, too. This is more discussed in the next lecture on 27.1.