Quiz 1 (slides 1):

1 a) Explain what is meant by affine equivariance in the context of statistics?

1 b) Give the definition of a location functional.

1 c) Give the definition of a scatter functional.

1 d) Is the covariance matrix affine equivariant (in the sense that scatter functionals are required to be)?

1 e) Is the expected value vector (the population mean vector) affine equivariant (in the sense that location functionals are required to be)?

1 f) Is the component wise multivariate median affine equivariant (in the sense that location functionals are required to be)?

1 g) What do we know about (proper affine equivariant) location functionals under central symmetry?

1 h) What do we know about (proper affine equivariant) scatter functional under multivariate ellipticity?

Quiz 2 (slides 2):

Assume that you have a p-variate random vector x. Assume that E(x) and Cov(x) exist as finite quantities. Assume that you have applied principal component transformation to x and obtained y. (That is, y is the random vector containing the principal components of x.)

2 a) How do you perform the transformation in practice?

2 b) What do you know about the correlation between the elements of y?

2 c) What do you know about Cov(y) and about the variances of the elements of y?

2 d) You goal is dimension reduction and you wish to keep enough components so that the components explain at least 90% of the variation (variance). How many components do you keep?

2 e) Is the principal component transformation uniquely defined?

Answers:

1 a) A functional or the corresponding estimator is called affine equivariant if it adapts naturally to affine transformations (changes in the coordinate system). What is "natural" depends on the functional at hands and it means different things for, e.g., location and scatter.

1 b) See page 15 of the slides 1.

1 c) See page 16 of the slides 1.

1 d) Yes, it is.

1 e) Yes, it is.

1 f) No it is not. However, there exists affine equivariant multivariate medians.

1 g) See pages 32 and 33 of the slides 1.

1 h) See pages 34-37 of the slides 1.

2 a) Calculate E[x] and the eigenvalues and orthonormal eigenvectors of Cov(x). Now y=G^T(x-u), where u=E[x] and the column vectors of G are the orthonormal eigenvectors of Cov(x) ordered such that the corresponding eigenvalues are in decreasing order.

2 b) The principal components are not correlated. The correlation coefficients are equal to zero.

2 c) See pages 7-11 of the slides 2.

2 d) Take a look at the eigenvalues. See page 13 of the slides 2.

2 e) No, the transformation is not uniquely defined. If the eigenvalues are distinct, then the transformation is uniquely defined up to the signs of the eigenvectors (column vectors of the matrix gamma). If there are multiple eigenvalues, then the transformation is far away from unique. However, there always exists orthonormal eigenvectors as covariance matrix is positive definite. Moreover, if the underlying distribution is continuous, and we consider the sample case, then, with probability 1, we do not have repeated eigenvalues. Note that even then some (or even all) of the eigenvalues of the population covariance matrix may be multiple. (Think about the case when Cov(x) is the identity matrix.)