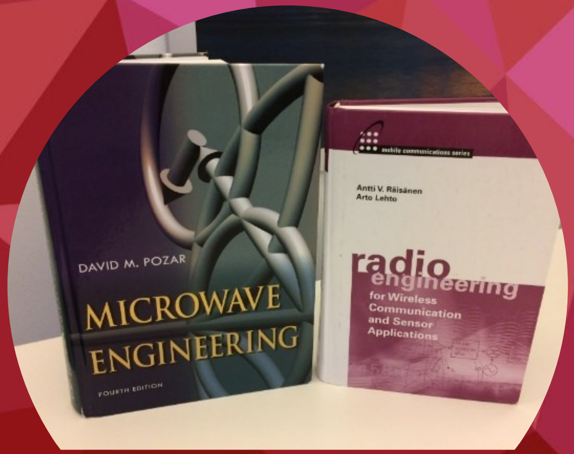
Microwave engineering I (MiWE I)

Interactive lecture 1 of Topic 2

The Smith chart and impedance matching

January 27, 2022

The main learning outcome of the course is to create readiness to work in microwave engineering related tasks and projects and enable further studies and continuous learning in microwave engineering.



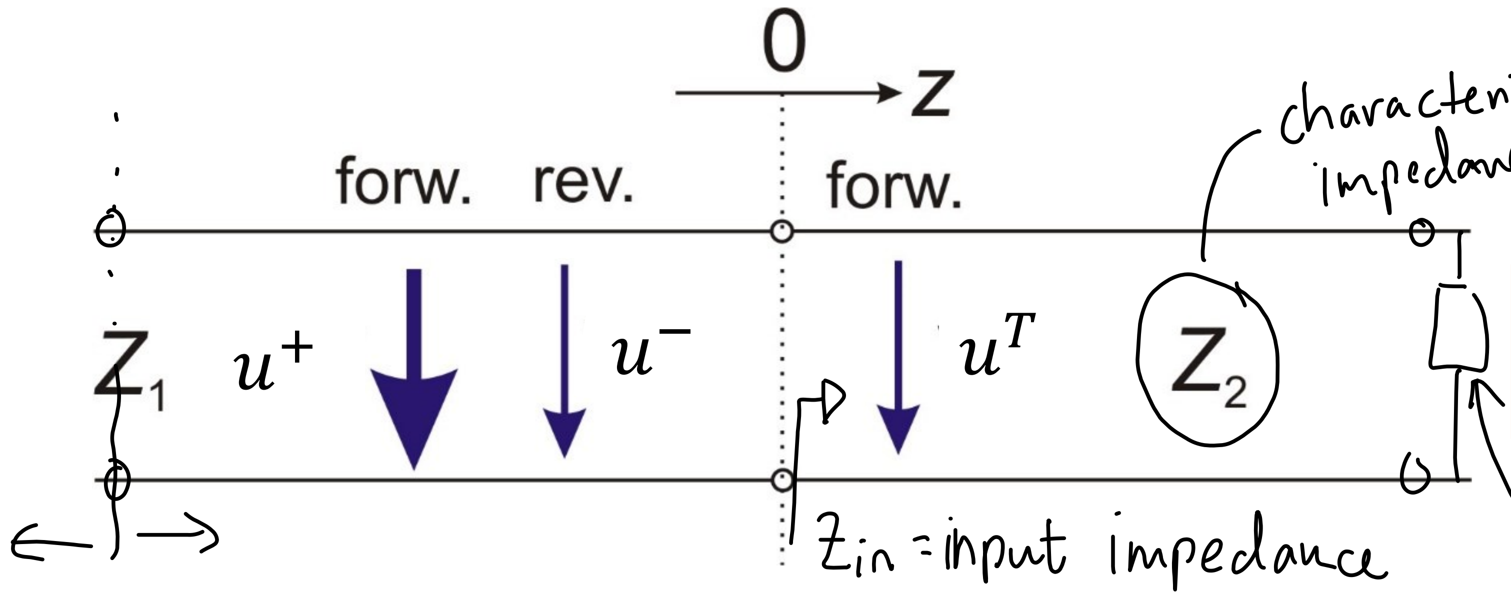
Topic 2: Learning outcomes and content

- The student can
  - **design** impedance matching circuits using the Smith chart and a simulator tool (AWRDE)
  - **explain** the design principles and bandwidth issues related to impedance matching.

- The terminated mismatched load impedance (Pozar Chapter 2.3) TODAY
- The Smith chart (Pozar Chapter 2.4)
- The quarter-wave transformer (Pozar Chapter 2.5 and Chapter 5.4) ← IN-CLASS TASK
- Matching with lumped elements (Pozar Chapter 5.1) ← SEE VIDEO IN MYCOURSES
- Single-stub tuning (Pozar Chapter 5.2)
- The Bode-Fano criterion (Pozar Chapter 5.9)

These lecture slides and notes are not designed for self-study.
Please, use the course book chapters 2 and 5 for self-study.

Recap of the last week session on one slide



At z =0, the total voltage and current are **continuous**!

$$\Gamma(z=0) = \frac{\text{reflected voltage at } z=0}{\text{incident voltage at } z=0} = \frac{u^-}{u^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$T(z=0) = u^T/u^+ = 1 + \Gamma$$

$$\rightarrow |u(z)| = |u^+e^{-j\beta z} + \underset{\uparrow\, \Gamma u^+}{u^-} e^{+j\beta z}| = \underset{\uparrow\ \text{real number}}{|u^+e^{-j\beta z}|}\,|1+\Gamma e^{+j2\beta z}| = |u^+|\,\overbrace{|1+\Gamma e^{+j2\beta z}|}$$

Q1: An example of the envelope $|u(z)|$ of a standing wave is shown in the figure. What is the period of the envelope – i.e., distance l between two adjacent maxima? λ = wavelength in the line

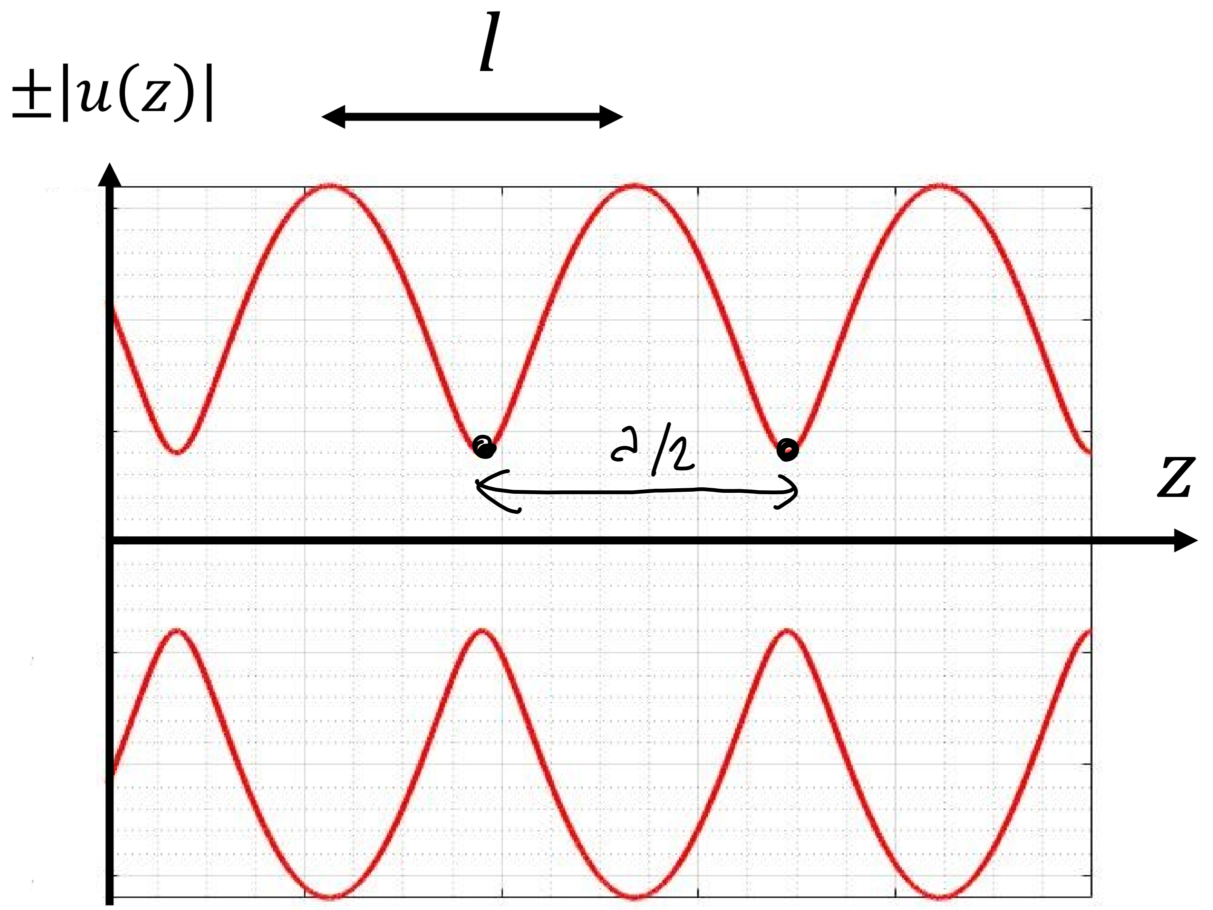
$$\boxed{|u(z)| = \pm|u^+|\left|1 + \Gamma e^{+j2\beta z}\right|}$$

1. $l = \frac{\lambda}{8}$
2. $l = \frac{\lambda}{4}$ — 19%
3. $l = \frac{\lambda}{2}$ — 52%
4. $l = \frac{2\lambda}{3}$ — 3%
5. $l = \lambda$ — 19%
6. I don't know — 6%

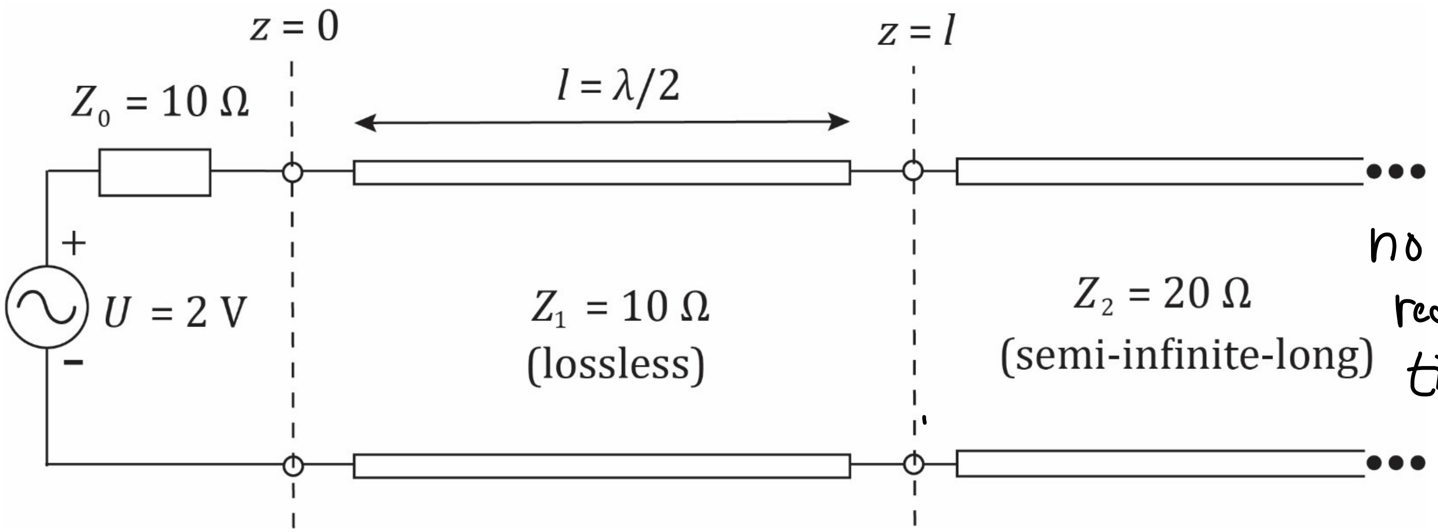
$$e^{j2\beta z} = e^{j2\cdot\frac{2\pi}{\lambda}z}$$

$$= e^{j\frac{2\pi}{\frac{\lambda}{2}}z}$$

period $\frac{\lambda}{2}$



Previous in-class task



no reflec- tion

Determine the numerical values of the

1) voltage of the forward-propagating wave u^+,
2) transmitted voltage amplitude u^T in region $z \geq l$,
3) total voltage $u(z=0)$ and total current $i(z=0)$,
4) input impedance $Z(z=0)=\frac{u(z=0)}{i(z=0)}$ looking into +z direction.
5) Answer the question: why the input impedance $Z(z=0) \neq Z_1$?

1) At z = 0, the forward-propagating wave sees only the impedance Z_1 of the first transmission line. Hence, we have a voltage division of U between Z_1 and Z_0. See further details in Chapter 2.8 ("Transients of transmission lines"). $u^+ = U\frac{Z_1}{Z_0+Z_1} = 1\ \text{V}$

2) First, we determine the value of the voltage reflection coefficient at $z = l$: $\Gamma = \frac{u^- e^{+\mathrm{j}\beta\cdot l}}{u^+ e^{-\mathrm{j}\beta\cdot l}} = \frac{u^- e^{+\mathrm{j}\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}}{u^+ e^{-\mathrm{j}\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}} = \frac{u^- e^{+\mathrm{j}\pi}}{u^+ e^{-\mathrm{j}\pi}} = \frac{u^-(-1)}{u^+(-1)} = \frac{u^-}{u^+} = \frac{Z_2-Z_1}{Z_2+Z_1} = \frac{1}{3}$.

   Then we can calculate the voltage $u^T = u\left(z=\frac{\lambda}{2}\right) = u^+ e^{-\mathrm{j}\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}} + u^- e^{+\mathrm{j}\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}} = u^+ e^{-\mathrm{j}\pi} + \Gamma u^+ e^{+\mathrm{j}\pi} = u^+[-1+\Gamma(-1)] = 1\ \text{V}\cdot\left[-1-\frac{1}{3}\right] = -\frac{4}{3}\ \text{V}$

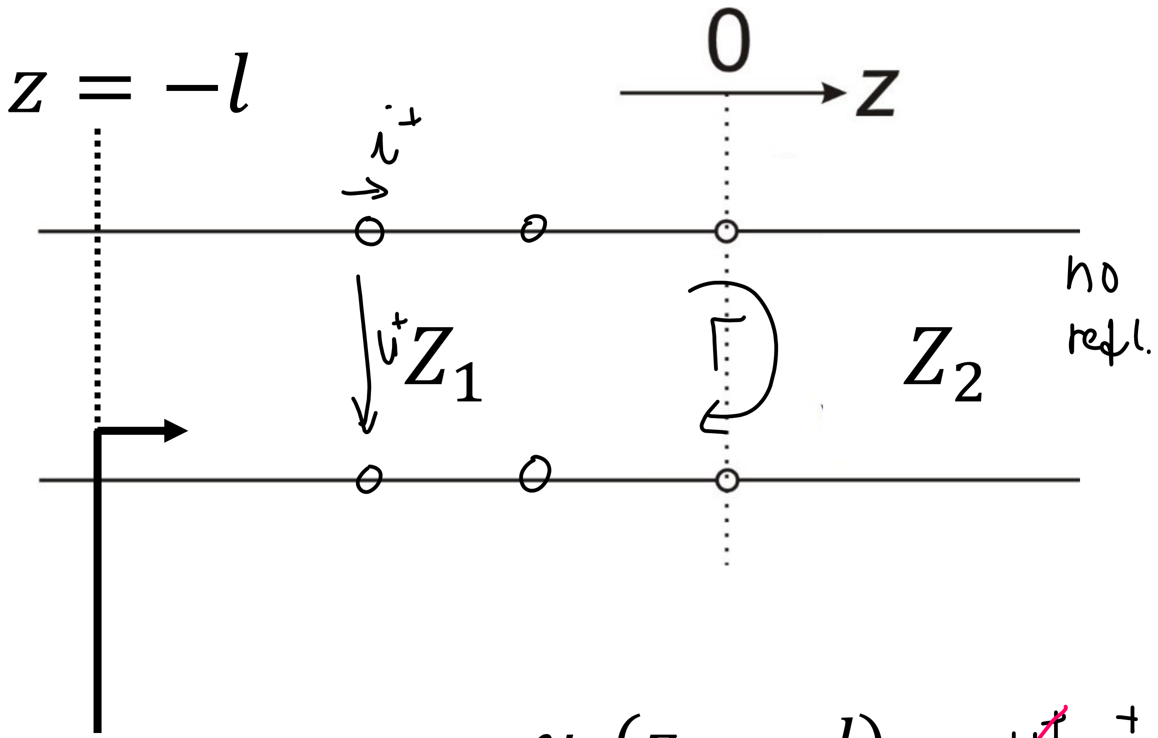
3) Total voltage at z = 0, $u(z=0) = u^+ e^{-\mathrm{j}0} + u^- e^{+\mathrm{j}0} = u^+ + \Gamma u^+ = u^+(1+\Gamma) = \frac{4}{3}\ \text{V}$

   Total current at z = 0, $i(z=0) = \frac{u^+}{Z_1}e^{-\mathrm{j}0} - \frac{u^-}{Z_1}e^{+\mathrm{j}0} = \frac{u^+}{Z_1} - \Gamma\frac{u^+}{Z_1} = \frac{u^+}{Z_1}(1-\Gamma) = \frac{1}{15}\ \text{A}$

4) The input impedance looking into +z direction $Z(z=0) = \frac{u(z=0)}{i(z=0)} = \frac{\frac{4}{3}\ \text{V}}{\frac{1}{15}\ \text{A}} = 20\ \Omega$.

5) The total input impedance is affected by the reflected wave, too. This is more discussed in the next lecture on 27.1.

Input impedance is affected by the reflected wave



$$\boxed{u_1(z) = u^+ e^{-j\beta z} + u^+ \Gamma e^{j\beta z}}$$

$$\boxed{i_1(z) = \frac{u^+}{Z_1} e^{-j\beta z} - \frac{u^+\Gamma}{Z_1} e^{j\beta z}}$$

$$\boxed{\Gamma(z=0) = \frac{\text{reflected voltage}}{\text{incident voltage}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$

$e^{-j\beta l}$

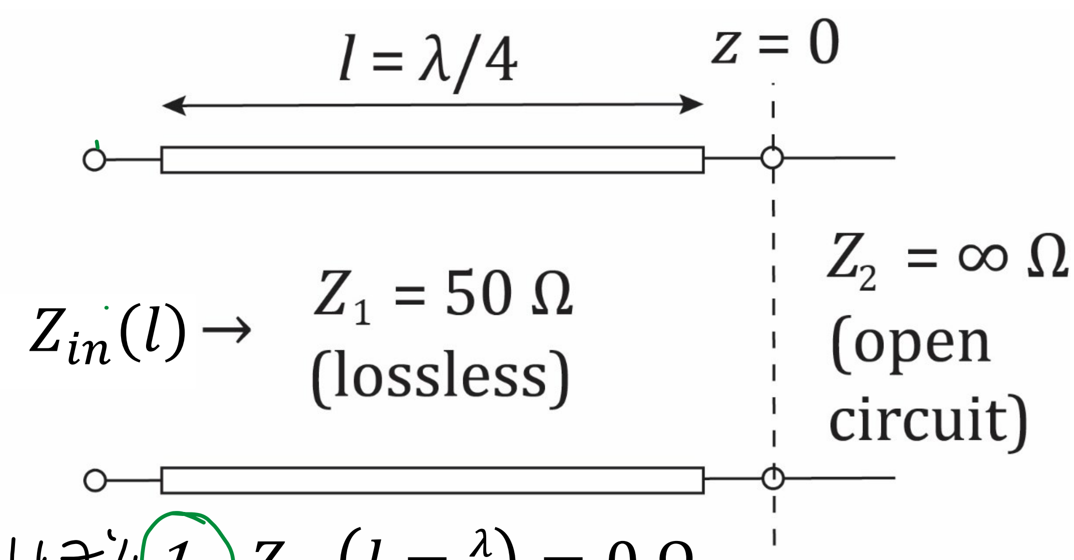
$$Z_{in}(z=-l) = \frac{u_1(z=-l)}{I_1(z=-l)} = \frac{\cancel{u^+} e^{+j\beta l} + \cancel{u^+}\Gamma e^{-j\beta l}}{\frac{\cancel{u^+}}{Z_1} e^{+j\beta l} - \frac{\cancel{u^+}}{Z_1}\Gamma e^{-j\beta l}} = Z_1 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$$

If reflection takes place $\Gamma \neq 0 \Rightarrow Z_{in} \neq Z_1$

IF $\Gamma = 0 \Rightarrow Z_{in} = Z_1$

INPUT IMPEDANCE AFFECTED BY THE REFLECTION

Q2: A **λ/4-long** line is terminated with open circuit – i.e., $Z_2 = \infty\ \Omega$. What is the input impedance Z_{in}?



47% 1. $Z_{in}\left(l = \frac{\lambda}{4}\right) = 0\ \Omega$

9% 2. $Z_{in}\left(l = \frac{\lambda}{4}\right) = \infty\ \Omega$

24% 3. $Z_{in}\left(l = \frac{\lambda}{4}\right) = Z_1$

6% 4. $Z_{in}\left(l = \frac{\lambda}{4}\right) = +jZ_1$

6% 5. $Z_{in}\left(l = \frac{\lambda}{4}\right) = -jZ_1$

9% 6. I don't know

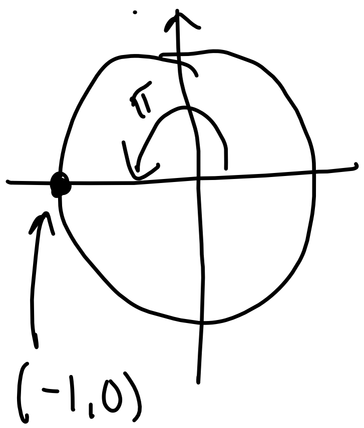
$$\Gamma(l=0) = \frac{Z_2 - Z_1}{Z_2 + Z_1} = 1 = \frac{\frac{Z_2}{Z_2} - \frac{Z_1}{Z_2}}{\frac{Z_2}{Z_2} + \frac{Z_1}{Z_2}} = \frac{1 - \overbrace{\frac{Z_1}{Z_2}}^{\to 0}}{1 + \underbrace{\frac{Z_1}{Z_2}}_{\to 0}} = 1$$

$Z_2 \to \infty$

$$\boxed{\beta = \frac{2\pi}{\lambda}} \qquad \boxed{Z_{in}(l) = Z_1 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}}$$

$$e^{-j2\beta l} = e^{-j\cancel{2}\cdot\frac{\cancel{2}\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{4}}} = e^{-j\pi}$$

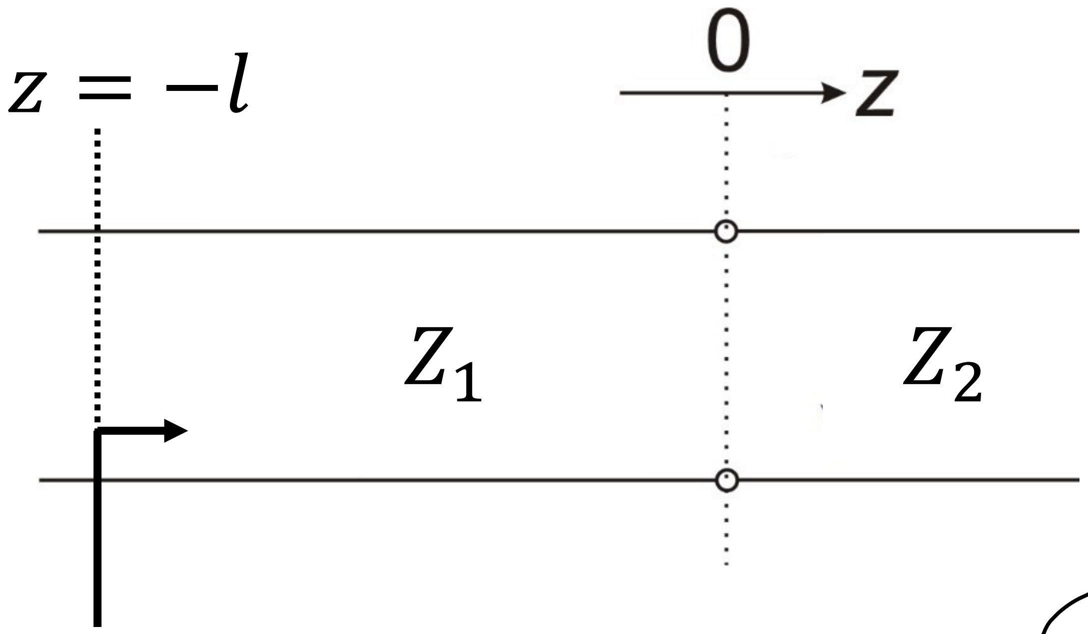
$$= -1$$



$$Z_{in} = Z_1 \frac{1 + 1\cdot(-1)}{1 - 1(-1)} = Z_1 \frac{0}{2}$$

$$Z_{in} = 0$$

Input reflection coefficient is affected by the reflected wave



$$\boxed{\Gamma(z=0) = \frac{\text{reflected voltage}}{\text{incident voltage}} = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$

$$\boxed{u_1(z) = \underbrace{u^+ e^{-j\beta z}}_{\text{incident voltage wave}} + \underbrace{\overbrace{u^+\Gamma}^{u} e^{j\beta z}}_{\text{reflected voltage wave}}}$$

$$\Gamma(z=-l) = \frac{\text{reflected voltage at } z=-l}{\text{incident voltage at } z=-l} = \frac{\cancel{u^+}\,\Gamma e^{-j\beta l}}{\cancel{u^+}\, e^{+j\beta l}} = \underbrace{\Gamma e^{-j2\beta l}}_{\text{phasor = complex number}}$$

REFLECTION COEFFICIENT IS A PHASOR

Q3: Which is the location of the voltage reflection coefficient Γ on the complex plane?

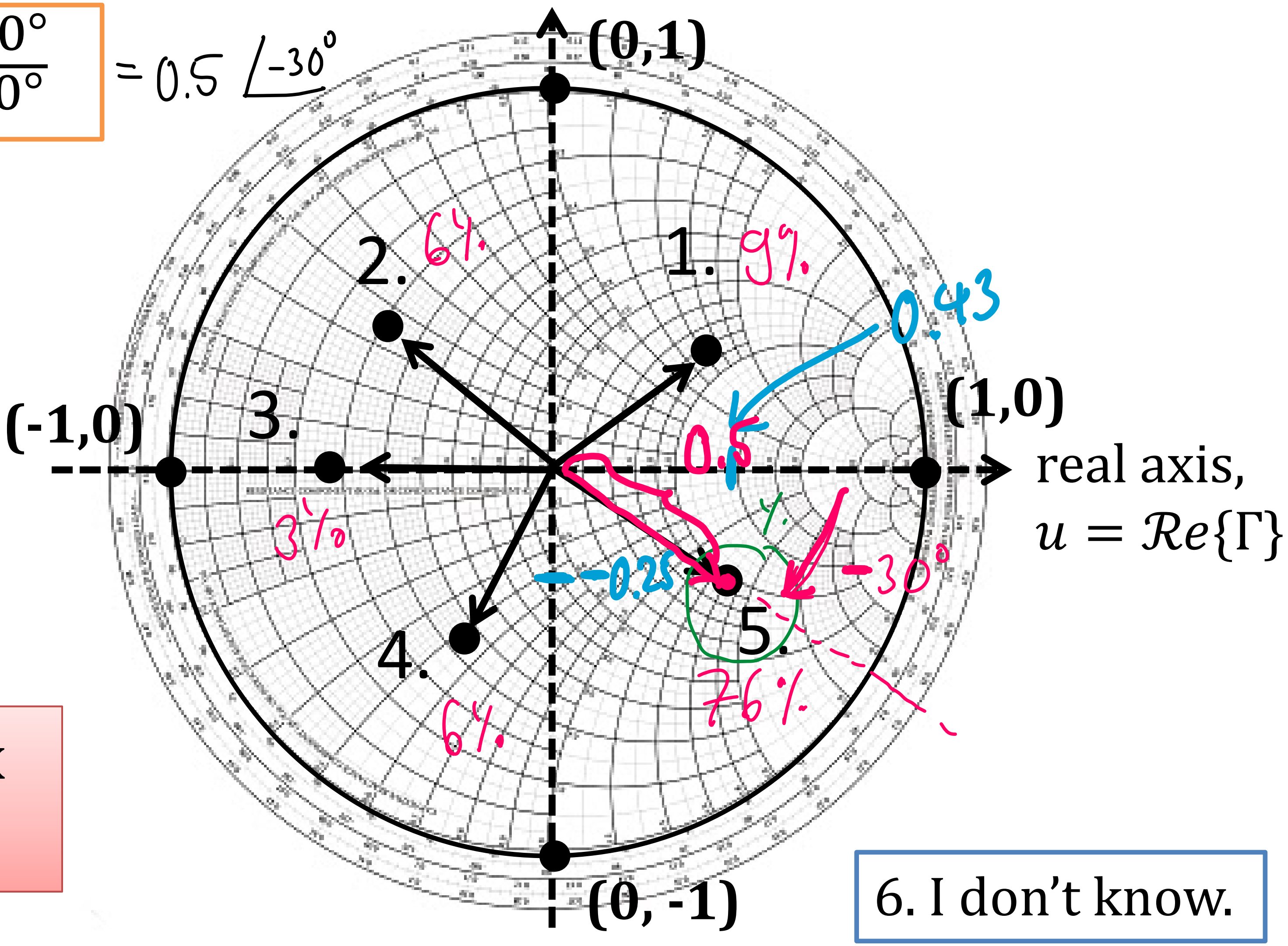
$$\Gamma = 0.5e^{-j\frac{\pi}{6}} = 0.5e^{j2\pi\cdot\frac{-30^\circ}{360^\circ}} = 0.5\angle -30^\circ$$

$$\Gamma = 0.5\left[\cos\left(-\frac{\pi}{6}\right) + j\sin\left(-\frac{\pi}{6}\right)\right]$$

$$= 0.5\left(\cos\frac{\pi}{6} - j\sin\frac{\pi}{6}\right)$$

$$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - j\frac{1}{4} \approx 0.43 - j\,0.25$$

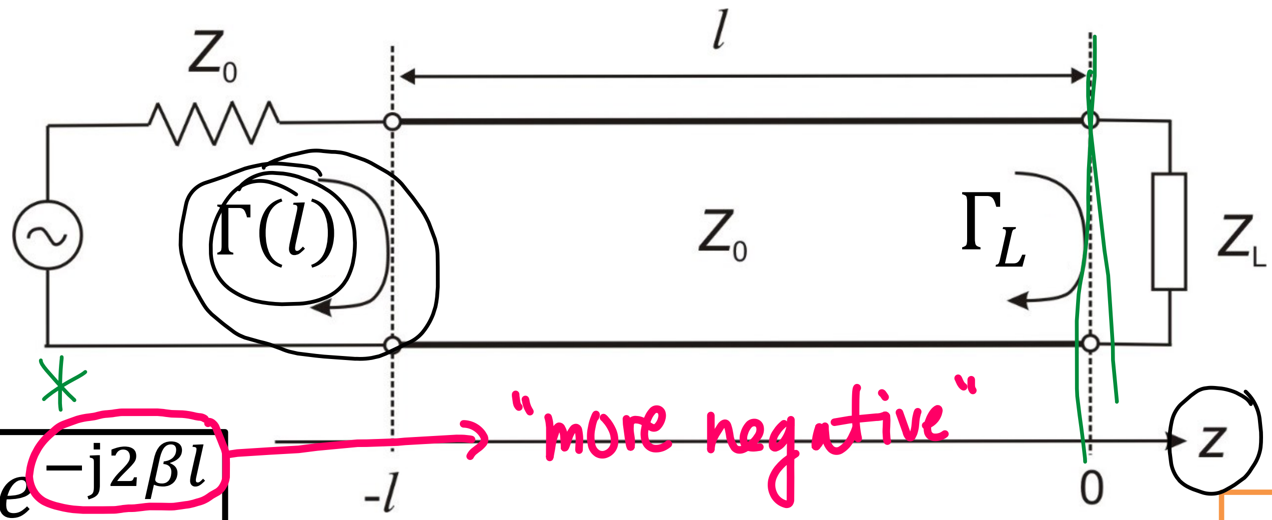


The Smith chart is a complex reflection coefficient plane.

6. I don't know.

Q4: We start from the load (at z=0) and move **towards generator** distance l, starting from Γ_L, the reflection coeffient $\Gamma(l)$ on the complex plane rotates …

generator here!

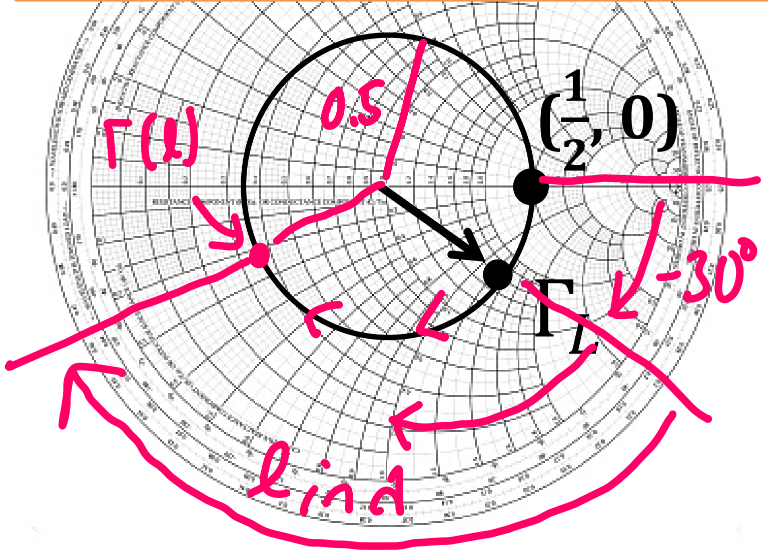


load here!

$$\Gamma_L = \Gamma(z=0) = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\boxed{\Gamma(l) = \Gamma_L e^{-j2\beta l}}$$

$$\Gamma_L = 0.5e^{-j\frac{\pi}{6}} = 0.5e^{-j2\pi\cdot\frac{30°}{360°}}$$



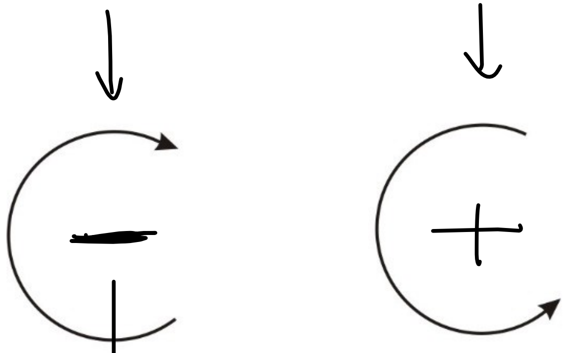
69% 1. clockwise

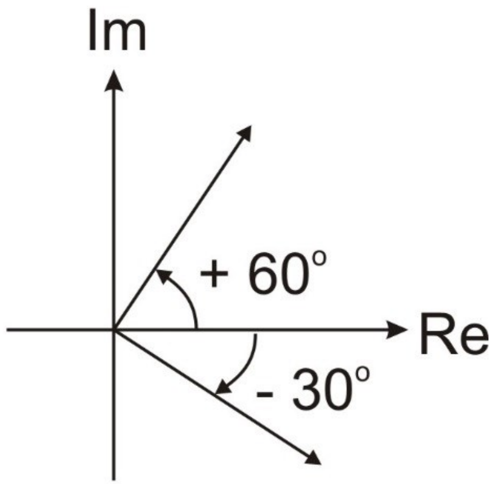
28% 2. anticlockwise (= counterclockwise)

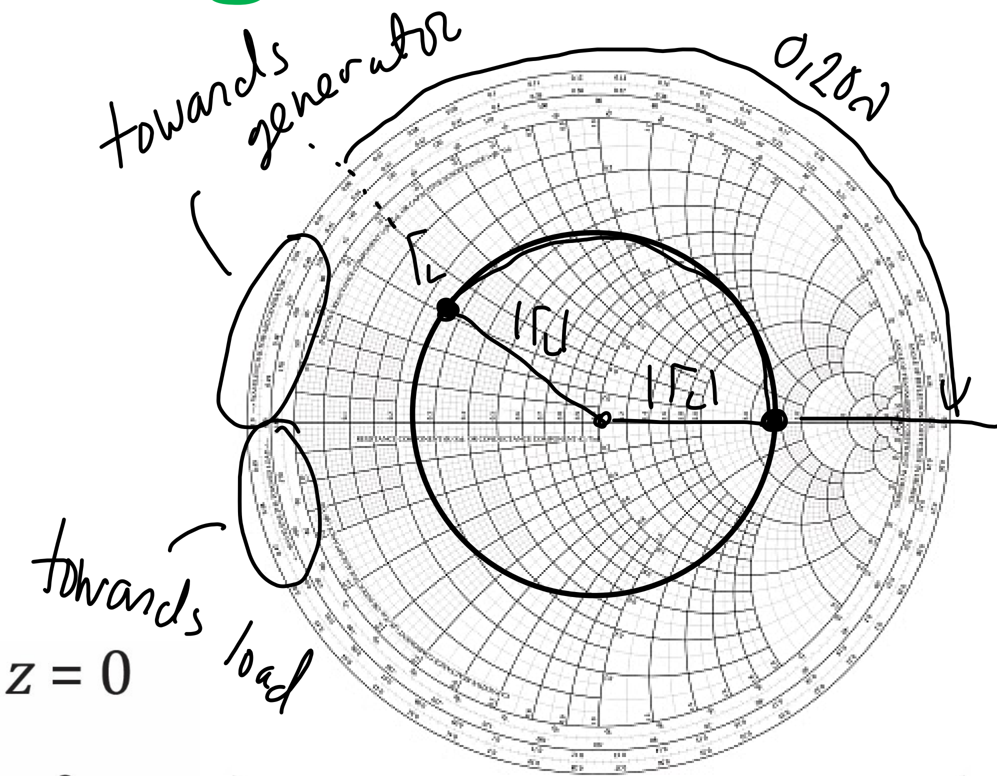
3% 3. It cannot be determined

4. I don't know

Towards generator is rotating clockwise

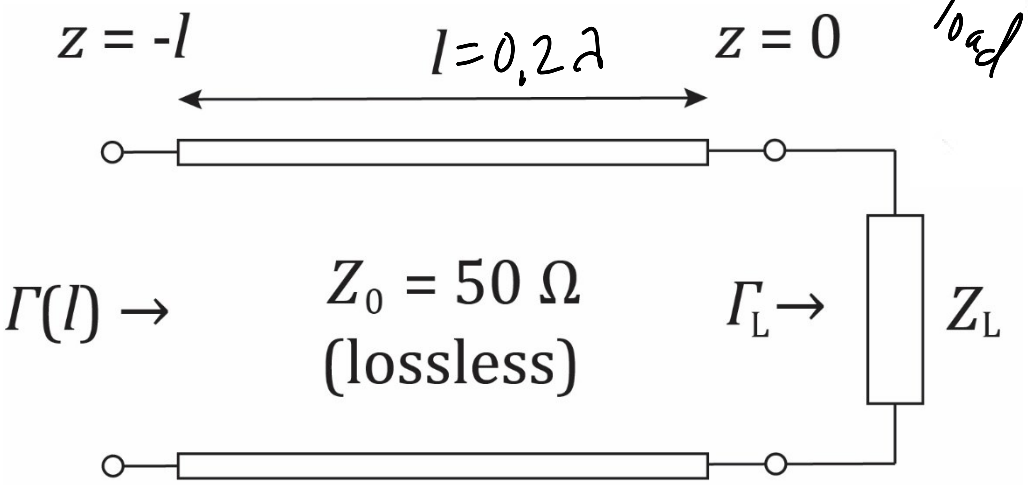






when l increases

$$\boxed{\Gamma(l) = \Gamma_L e^{-j2\beta l}}$$



Q5: How long is **one full round** on the Smith chart?

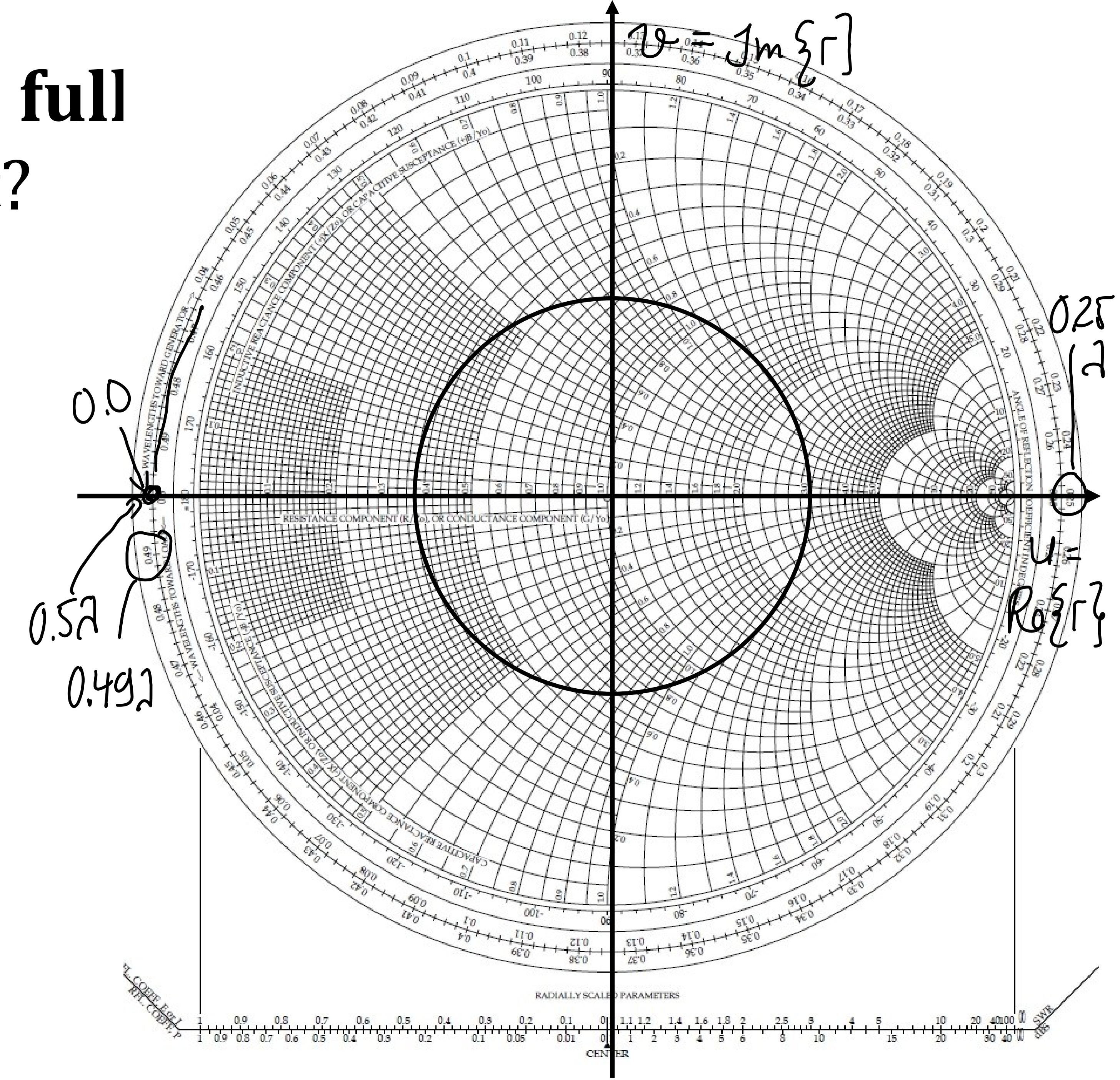
$$\boxed{\Gamma(l) = \Gamma_L e^{-j2\beta l}}$$

1. $l = \frac{\lambda}{8}$
2. $l = \frac{\lambda}{4}$ 3%
3. $l = \frac{\lambda}{2}$ 65%
4. $l = \frac{2\lambda}{3}$
5. $l = \lambda$ 32%
6. I don't know

$2\beta l = 2\pi$

$2 \cdot \frac{\cancel{2\pi}}{\lambda} l = \cancel{2\pi}$

$l = \frac{\lambda}{2}$



The Smith chart is the normalized impedance scale on top of the complex reflection coefficient plane

Z_0

normalized impedance (small letter)

$$\underline{\Gamma} = \boldsymbol{u} + \mathrm{j}\boldsymbol{v} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1} = \frac{\boldsymbol{r} + \mathrm{j}\boldsymbol{x} - 1}{\boldsymbol{r} + \mathrm{j}\boldsymbol{x} + 1}$$

$$z_L = \frac{Z_L}{Z_0}$$

1) $$\left(\boldsymbol{u} - \frac{\boldsymbol{r}}{\boldsymbol{r}+1}\right)^2 + (\boldsymbol{v} - 0)^2 = \frac{1}{(\boldsymbol{r}+1)^2}$$

2) $$(\boldsymbol{u} - 1)^2 + \left(\boldsymbol{v} - \frac{1}{\boldsymbol{x}}\right)^2 = \frac{1}{\boldsymbol{x}^2}$$

general circle equation: $(\boldsymbol{u} - u_r)^2 + (\boldsymbol{v} - v_r)^2 = a_r{}^2$

1) radius: $\frac{1}{r+1}$

centre: $\left(\frac{r}{r+1}, 0\right)$

2) radius $= \frac{1}{x}$

centre: $\left(1, \frac{1}{x}\right)$

Constant resistance lines are full circles whose centre is located on the real axis

Normalized impedance: $z_L = r + \mathrm{j}\,x$

Constant resistance circles:

centre $(u_r, v_r) = \left(\frac{r}{r+1}, 0\right)$

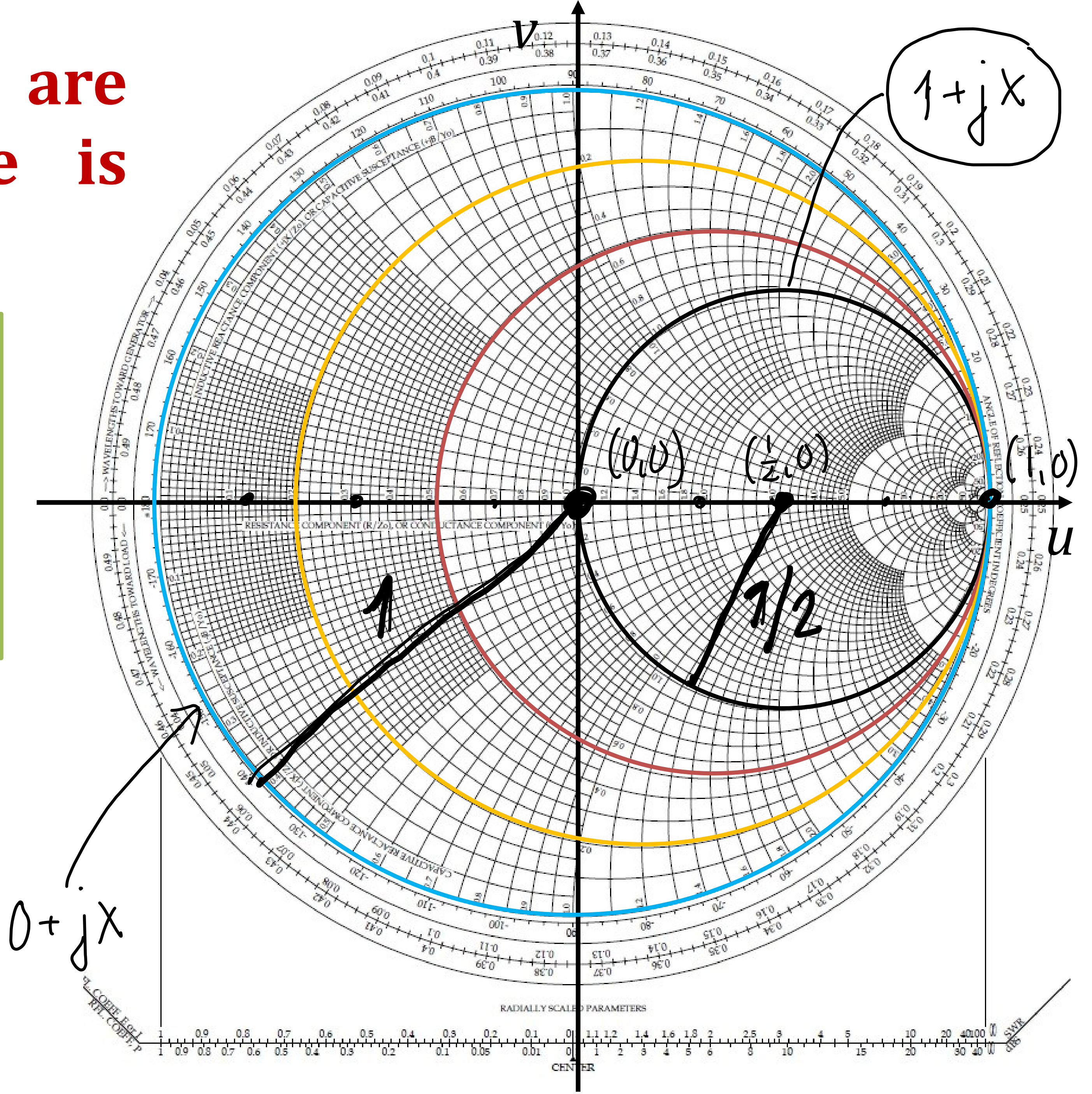
radius $a_r = \frac{1}{r+1}$

$\underline{r=1}$ centre $\left(\frac{1}{1+1}, 0\right) = \left(\frac{1}{2}, 0\right)$

radius $\frac{1}{1+1} = \frac{1}{2}$

$\underline{r=0}$ centre $(0, 0)$

radius $\frac{1}{1} = \underline{1}$



Q6: Which circle corresponds to $r = 2.0$ – i.e., where $z_L = 2.0 + jx$?

Normalized impedance: $z_L = r + \mathrm{j}\,x$

Constant resistance circles:

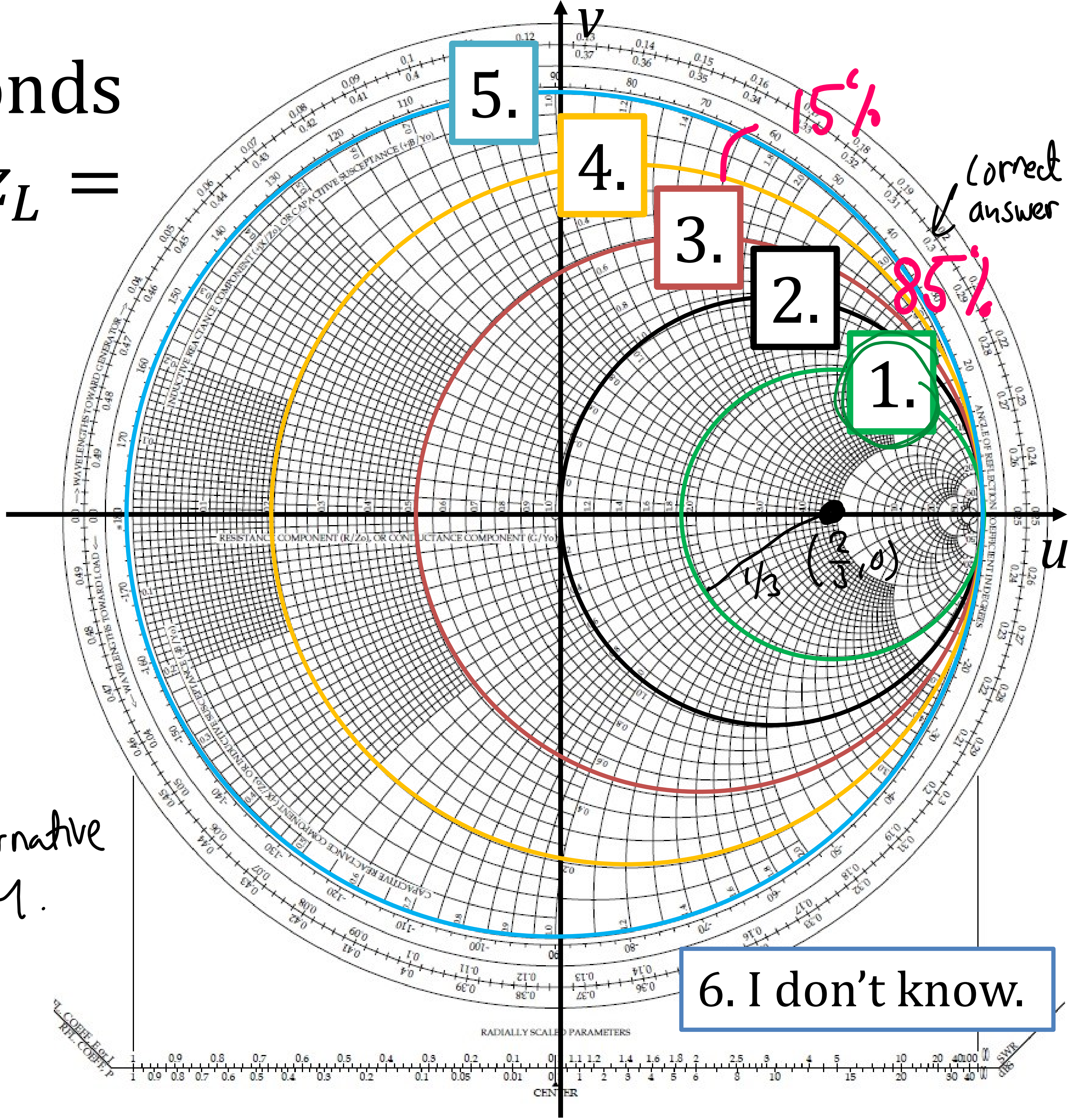
centre $(u_r, v_r) = \left(\frac{r}{r+1}, 0\right)$

radius $a_r = \frac{1}{r+1}$

centre $\left(\frac{2}{2+1}, 0\right) = \left(\frac{2}{3}, 0\right)$

radius $\frac{1}{2+1} = \frac{1}{3}$

alternative 1.



6. I don't know.

Constant reactance lines are sections of circles whose centre is located on the $u = 1$ axis

Normalized impedance: $z_L = r + \mathrm{j}x$

constant reactance circles:

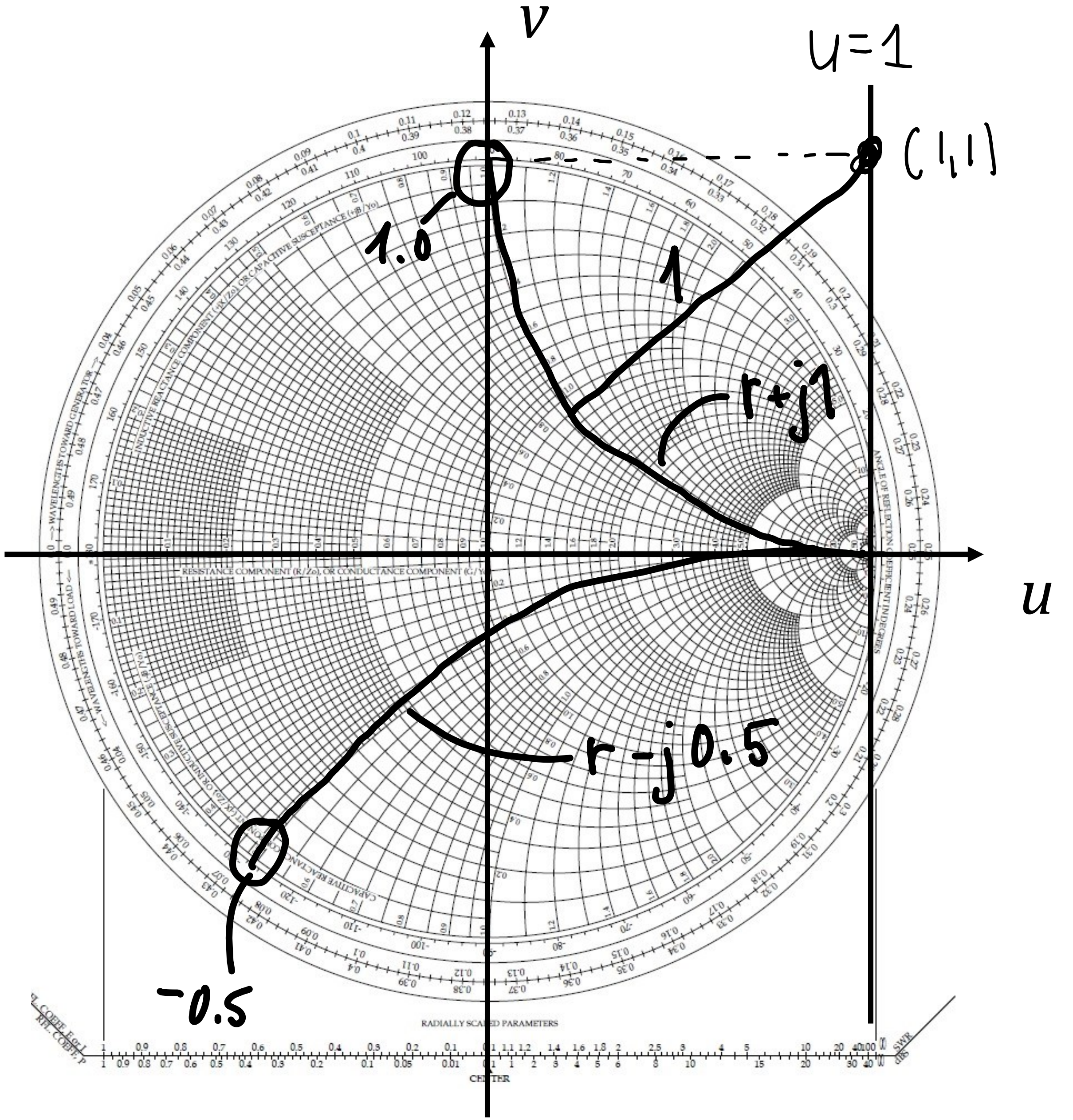
centre $(u_x, v_x) = (1, \frac{1}{x})$

radius $a_x = \frac{1}{x}$

x = 1 centre (1, 1)

radius = 1

x = -0.5

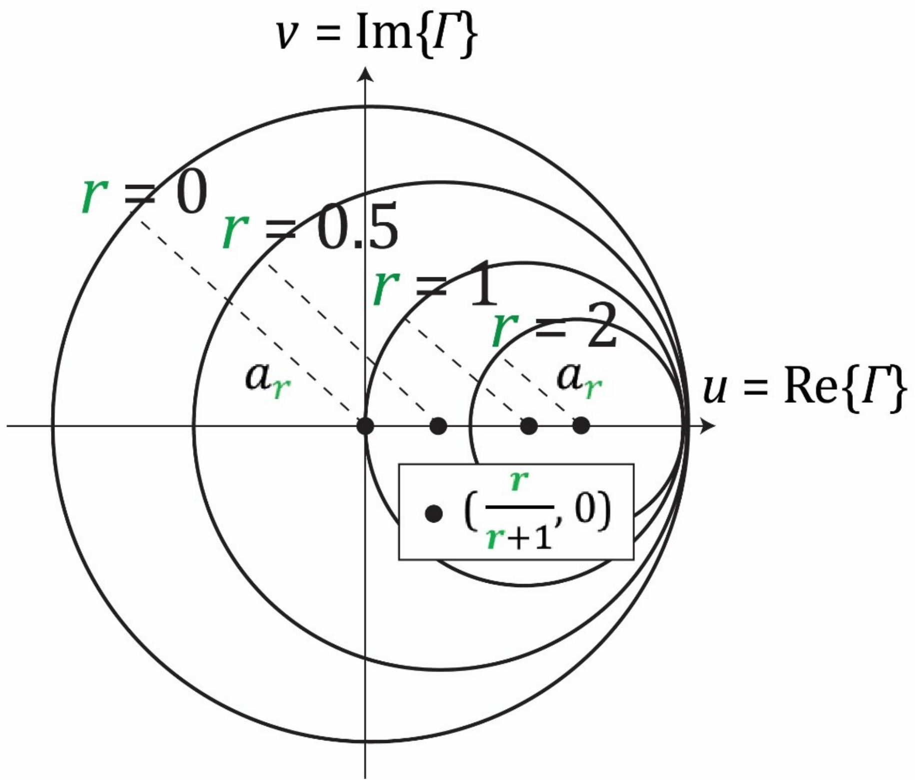


Constant reactance lines are sections of circles whose centre is located on the $u = 1$ axis

constant resistance circles:

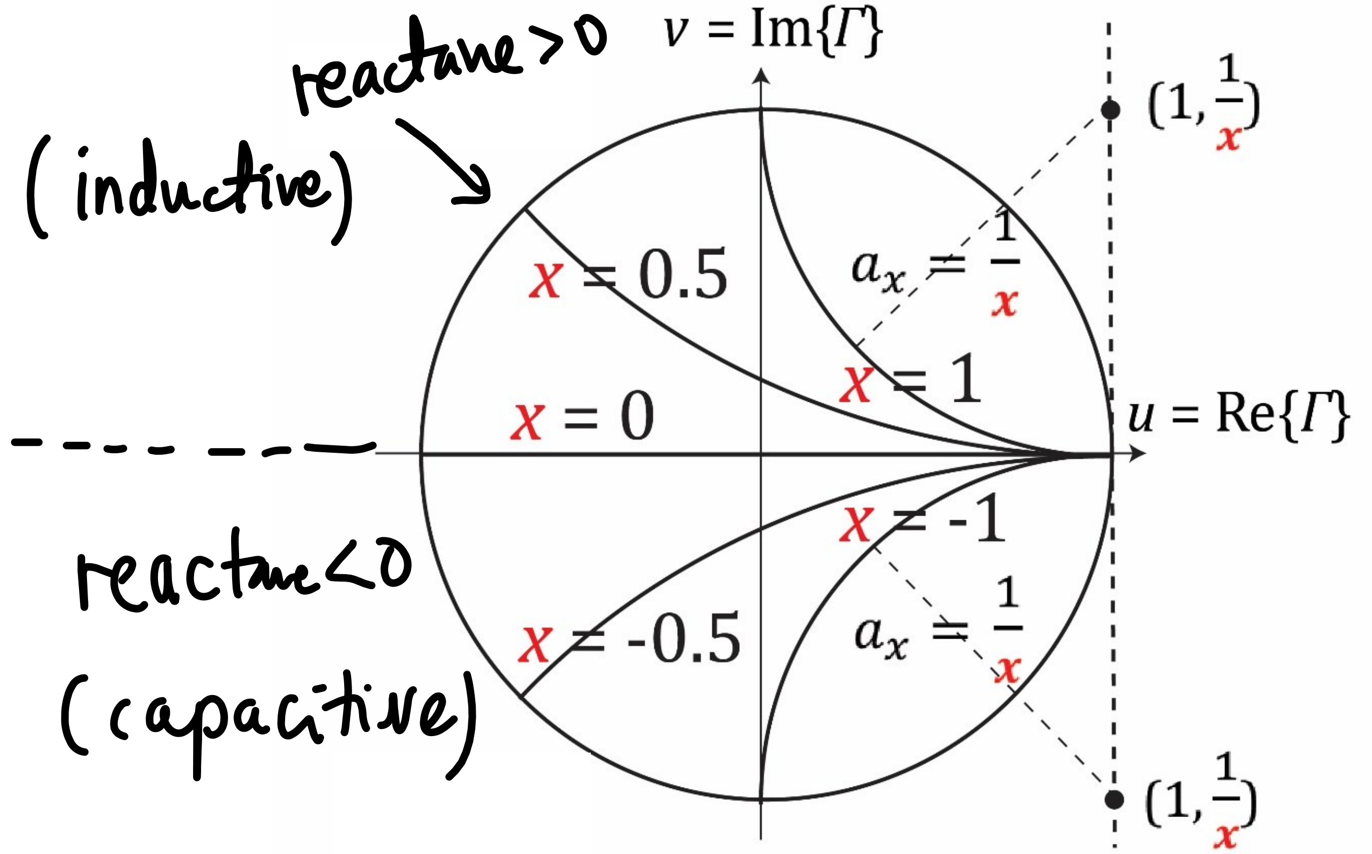
centre $(u_r, v_r) = (\frac{r}{r+1}, 0)$

and radius $a_r = \frac{1}{r+1}$



constant reactance circles:

centre $(u_x, v_x) = (1, \frac{1}{x})$ and radius $a_x = \frac{1}{x}$



The Smith chart is also a combined impedance – admittance plane

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

$$y_L = \frac{1}{z_L}$$

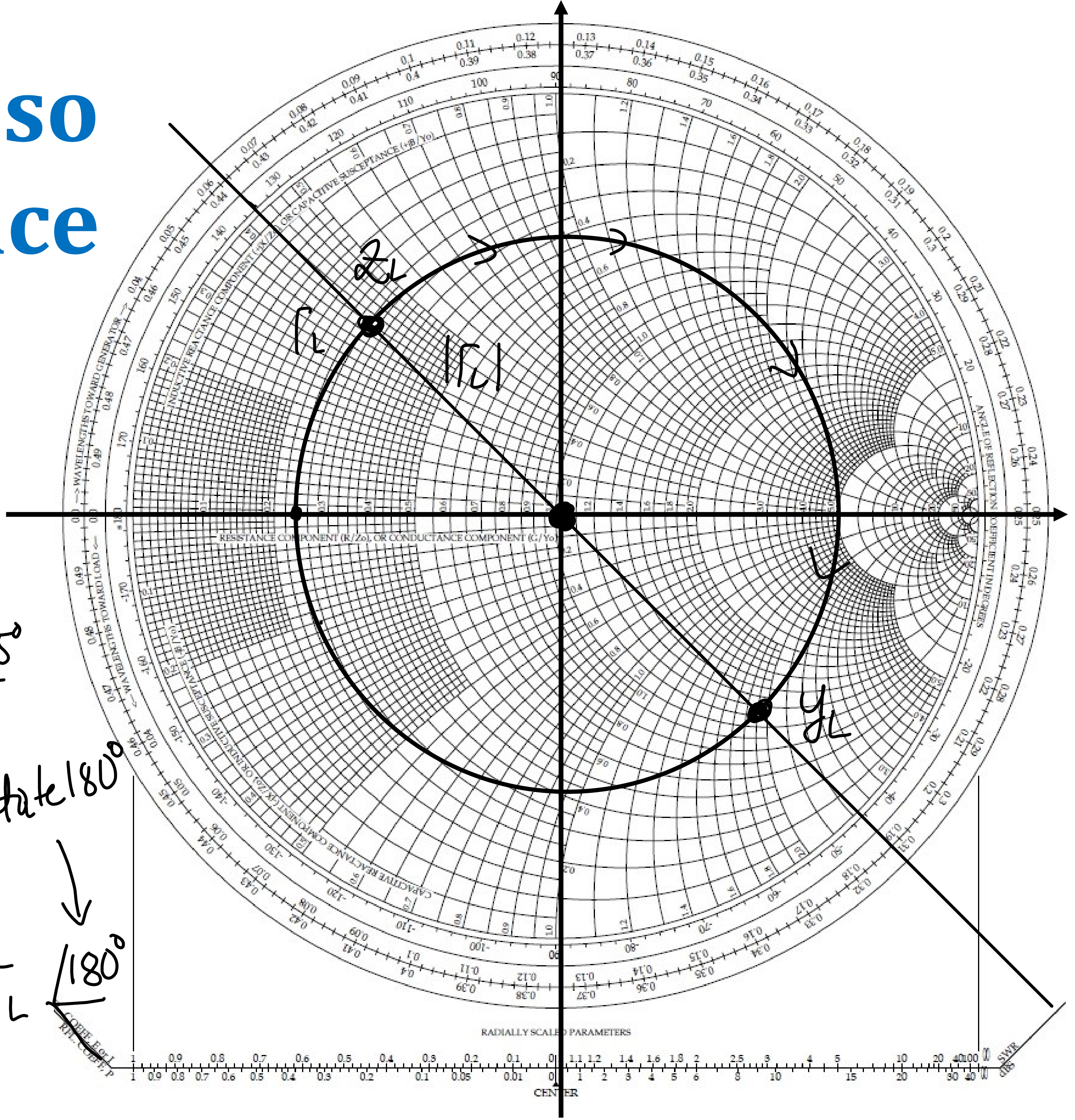
$$z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

$$\Gamma_L \simeq 0.6 \angle 135^\circ$$

$$y_L = \frac{1}{z_L} = \frac{1 - \Gamma_L}{1 + \Gamma_L} = \frac{1 + (-\Gamma_L)}{1 - (-\Gamma_L)}$$

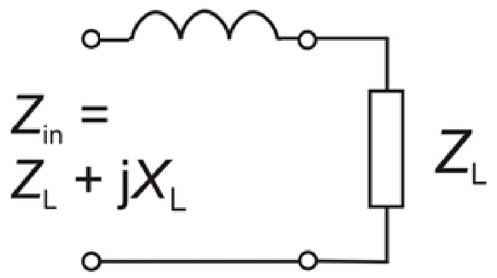
rotate 180°

From $z_L \to y_L$: $\Gamma_L \to -\Gamma_L = \Gamma_L \cdot e^{j\pi} = \Gamma_L \angle 180^\circ$

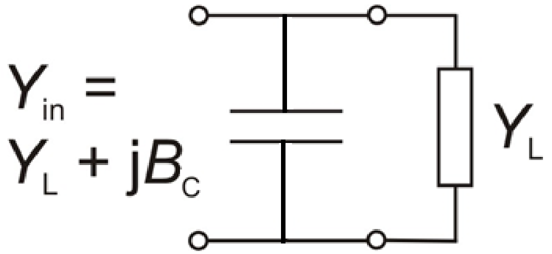


Impedance matching circuits are designed with the Smith chart

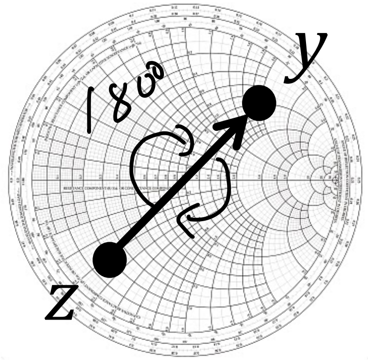
Handle components in SERIES configuration with the IMPEDANCE scale!



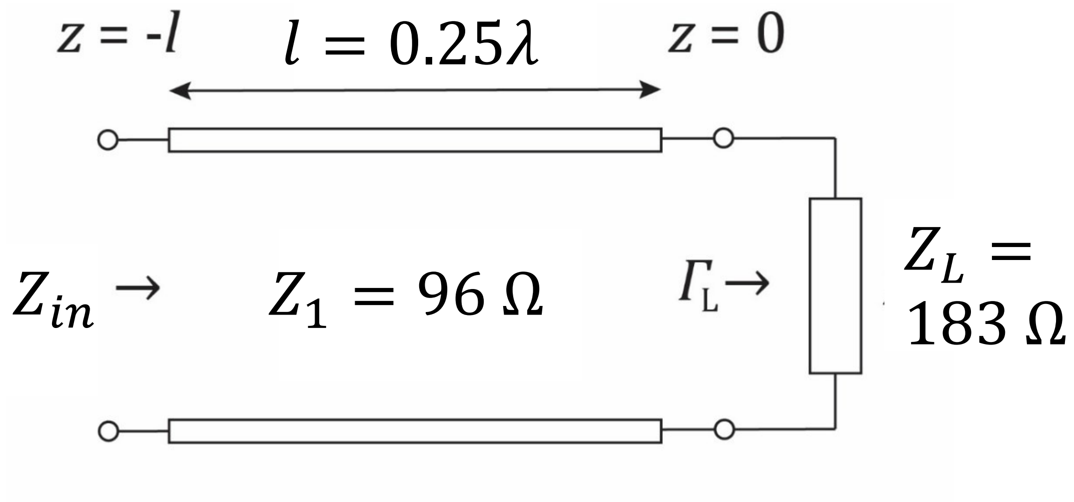
Handle components in PARALLEL configuration with the ADMITTANCE scale!

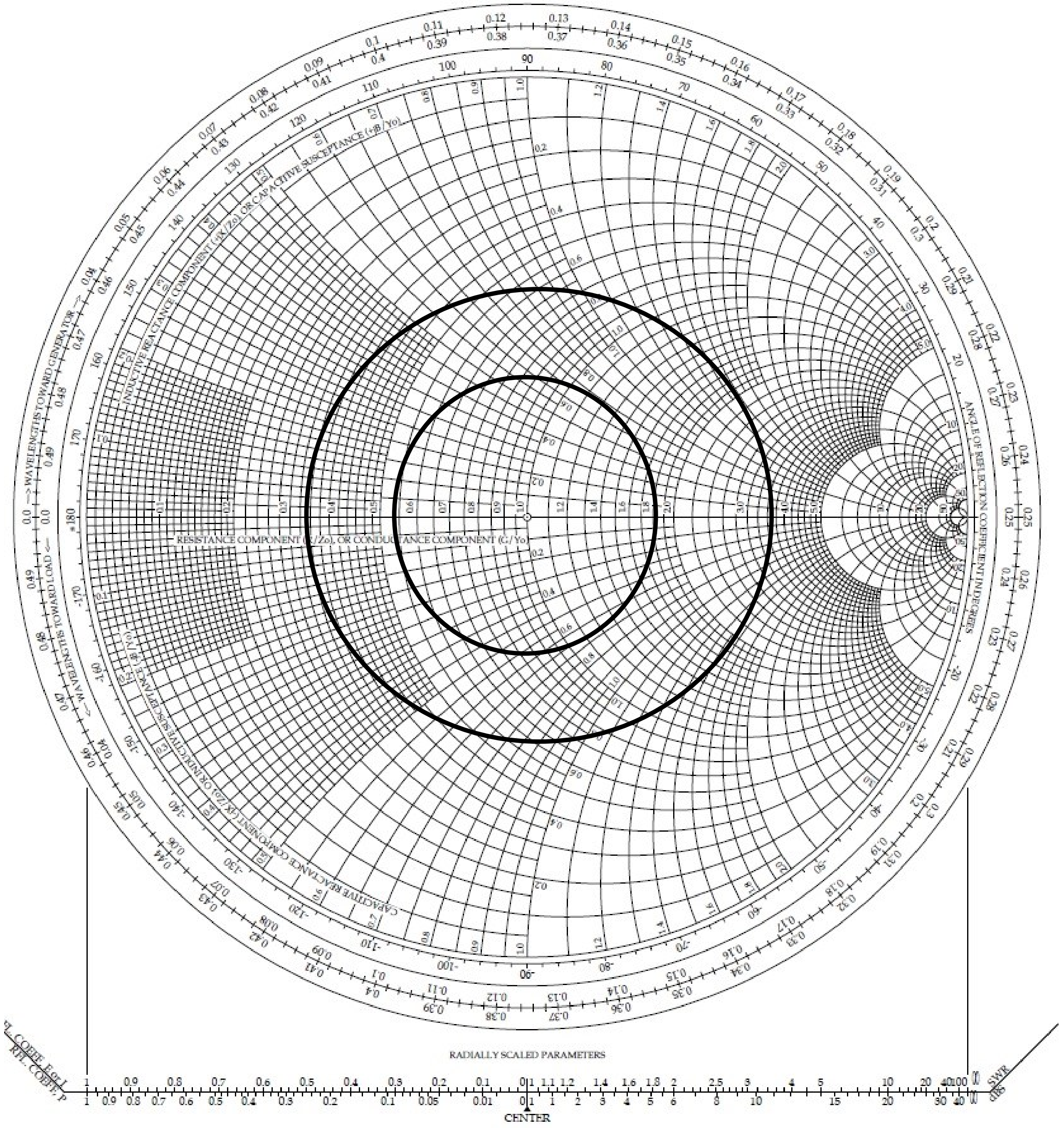


Switch between the two scales: rotate 180°



In-class task





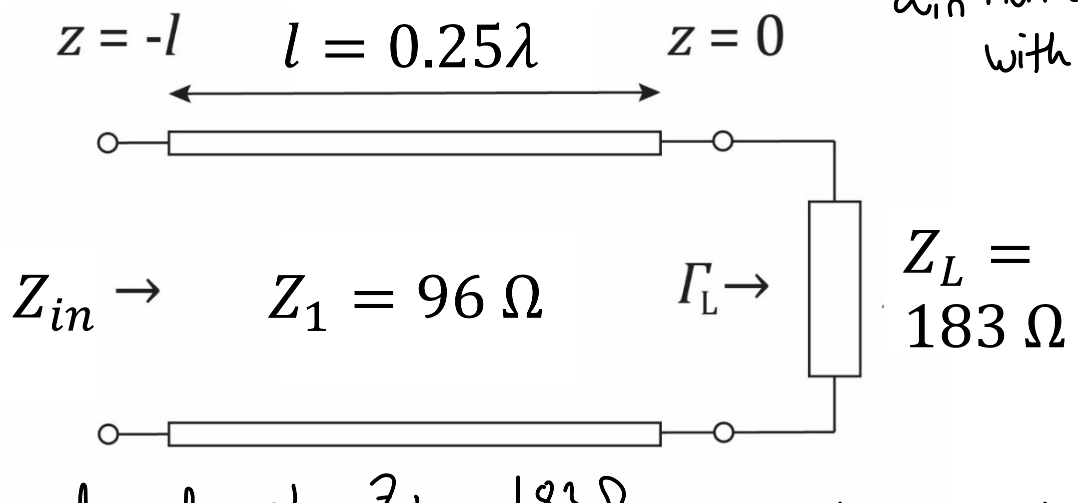
A lossless transmission line, whose characteristic impedance $Z_1 = 96\ \Omega$ and length $l = 0.25\lambda$, is terminated with the load impedance $Z_L = 183\ \Omega$ (← the same impedance as in pre task item 5.).

a. Calculate the input impedance Z_{in} and the reflection coefficient Γ_{in} (with respect to 50 Ω)of the circuit graphically with the Smith chart
   - Hint: normalize Z_L to Z_1, move 0.25λ, denormalize…

b. Explain the result of part. a.

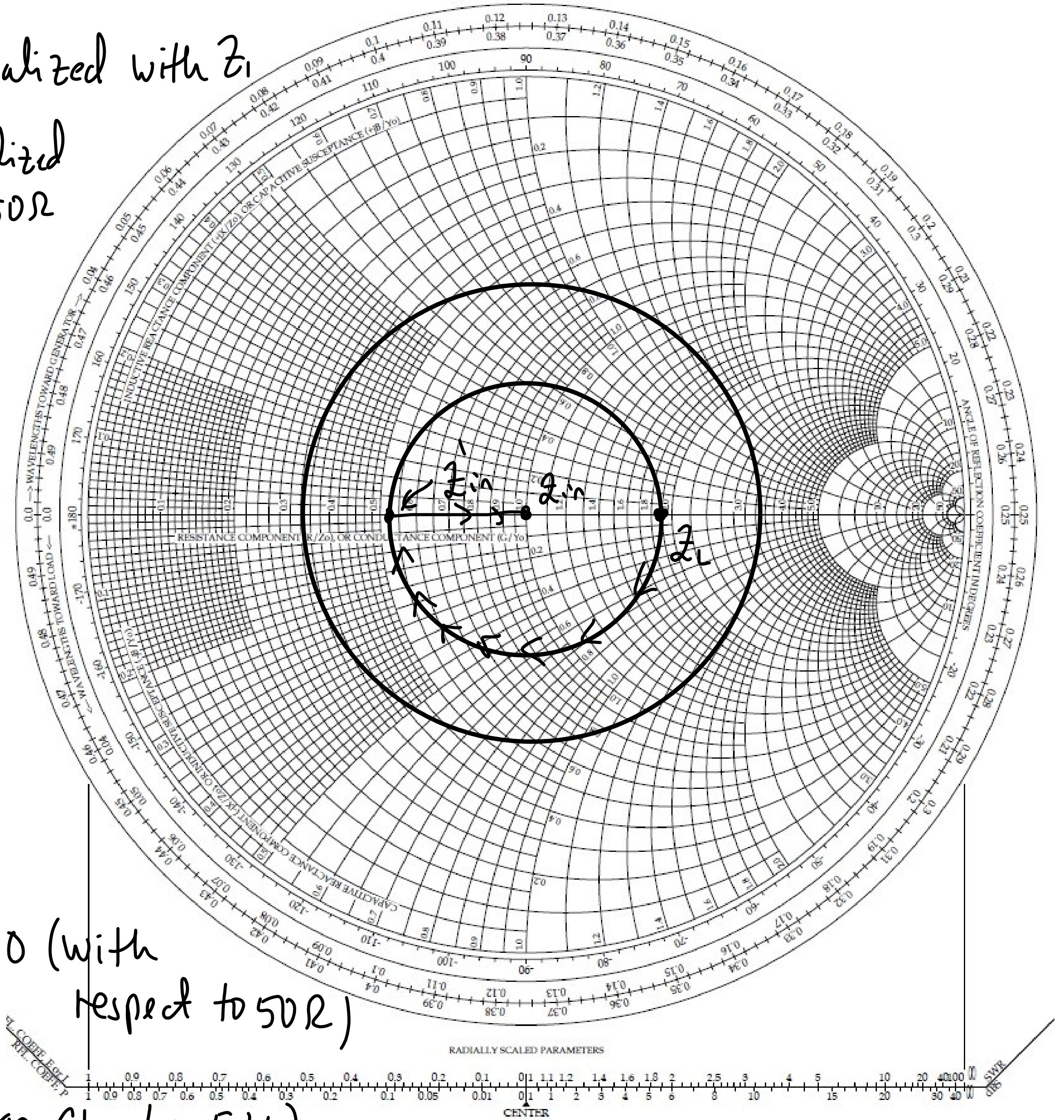
Return your effort in MyCourses latest at 12:30 pm.

In-class task



z'_{in} normalized with Z_1

z_{in} normalized with 50Ω



a) normalized $z'_L = \frac{Z_L}{Z_1} = \frac{183\Omega}{96\Omega} = 1.9$ (norm. 96Ω)

next rotate 0.25λ towards generator – i.e., clockwise

come to $z'_{in} = 0.52$ (norm. 96Ω)

denormalize $Z_{in} = z'_{in} \cdot Z_1 = 0.52 \cdot 96\Omega = 50\Omega$

The input impedance $Z_{in} = 50\,\Omega$ (matched); $\Gamma_{in} = 0$ (with respect to 50Ω)

b) This is called quarter-wavelength transformer (Pozar Chapter 5.4)

$Z_1 = \sqrt{183\Omega \cdot 50\Omega} = 96\Omega$