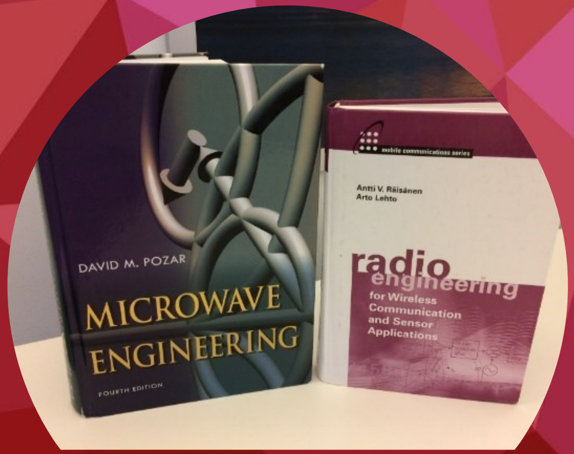
Microwave engineering I (MiWE I)

Interactive lecture 2 of Topic 2
The Smith chart and impedance matching
February 3, 2022

The main learning outcome of the course is to create readiness to work in microwave engineering related tasks and projects and enable further studies and continuous learning in microwave engineering.



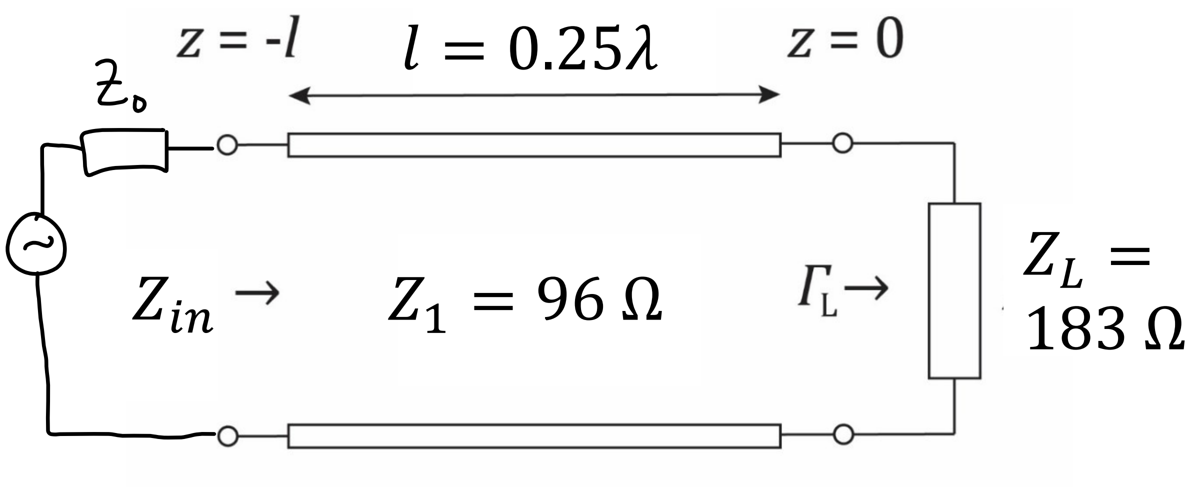
Topic 2: Learning outcomes and content

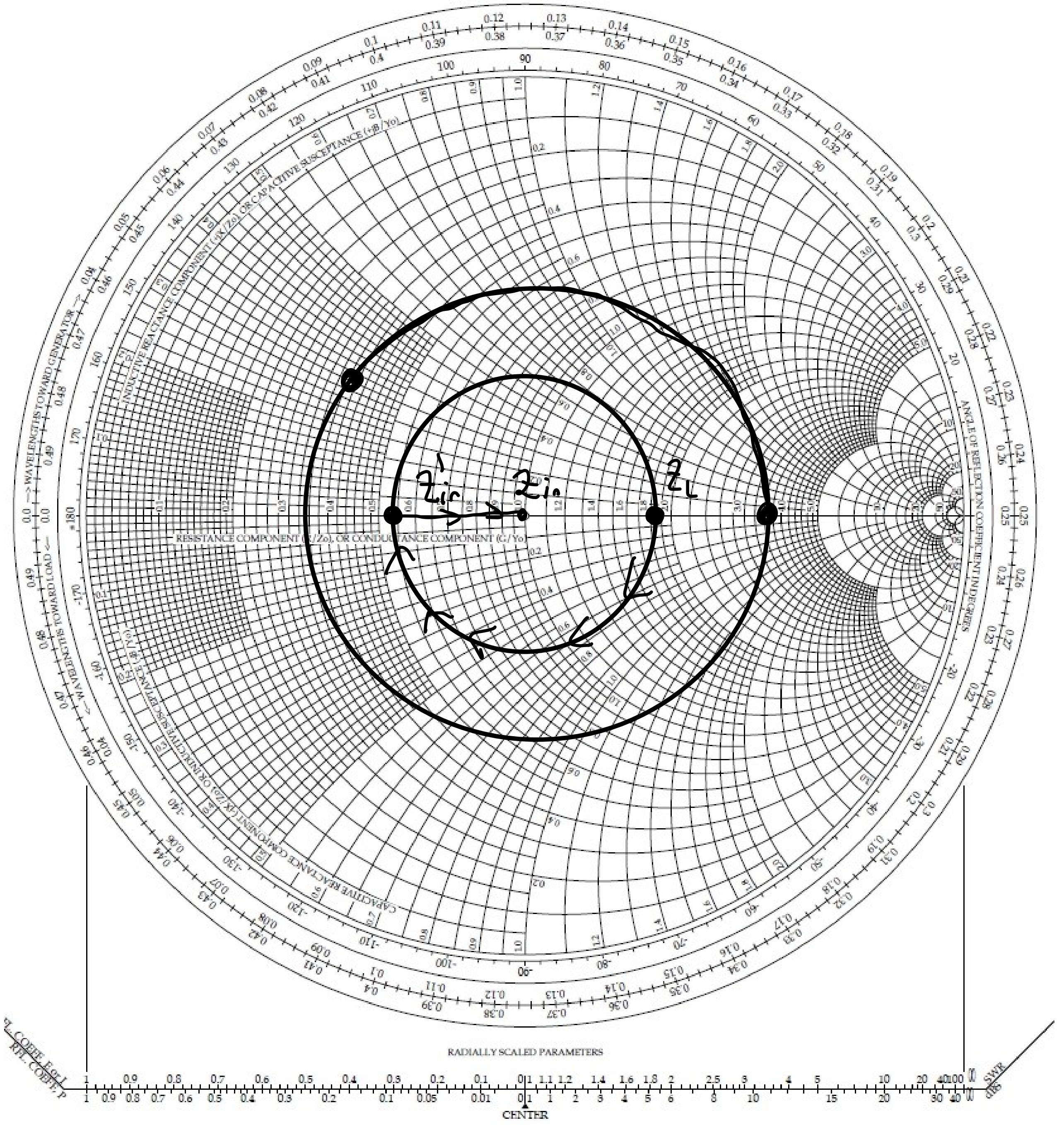
- The student can
  - **design** impedance matching circuits using the Smith chart and a simulator tool (AWRDE)
  - **explain** the design principles and bandwidth issues related to impedance matching.

- The terminated mismatched load impedance (Pozar Chapter 2.3)
- The Smith chart (Pozar Chapter 2.4)
- The quarter-wave transformer (Pozar Chapter 2.5 and Chapter 5.4)
- Matching with lumped elements (Pozar Chapter 5.1)
- Single-stub tuning (Pozar Chapter 5.2)
- The Bode-Fano criterion (Pozar Chapter 5.9)

These lecture slides and notes are not designed for self-study.
Please, use the course book chapters 2 and 5 for self-study.

Last week's in-class task





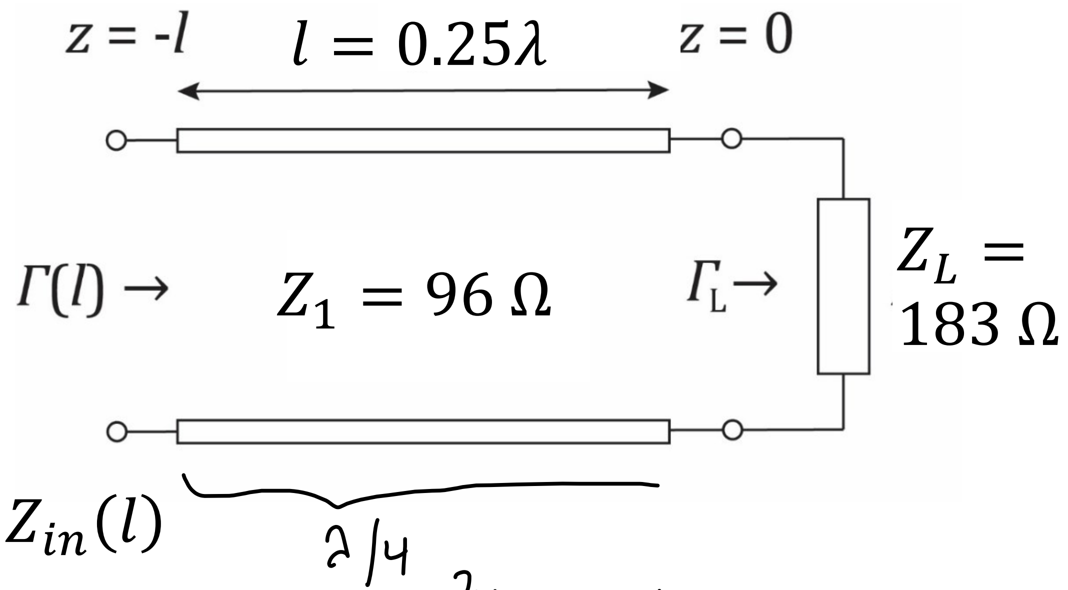
normalized impedance $z_L = \frac{Z_L}{Z_1} = \frac{183\Omega}{96\Omega} = 1.9$ (96Ω)

rotate length 0.25λ clockwise

$z'_{in} = 0.52$ (96Ω) ; $Z_{in} = z'_{in} \cdot Z_1 = 0.52 \cdot 96\Omega = 50\Omega$

$z_{in} = \frac{Z_{in}}{Z_0} = 1.0$; $\Gamma_{in} = 0$ (perfectly matched)

Last week's in-class task



$$\Gamma_L = \frac{Z_L - Z_1}{Z_L + Z_1}$$

$$e^{\pm j\beta l} = \cos(\beta l) \pm j\sin(\beta l)$$

"tangent formula"

$$Z_{in}(l) = Z_1 \frac{1 + \Gamma_L e^{-j2\beta l}}{1 - \Gamma_L e^{-j2\beta l}} = Z_1 \frac{Z_L + jZ_1 \tan(\beta l)}{Z_1 + jZ_L \tan(\beta l)}$$

$$\tan(\beta l) = \tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$Z_{in} = Z_1 \frac{\frac{Z_L}{\tan(\beta l)} + jZ_1}{\frac{Z_1}{\tan(\beta l)} + jZ_L} \xrightarrow{\tan(\beta l)\to\infty} Z_1 \cdot \frac{jZ_1}{jZ_L} = \frac{Z_1^2}{Z_L} \Rightarrow Z_1 = \sqrt{Z_L \cdot Z_{in}} =$$

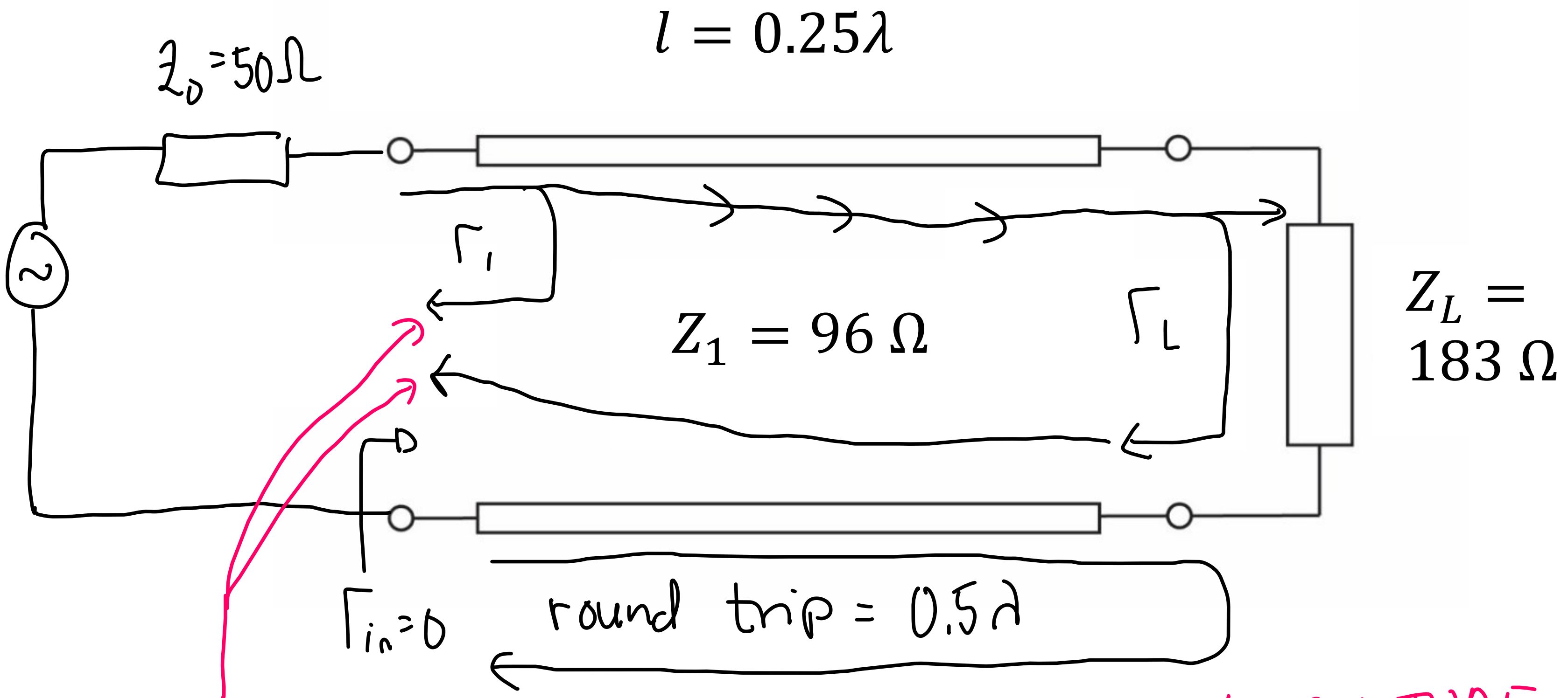
$$Z_1 = \sqrt{183\,\Omega \cdot 50\,\Omega} = \underline{\underline{96\,\Omega}}$$

"quarter-wavelength transformer"

Last week's in-class task

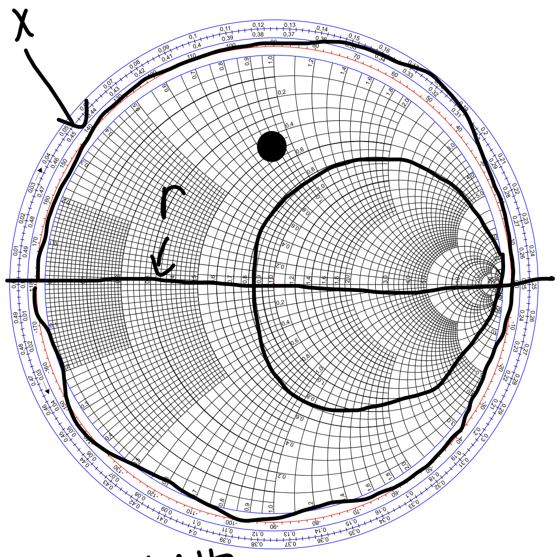
$$\Gamma_1 = \frac{96\,\Omega - 50\,\Omega}{96\,\Omega + 50\,\Omega} = 0.31$$

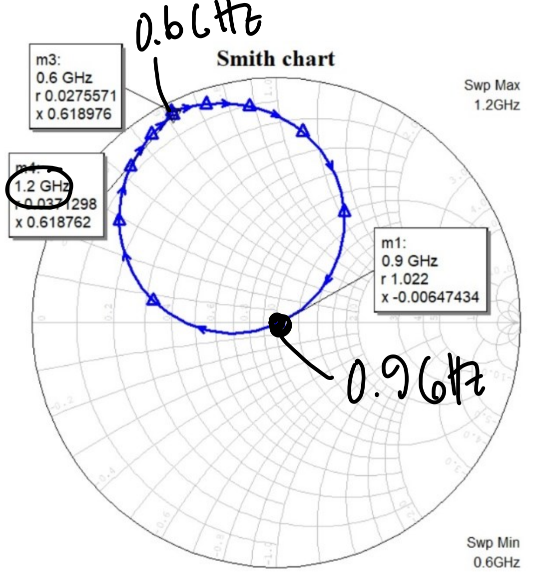
$$\Gamma_L = \frac{183\,\Omega - 96\,\Omega}{183\,\Omega + 96\,\Omega} = 0{,}31$$



TWO REFLECTIONS ARE EQUAL IN MAGNITUDE, BUT THEY ARE OUT-OF-PHASE → THEY CANCEL EACH OTHER OUT → $\Gamma_{in} = 0$

Place a word into an empty space. There are a few extra words.

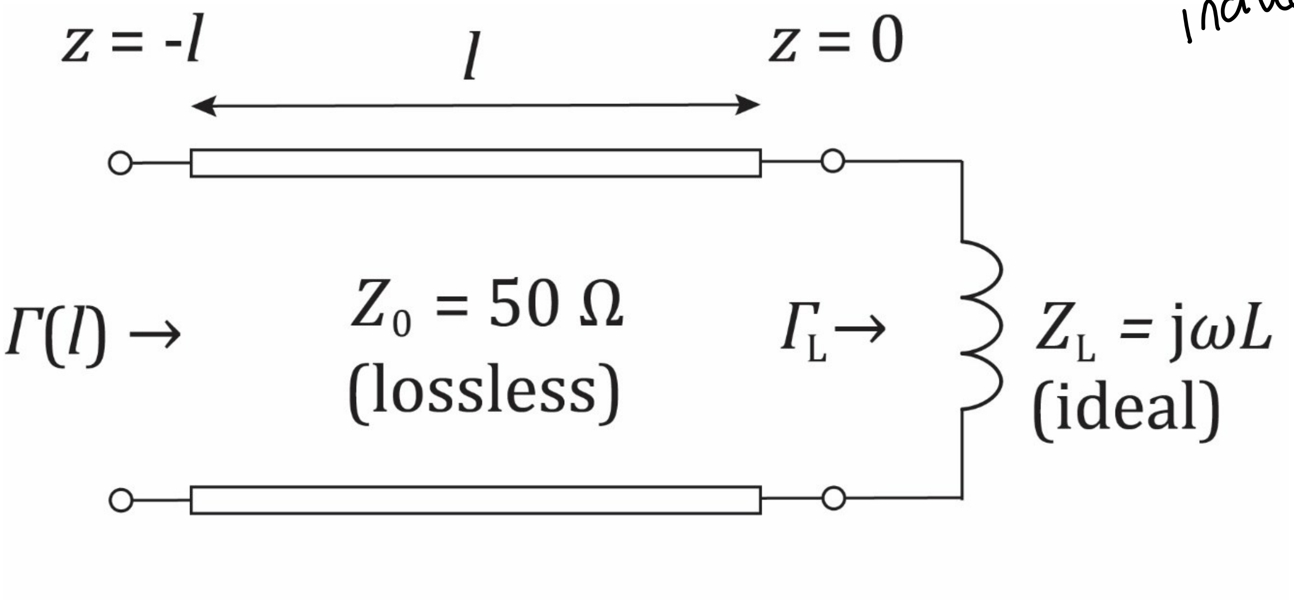


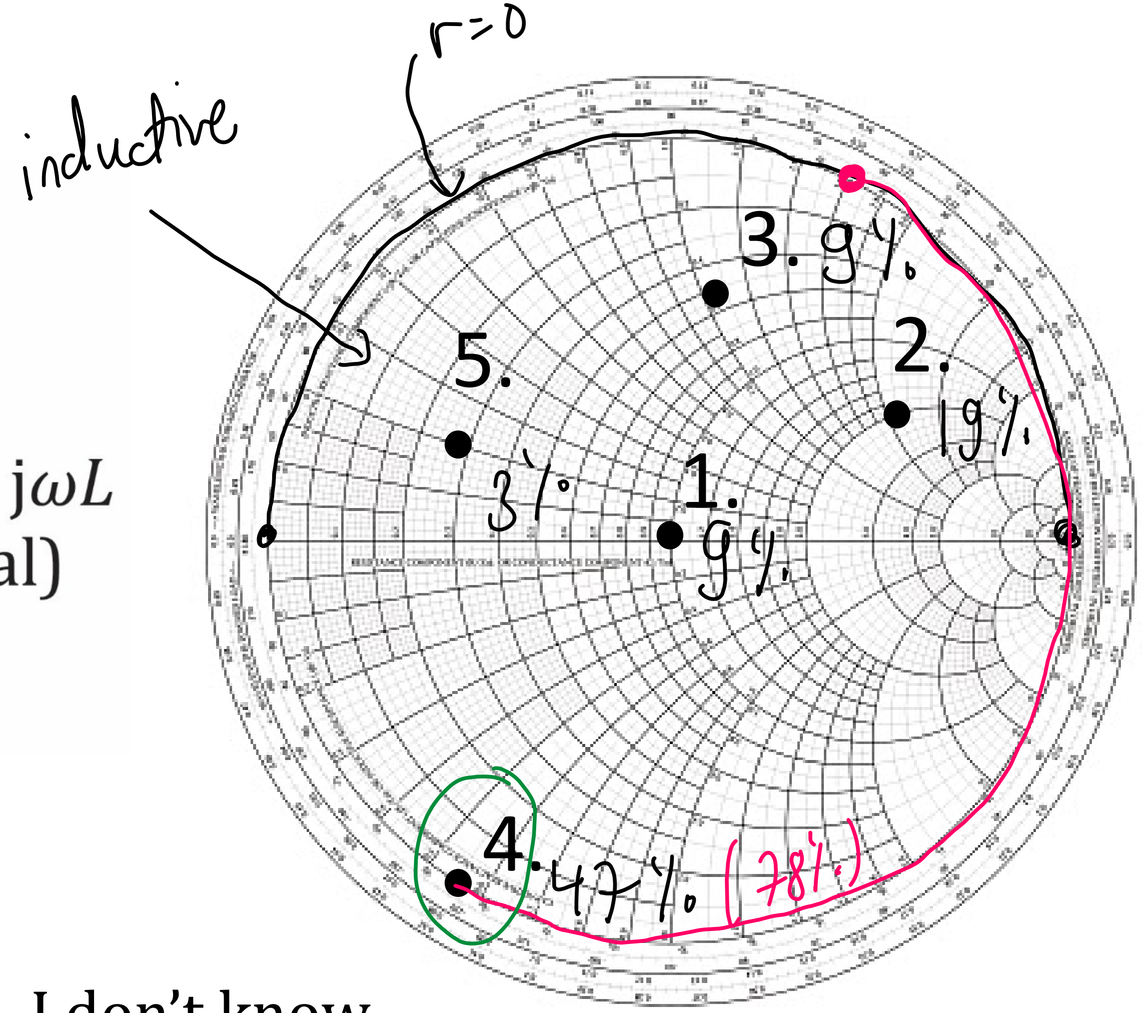


1. Smith chart **visualizes** ___complex___ impedances or admittances.
2. All reactance values in the upper half are ___inductive___.
3. All susceptance values in the upper half are ___capacitive___.
4. The **closer** the normalized impedance z_L to the centre, the better a device is ___matched___ .
5. Every point that lies along the same circle has the **same** normalized ___resistance___ **or** ___conductance___ .
6. Normalized resistance value can be read from the ___horizontal___ axis.
7. Normalized reactance value can be read from the ___circumference___ .
8. Often want to move z_{in} closer to the centre, using a ___matching circuit___ .
9. z_{in} is often shown as a function of frequency. A device is **resonant** at the ___frequency___ , where the impedance moves **through** the centre.

susceptance   vector network analyzer   reflection coefficient

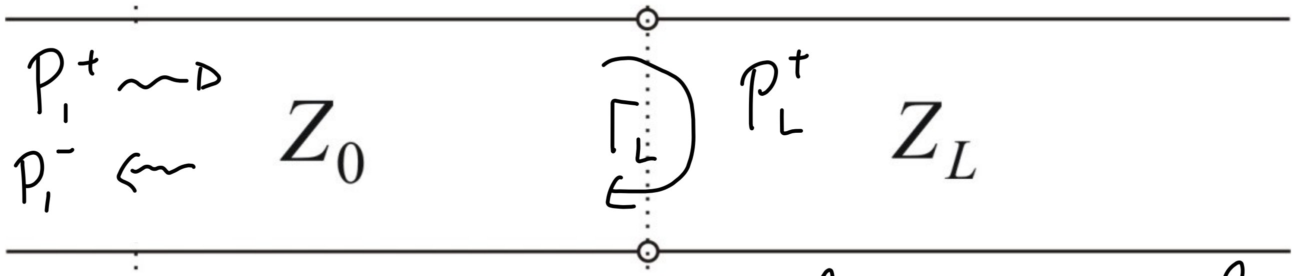
Q1: The line is terminated with an ideal inductor (ideal = no losses, no parasitics). Which of the following is a **possible** reflection coefficient seen at distance *l*?





6. I don't know
13%

Q2: The signal propagates to the positive z direction. $Z_0 \neq Z_L$. How much (%) of the **power** is **transmitted** to the line whose impedance is Z_L



$Z_L = 2\ \Omega$
$Z_0 = 1\ \Omega$

$\Gamma_L = \frac{2\Omega - 1\Omega}{2\Omega + 1\Omega} = \frac{1}{3}$

3% 15% 1. 11% (1/9)
0% 18% 2. 33% (1/3)
6% 15% 3. 66% (2/3)
74% 30% 4. 89% (8/9)
16% 9% 5. 133 % (4/3)
12% 6. I don't know

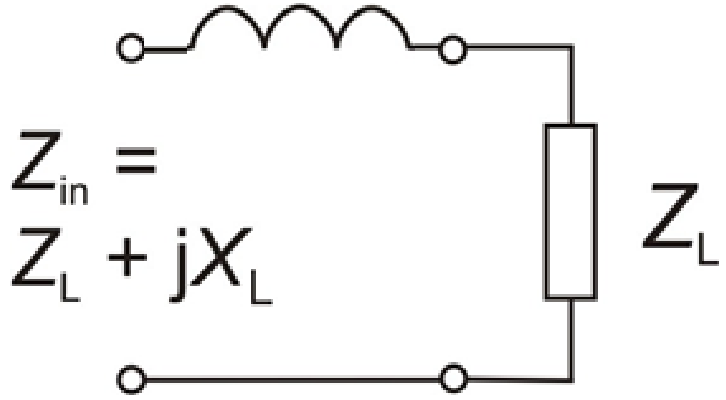
$P_1^+ = \frac{|V_1^+|^2}{2Z_0}$ $\quad P_1^- = \frac{|\Gamma V_1^+|^2}{2Z_0}$ $\quad P^T = \frac{|V^T|^2}{2Z_L} = \frac{|T \cdot V_1^+|^2}{2Z_L}$ $\quad T = \Gamma + 1 = \frac{4}{3}$

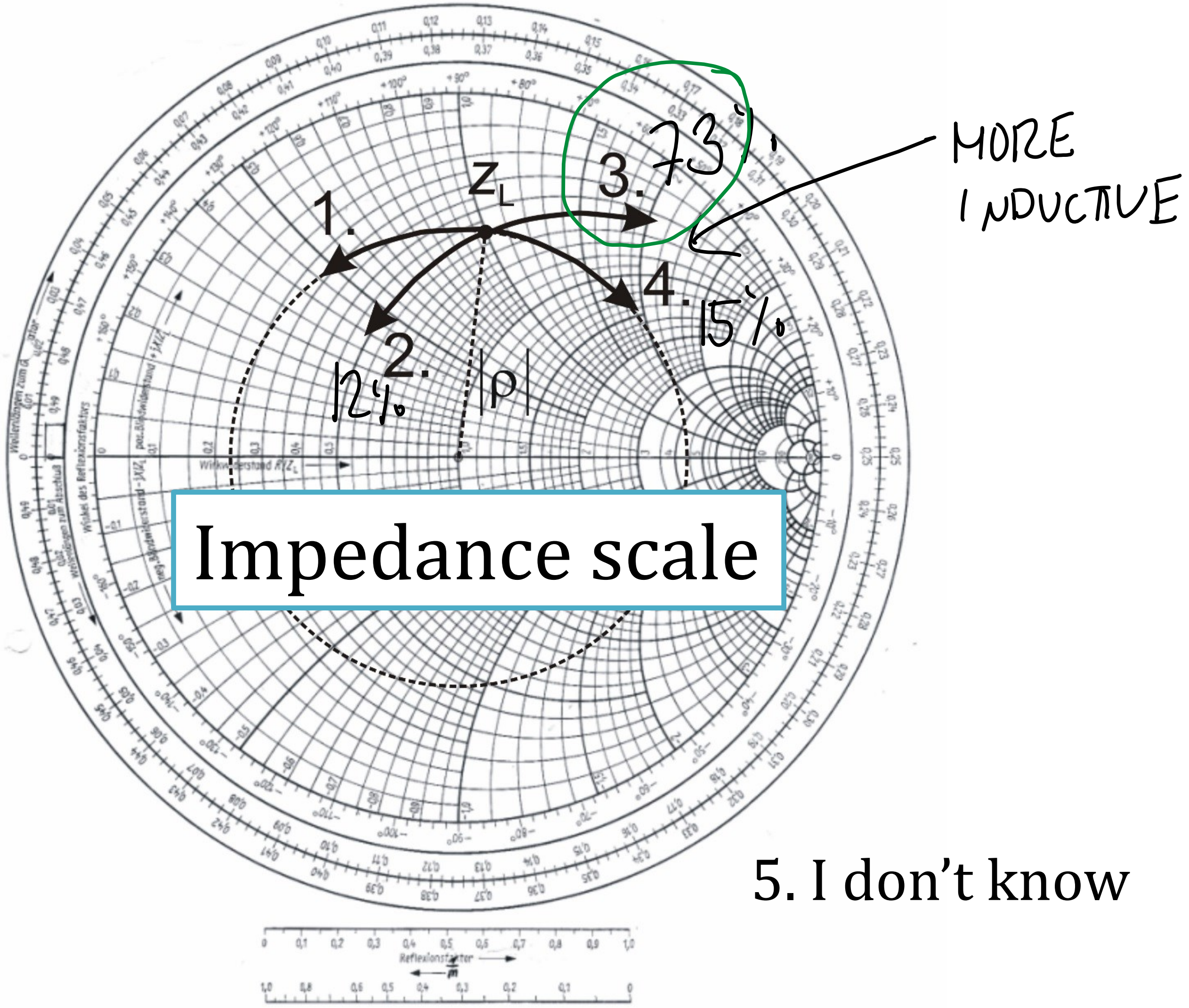
$P_1^- = |\Gamma|^2 \cdot P_1^+ = \left|\frac{1}{3}\right|^2 \cdot P_1^+ = \frac{1}{9} P_1^+$

conservation of energy: $P^T = P_1^+ - P_1^- = P_1^+ - \frac{1}{9} P_1^+ = \frac{8}{9} P_1^+$

$\frac{P^T}{P_1^+} = \frac{\frac{|T \cdot V_1^+|^2}{2Z_L}}{\frac{|V_1^+|^2}{2Z_0}} = |T|^2 \cdot \frac{Z_0}{Z_L} = \left|\frac{4}{3}\right|^2 \cdot \frac{1\Omega}{2\Omega} = \frac{16}{9} \cdot \frac{1}{2} = \frac{8}{9}$ SAME

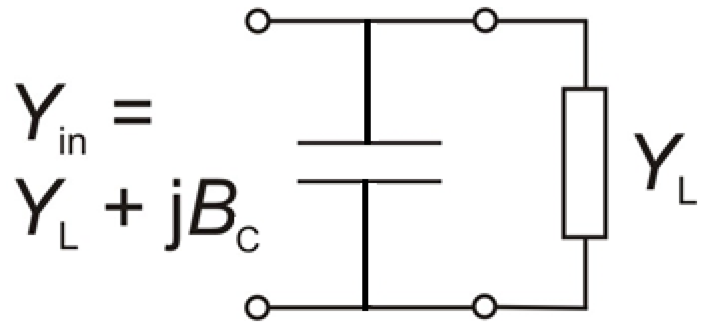
Q3: Which of the following transitions (1-4) corresponds to adding **a series inductor** in the **impedance** scale?



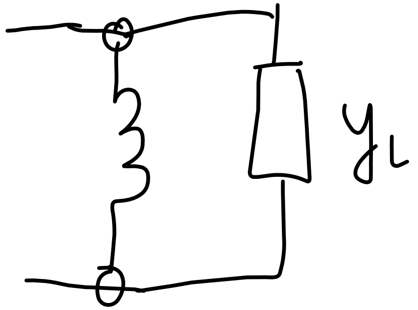


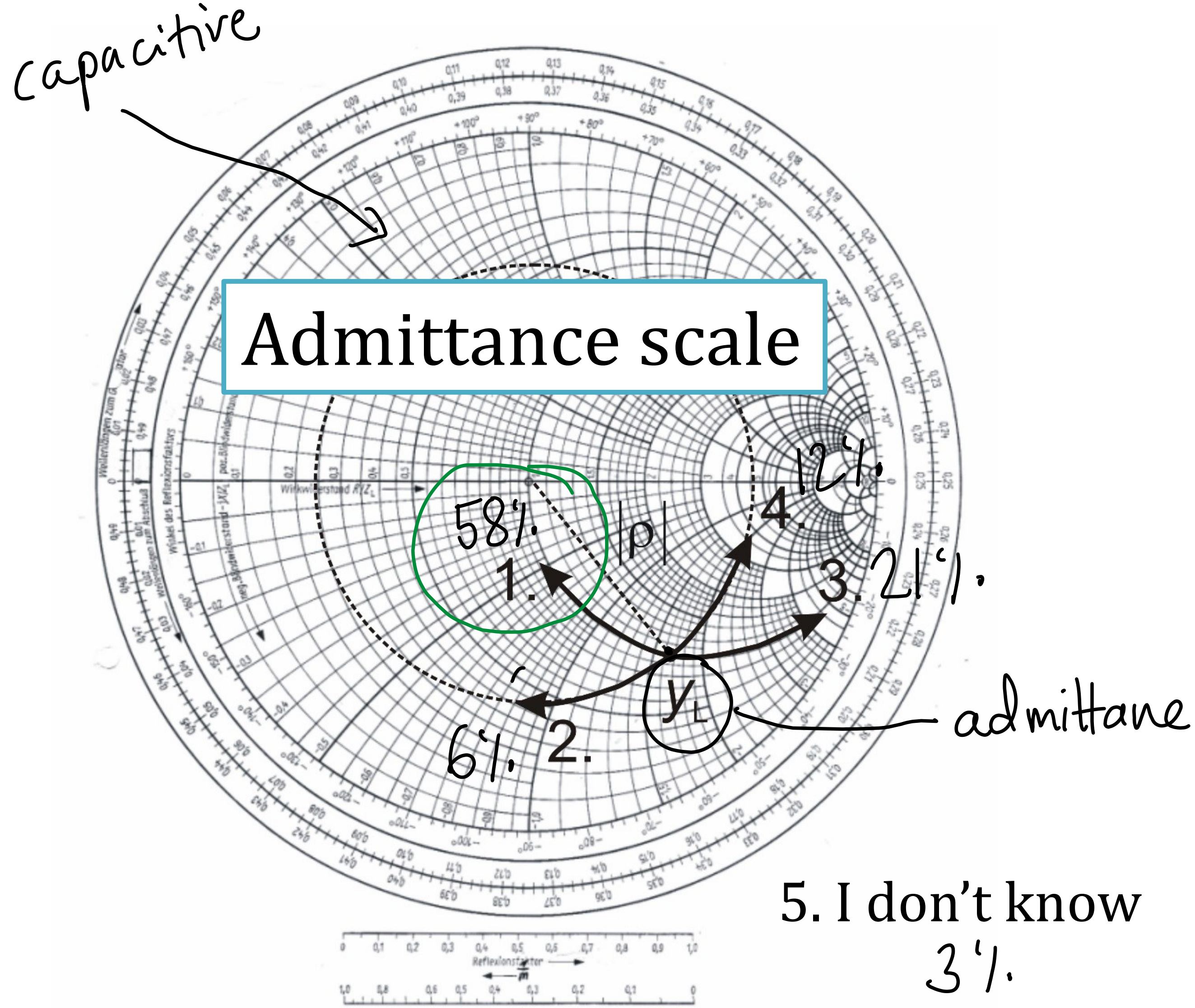
5. I don’t know

Q4: Which of the following transitions on the Smith chart (1-5) corresponds to adding a **shunt capacitor** in the **admittance** scale?



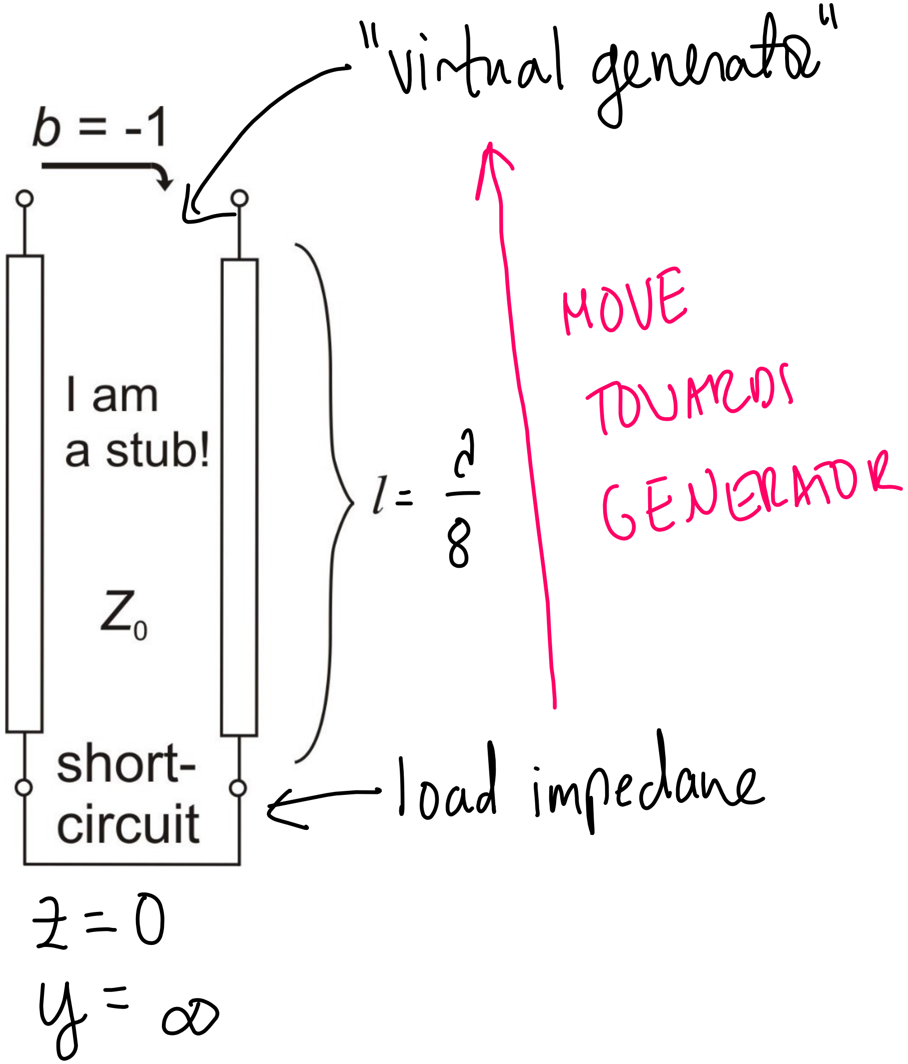
alternativ 3.

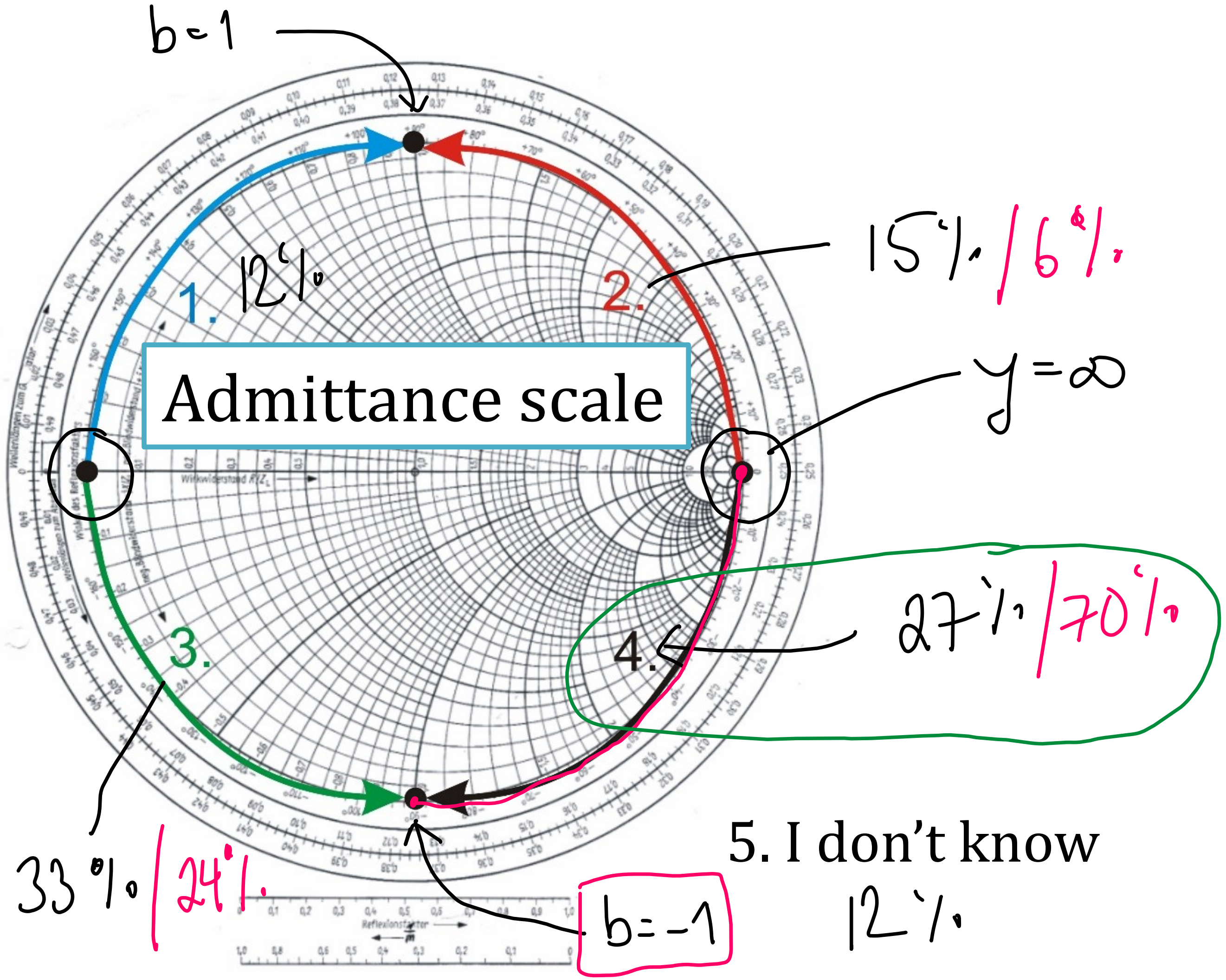




5. I don't know
3%

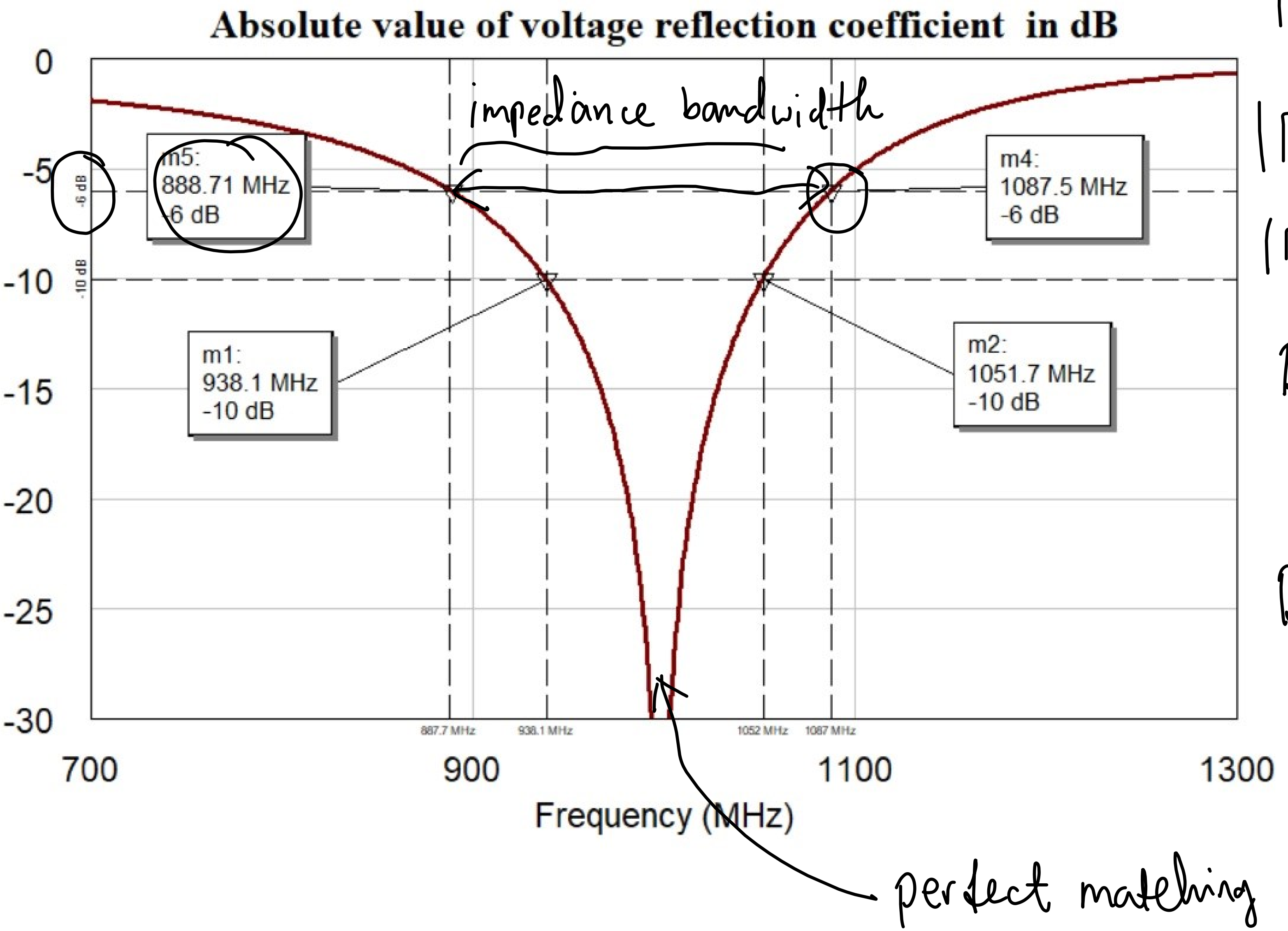
Q5: We want to design a lossless **shorted shunt** stub (with length l) that has the admittance $y = jb = -j \cdot 1.0$. Which of the following transitions (1-4) correspond to that stub in the **admittance** scale?





5. I don't know
12%

The impedance bandwidth depends on the matching level



$$\Gamma(\mathrm{dB}) = 10\log_{10}|\Gamma|^2 = 20\log_{10}|\Gamma|$$

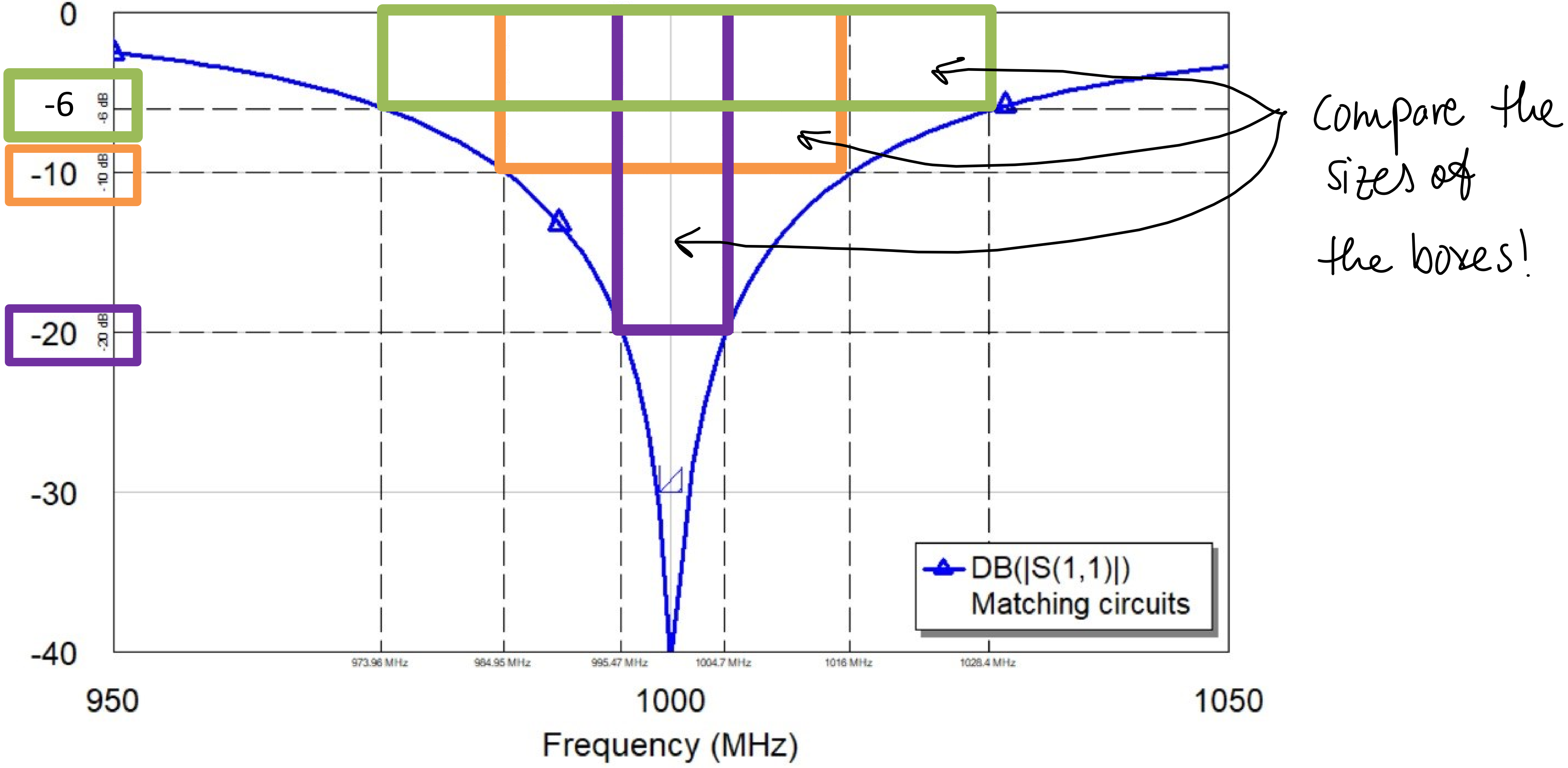
$$|\Gamma_m| = -6\,\mathrm{dB} \mathrel{\hat{=}} 10^{-\frac{6}{20}} = 0{,}501$$

$$|\Gamma_m| = -10\,\mathrm{dB} \mathrel{\hat{=}} 0{,}316$$

$$BW_{-6\mathrm{dB}} = 1087{,}5\,\mathrm{MHz} - 888{,}71\,\mathrm{MHz} = 198{,}8\,\mathrm{MHz} = \underline{\underline{199\,\mathrm{MHz}}}$$

$$BW_{-10\mathrm{dB}} = 113{,}6\,\mathrm{MHz} = \underline{\underline{114\,\mathrm{MHz}}}$$

The impedance bandwidth and matching level are inversely proportional

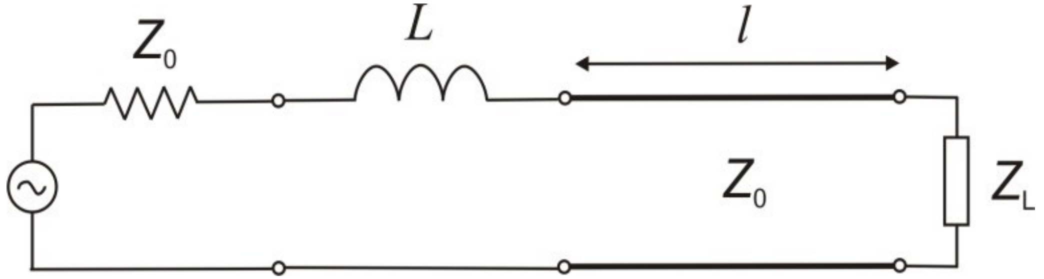


Today's in-class task

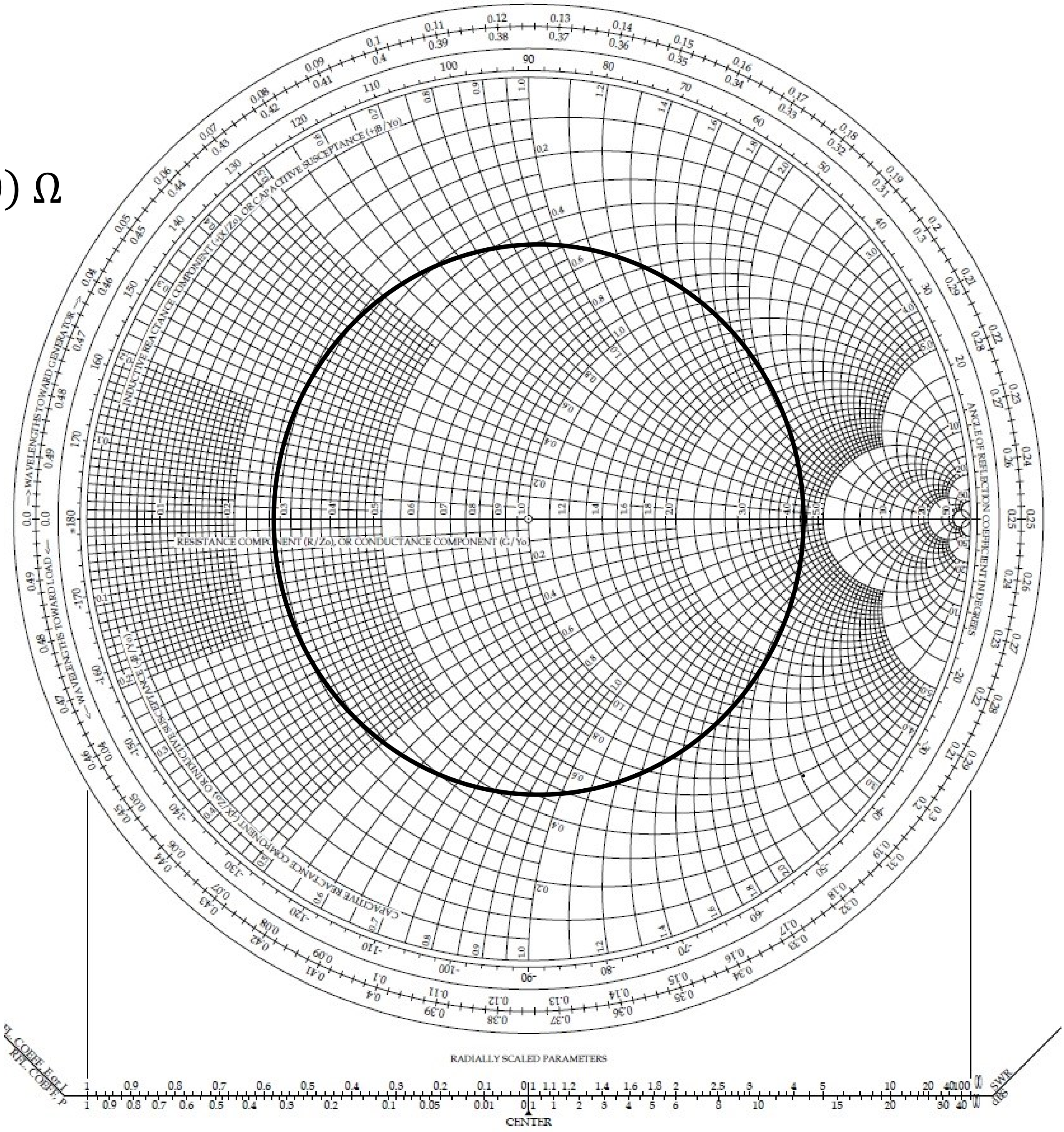
The input impedance of a load is $Z_L = (100.0 + j100.0)\ \Omega$ at 1.5 GHz. $Z_0 = 50\ \Omega$.

1) The load is attached directly to the 50-Ω feed. What percentage of the feed power is **reflected** from the load?
2) Match the load to the feed with the matching circuit shown graphically with the Smith chart – i.e., calculate l (in wavelengths) and L [nH].

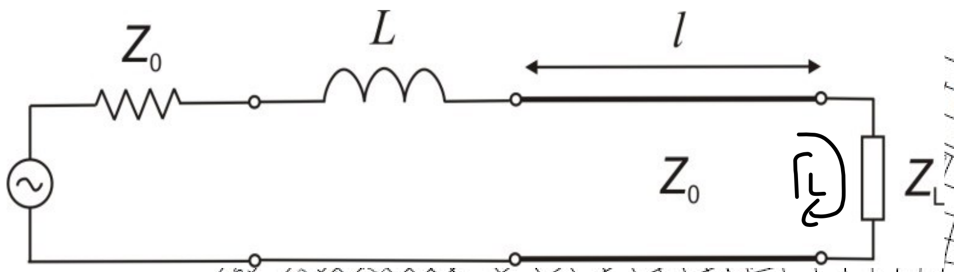
Hint: you need only the impedance scale.



Return your effort at 12:30 in MyCourses.



Today's in-class task



The input impedance of a load is $Z_L = (100.0 + j100.0)\ \Omega$ at 1.5 GHz. $Z_0 = 50\ \Omega$.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100+j100)\Omega - 50\Omega}{(100+j100)\Omega + 50\Omega} = 0{,}539 + j0{,}308$$

$|\Gamma_L| = 0{,}602$ reflected-% $= 100\% \cdot (1 - |\Gamma_L|^2) = 38\%$

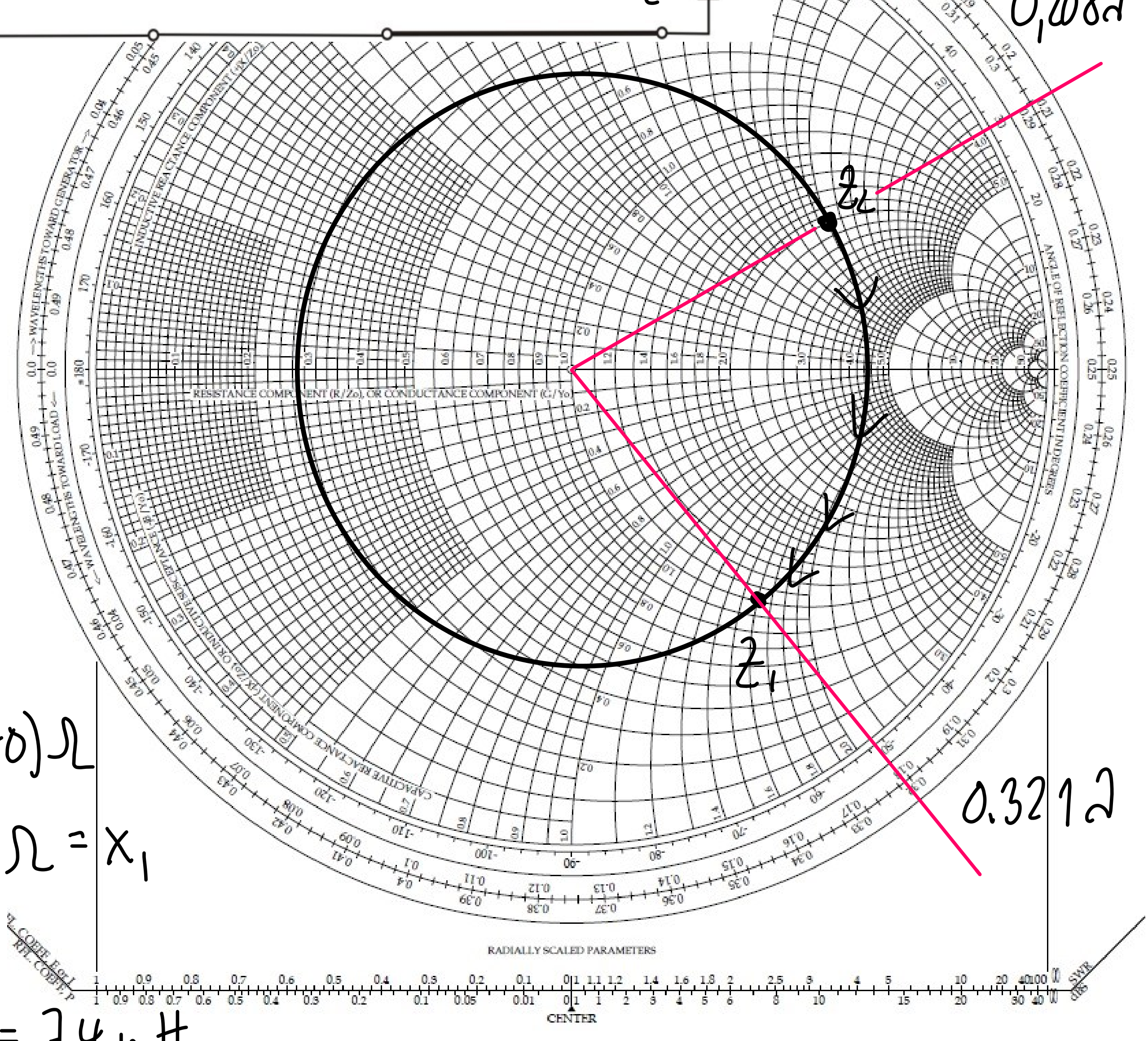
$$z_L = \frac{Z_L}{Z_0} = \frac{(100+j100)\Omega}{50\Omega} = 2.0 + j2.0\ (50\Omega)$$

rotate $0{,}321\lambda - 0{,}208\lambda = 0{,}113\lambda = l$

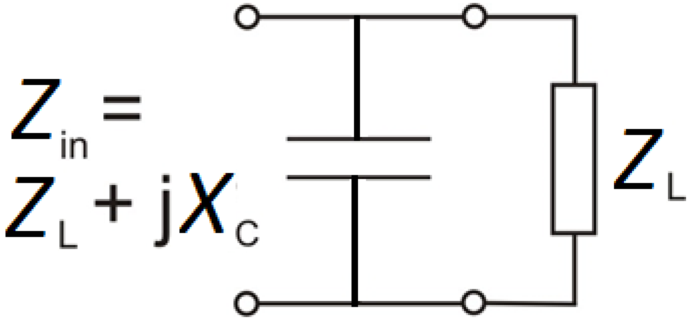
$z_1 = 1{,}0 - j1{,}4$; $Z_1 = z_1 \cdot Z_0 = (1 - j1{,}4) \cdot 50\Omega = (50 - j70)\Omega$

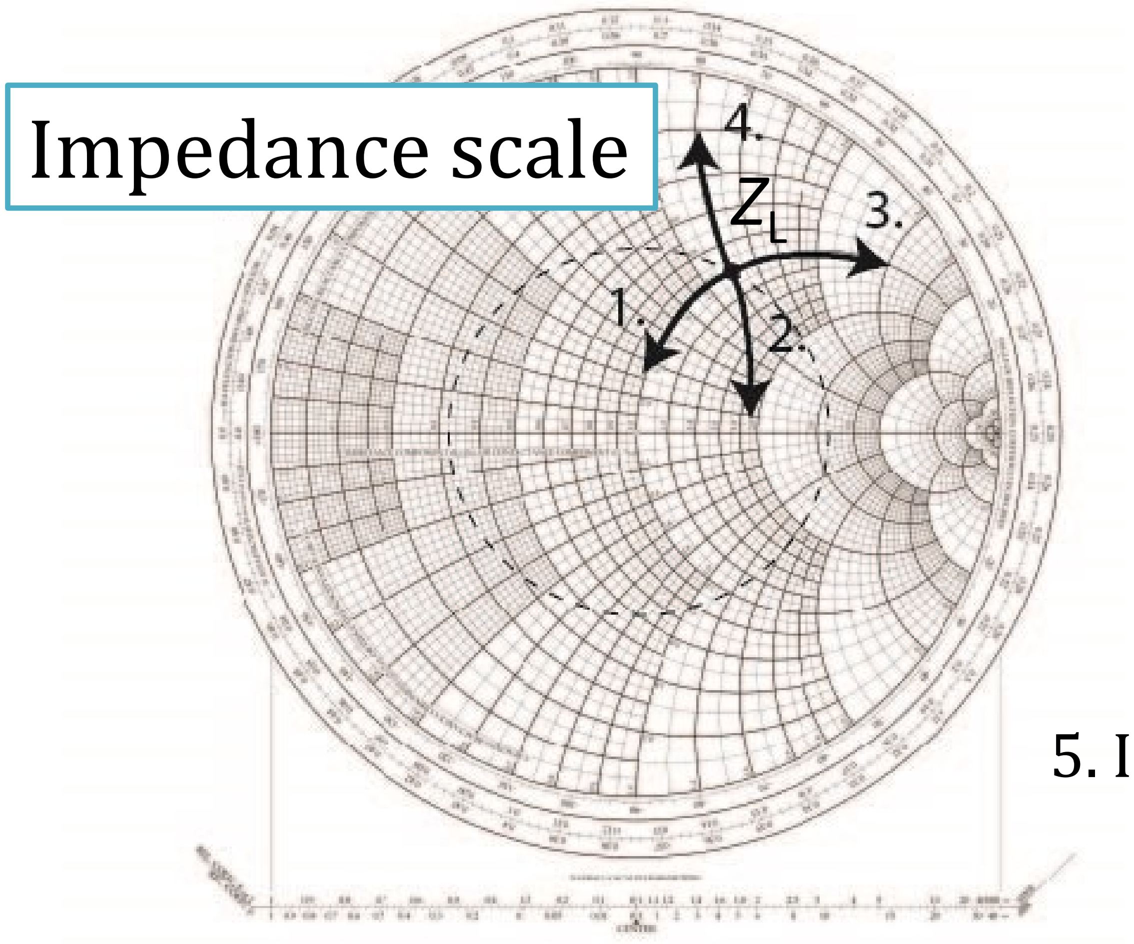
The needed inductance has a reactance of $70\Omega = X_1$

$$\omega L = X_1 \Rightarrow L = \frac{X_1}{2\pi f} = \frac{70\,\Omega}{2\pi \cdot 1.5\,\text{GHz}} = \frac{70}{2\pi \cdot 1.5} \cdot 10^{-9}\,\text{H} = 7.4\,\text{nH}$$



Q5: Which of the following transitions on the Smith chart (1-4) corresponds to adding **a shunt capacitor** in the **impedance** scale?

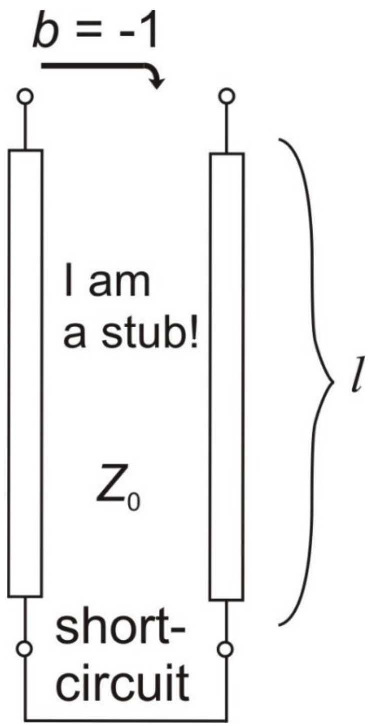


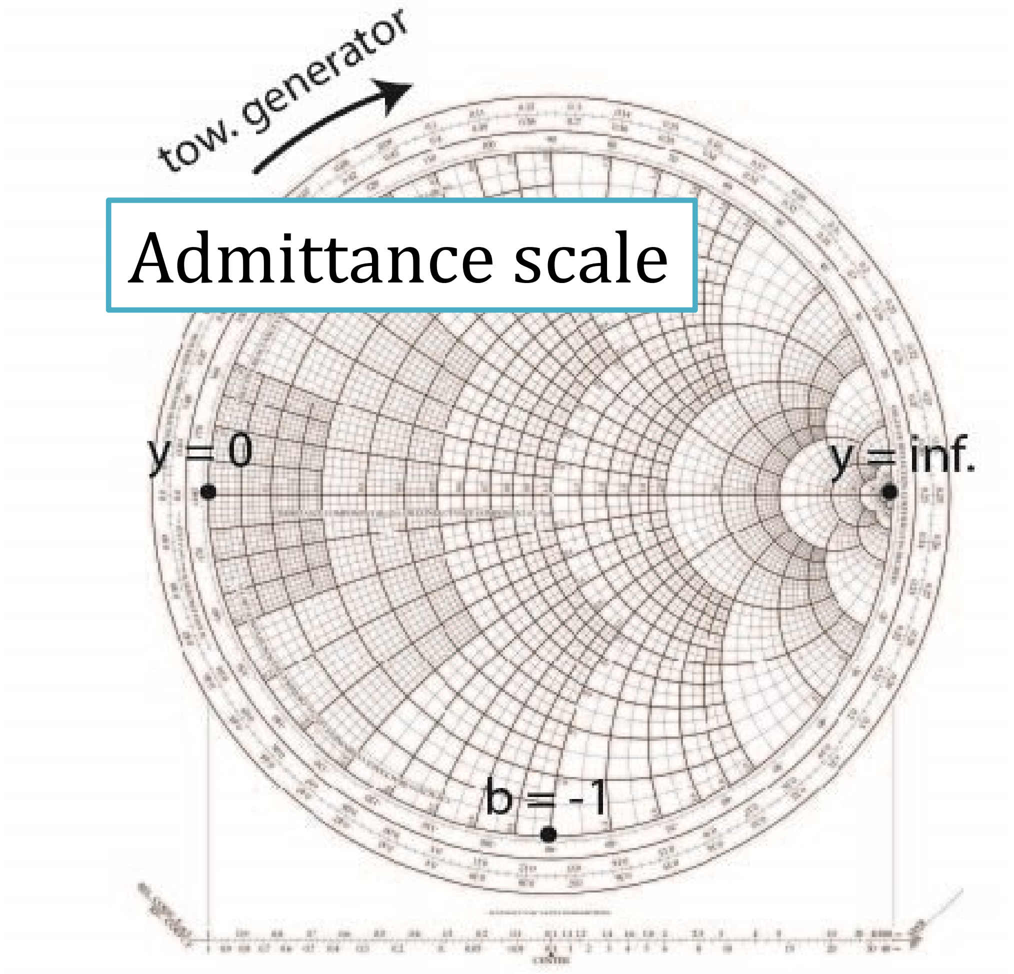


5. I don't know

Q7: The previous problem continues. What is the length of the stub in λ? ($y = \mathrm{j}b = -\mathrm{j} \cdot 1.0$)

1. $\lambda/32$
2. $\lambda/16$
3. $\lambda/8$
4. $\lambda/4$
5. $\lambda/2$
6. I don't know

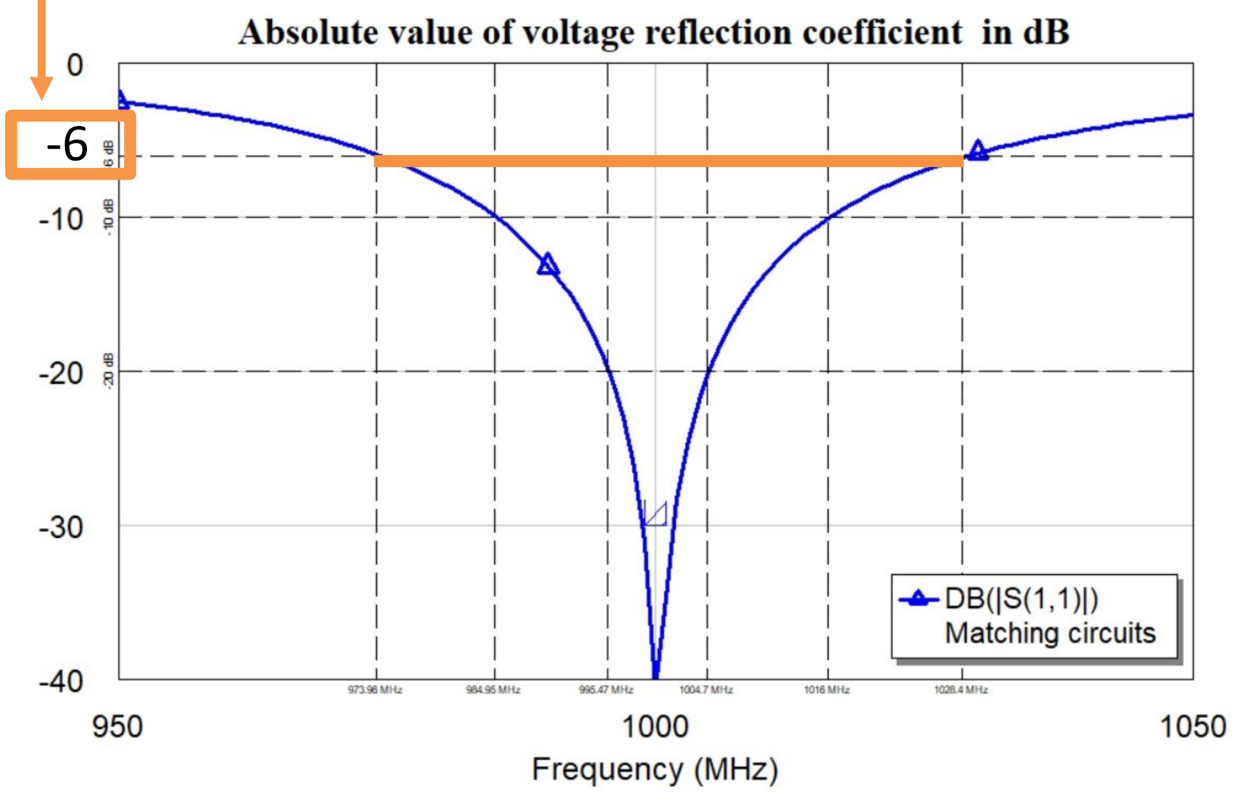




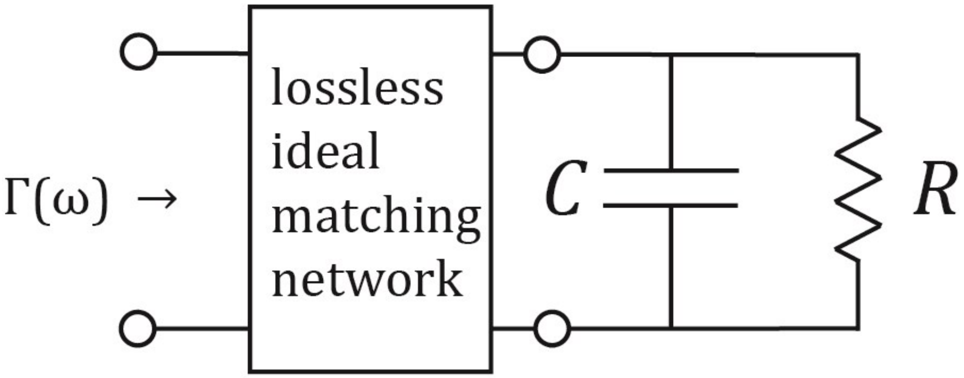
Q8: How much **power** (in %) is **delivered to the load** if the reflection coefficient is **-6 dB**? (Neglect any other losses.)

1. 25%
2. 50%
3. 75%
4. 90%
5. 99%
6. I don't know

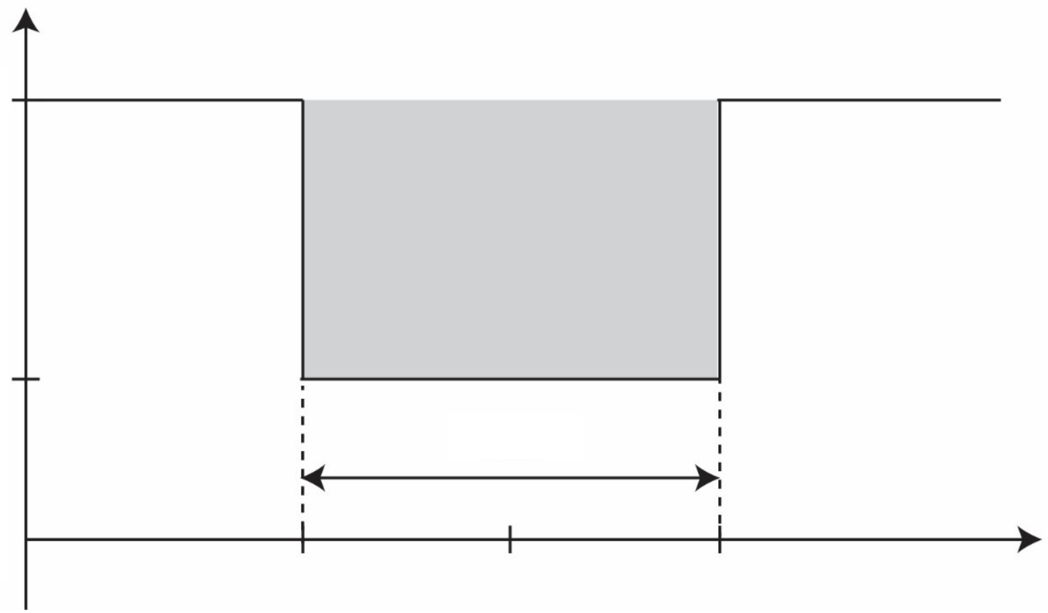
$$\Gamma(\text{dB}) = 10\log_{10}|\Gamma|^2 = 20\log_{10}|\Gamma|$$



The Bode-Fano criterion is related to matching level and impedance bandwidth

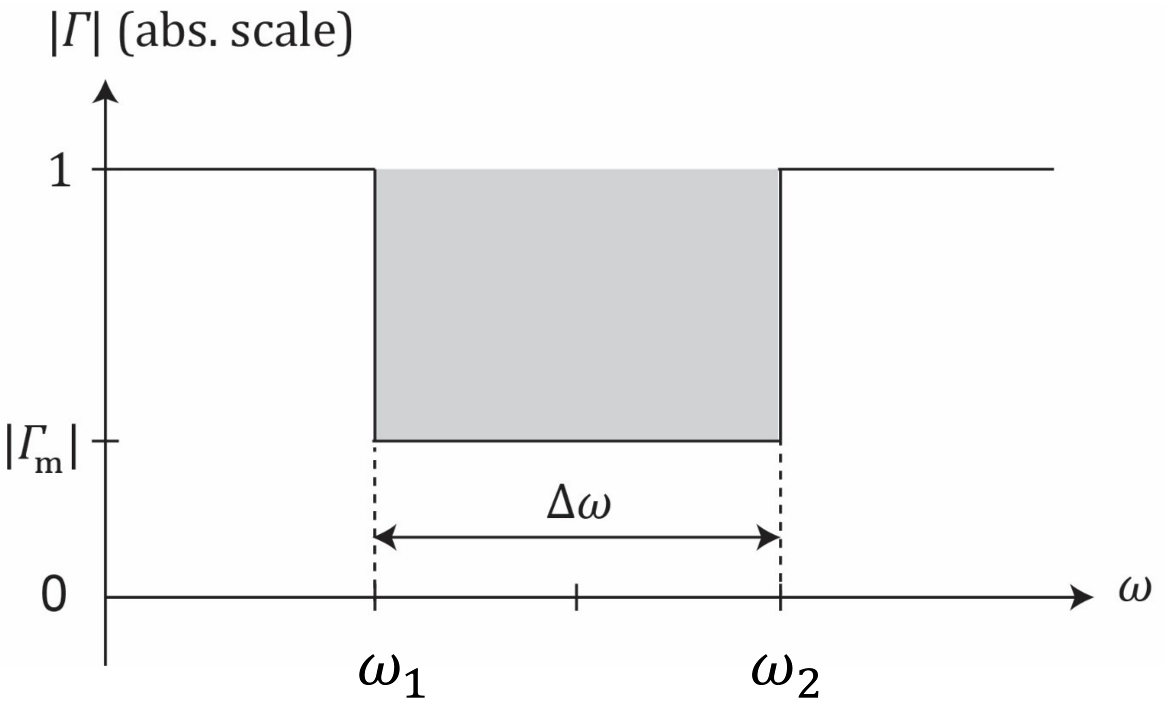


$$\int_0^\infty \ln\frac{1}{|\Gamma(\omega)|}\,d\omega < \frac{\pi}{RC}$$

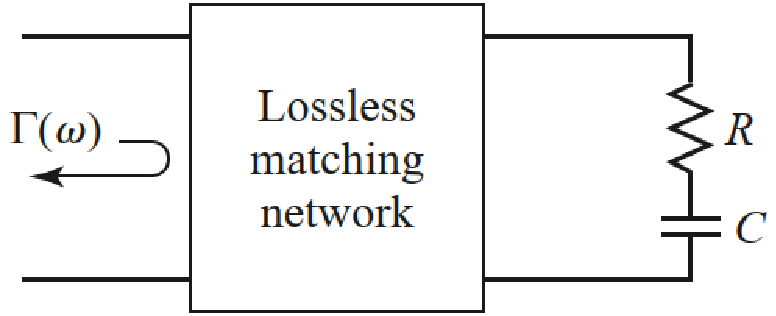


Perfect matching over finite bandwidth is impossible

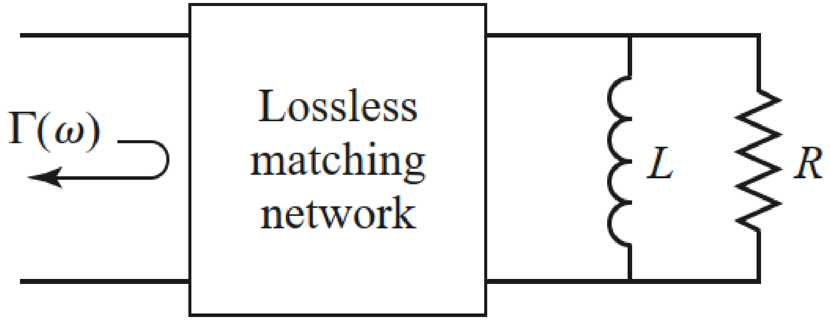
$$\Delta\omega \ln \frac{1}{|\Gamma_m|} < \frac{\pi}{RC}$$



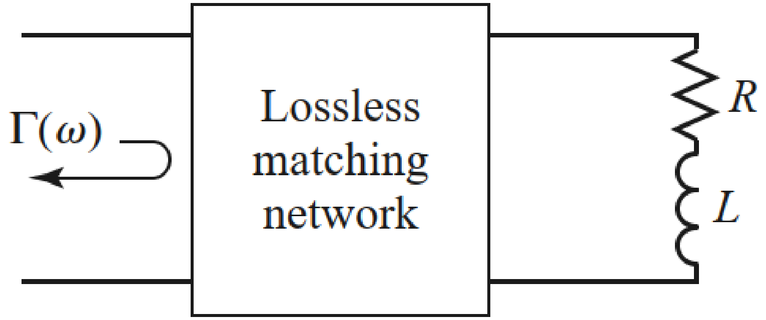
The Bode-Fano formula depends on the load impedance



$$\int_0^\infty \frac{1}{\omega^2} \ln \frac{1}{|\Gamma(\omega)|} \, d\omega < \pi RC$$



$$\int_0^\infty \frac{1}{\omega^2} \ln \frac{1}{|\Gamma(\omega)|} \, d\omega < \frac{\pi L}{R}$$



$$\int_0^\infty \ln \frac{1}{|\Gamma(\omega)|} \, d\omega < \frac{\pi R}{L}$$

Takes homes of the Bode-Fano criterion