Exercise problems of topic 3: Analysis of microwave circuits

Write your answers clearly, so that the answer proceeds logically and includes necessary intermediate steps and enough explanations. Your answer should be understandable without oral explanations, too. See further instructions for systematic problem solving in MyCourses.

The exercise problem answers are to be returned during a pre-booked appointment on Mondays or in a separately agreed appointment. The minimum is to return at least one answer per week. If you cannot meet this, you lose a chance to earn those points also later. It is good to note that average of two returned answers per week would be optimal to complete the course with flat speed.

Topic 3 consists of **four (4)** problems and an extra problem (3.E). Be prepared to explain your answer to the teacher. The purpose of this returning method is to enhance your learning through two-way communication and constructive feedback. Your answer will be graded in the scale of 0-3 points.

Exercise problem 3.1. The impedance of a surface-mounted lumped capacitor can be modelled with a series RLC equivalent circuit, see the figure below. The datasheet of the capacitor provides the following information:

- The nominal capacitance of the capacitor is C = 3.0 pF,
- The resistor R models the resistive losses,
- The inductor L is the parasitic inductance,
- The self-resonant frequency of the capacitor is f_r = 2.46 GHz
- The unloaded quality factor Q_0 (defined at the resonant frequency f_r) is 43.

a. Solve the component values L and R of the equivalent circuit of the capacitor. Use units in your all calculations.

b. At which frequencies is the capacitor capacitive? When it is inductive? Discuss insight of this question.

c. The capacitor is used as a filter to block a strong Wi-Fi signal at 2.4-2.5 GHz, see figure below. Hence, the capacitor is soldered between the lines of a transmission line (Z_0=50 Ω). At which frequency does the capacitor attenuate the most and what is the attenuation in dB?

d. Simulate the filter of part c. with AWRDE. Use the component called "CHIPCAP" and give the parameters C, f_r and Q_0 as defined in this problem. Do you get the same result as in c. part? If not, try to explain why.

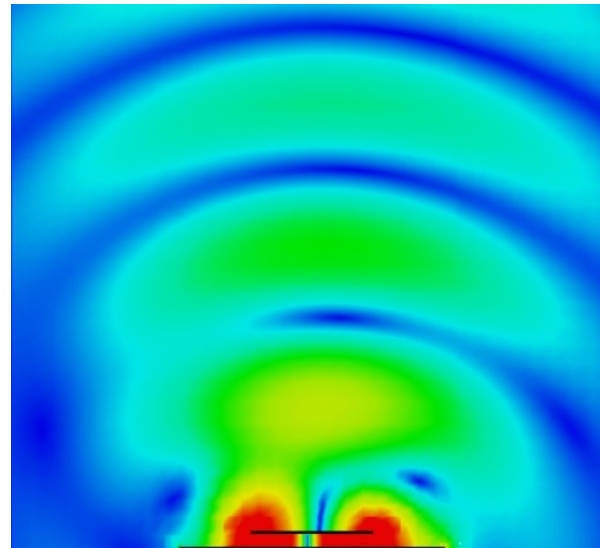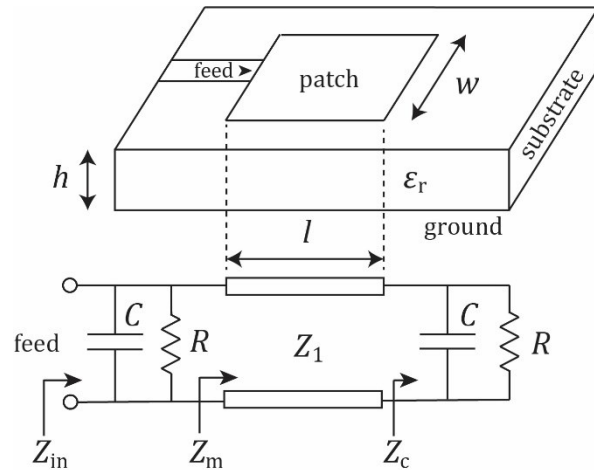*Hint: simulate the S parameter DB|S(2,1)| that gives the coupling between the ports 1 and 2.*
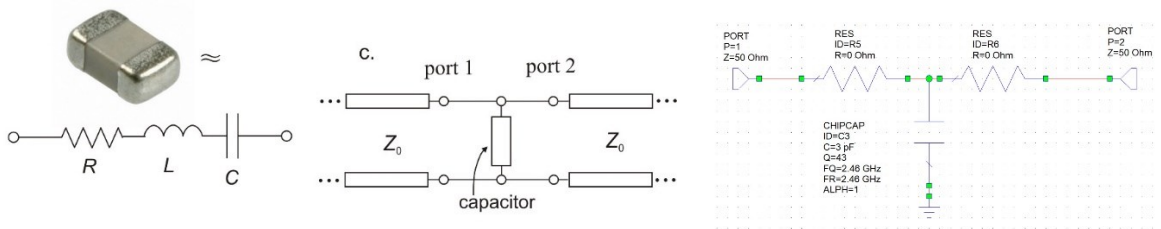
Exercise problem 3.2. In this problem, you can use analytic formulas, Matlab/Mathematica and/or AWRDE.

The upper figure shows a sketch and an equivalent circuit of a microstrip patch antenna whose operation is based on the resonance phenomenon. The microstrip patch antennas are widely used in planar applications.

The parallel RC circuits model the *radiation* ($R = 100\ \Omega$) of the antenna and the *fringing electric fields* ($C = 1.6$ pF) that occur at the open ends of the patch [1], see the lower figure. The patch itself can be modelled as a piece of microstrip line whose length $l = 40$ mm and width $w = 60$ mm. The parameters of the FR-4 substrate are the relative dielectric constant $\varepsilon_r = 4.3$ and the thickness of the substrate $h = 5.0$ mm. Let us assume that the substrate is lossless and non-dispersive for simplicity.

In case you select to use AWRDE, describe the main steps for solving the problem analytically and give the essential formulas for solving Z_c, Z_m and Z_{in}, too.

a. Define the characteristic impedance Z_1 and the effective dielectric constant ε_e of the patch (microstrip line). The antenna operates around 1–2 GHz.

b. Define the input impedance Z_{in} of the equivalent circuit of the antenna and plot the magnitude of the input reflection coefficient $|\Gamma(f)|$ as a function of frequency in the dB scale in the frequency range of 1–2 GHz. The reference impedance is $Z_0 = 50\ \Omega$.

The magnitude of the electric fields of a microstrip patch antenna. Red colour = strong E field, blue = weak E field.

c. Read from the $|\Gamma(f)|$ plot of b-part: at which frequency (within 1–2 GHz) the antenna is in resonance – i.e., matched to a 50-Ω feed? What is the -10-dB frequency range and bandwidth?

d. How is it possible that the circuit has a strong resonance even though we cannot see any inductor component in the equivalent circuit? Discuss any additional insight that you learnt with the problem.

[1] W.L. Stutzman, G. A. Thiele, *Antenna Theory and Design*, Hoboken, New Jersey, USA: John Wiley & Sons Inc., 2013.

Exercise problem 3.3. An amplifier consists of two *quadrature hybrids* (also called 90 degrees or branch-line hybrid) and two transistors. The hybrids are modelled with the S-parameter matrix shown below. For the transistors a and b, S_{11a} and S_{11b} denote the input reflection coefficients and S_{21a} and S_{21b} the voltage transmission coefficients (voltage amplification), respectively. All the S-parameters are defined with respect to $Z_0 = 50$ Ω. For simplicity, assume that the transistors are unilateral – i.e., $S_{12a} = S_{12b} = 0$. In addition, we denote V_i^+ the amplitude of the **in**cident wave in port i, P_i^+ the corresponding power, V_i^- the amplitude of the wave coming **out** in port i and P_i^- the corresponding power. Port 1 is the input and port 7 is the output of the amplifier.

a. Is the quadrature hybrid ideally lossless?

b. Define the expressions for the input reflection coefficient Γ_{in} (as a function of S_{11a} and S_{11b}) and the power gain G (as a function of S_{21a} and S_{21b}) for the amplifier.

$$\Gamma_{in}(S_{11a}, S_{11b}) = \frac{V_1^-}{V_1^+} \quad \text{and} \quad G(S_{21a}, S_{21b}) = \frac{P_7^-}{P_1^+} = \left|\frac{V_7^-}{V_1^+}\right|^2$$

c. Derive expression for the powers P_4^- and P_6^- that terminate at the matched loads in ports 4 and 6.

d. What do the expressions for Γ_{in}, G, P_4^- and P_6^- become if the transistors are identical – i.e., the S-parameters of the transistors satisfy $S_{11a} = S_{11b}$ and $S_{21a} = S_{21b}$?
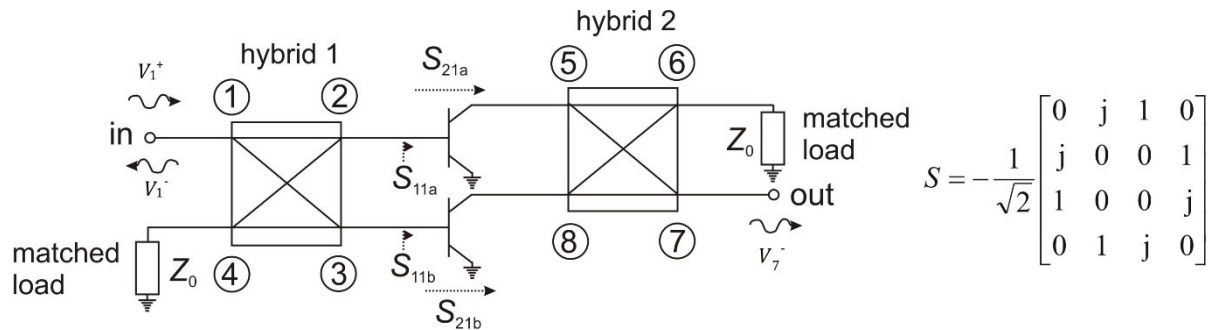
Exercise problem 3.4. The output power of a matched generator is 5 dBm. That power is split into two ports with a resistive power splitter whose S-parameter (Z_0 = 50 Ω) matrix (see Pozar Chapter 7.2) is

$$S = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

a. Is the power splitter ideally lossless?

b. Both the output ports of the power splitter are terminated with identical half-wave dipole antennas, whose radiation efficiency is 0.79 and load impedance Z_L = 70 Ω. Assume that the electromagnetic isolation between the antennas is very high. Calculate analytically the power radiated (dBm) by one antenna. What is the radiated power of the whole system (dBm)?

Hint: this can be solved with the help of the matrix equation: $[V^-] = [S]\cdot[V^+]$, in which the reflection coefficient Γ_L of the load is written with the help of V^+ and V^-.

c. Simulate the circuit system with AWRDE. Find the circuit schematic of the power splitter in Pozar Chapter 7.2. The antenna impedance (resistance) can be modelled as the loss R_{loss} and radiation resistance R_{rad} in series.

d. Change in AWRDE: Z_L = 50 Ω (perfect impedance matching). Simulate the power radiated by one antenna. What surprising observation can be done compared to the results of a. and b. parts? Try to find an explanation.
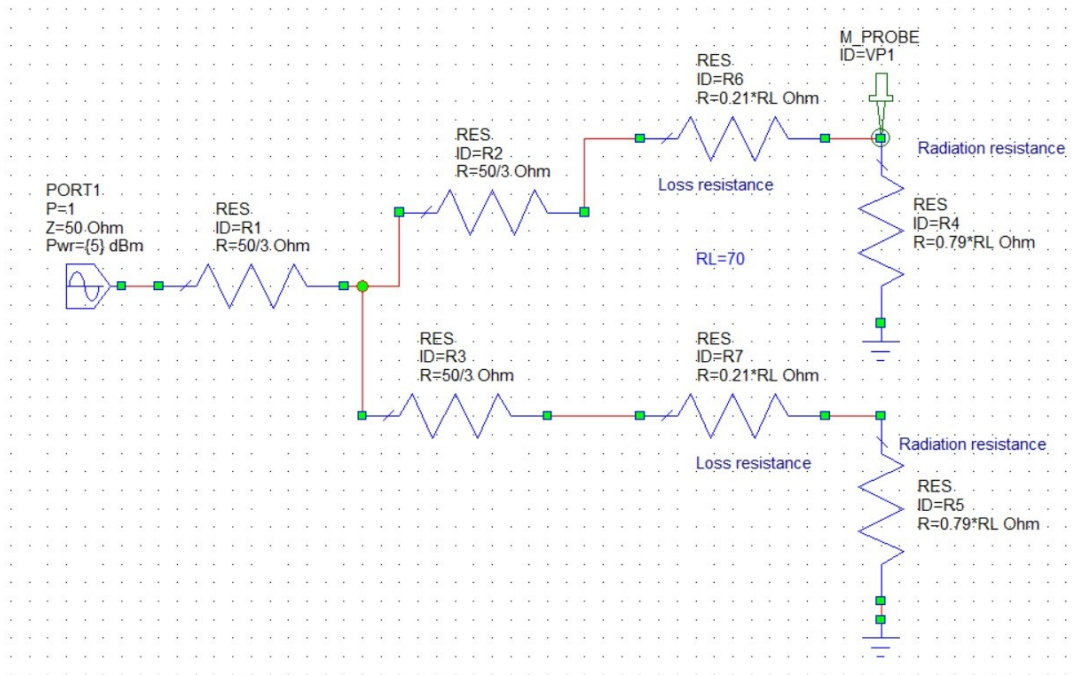
Exercise problem 3.EXTRA. You can earn three (3) **extra** points to exercise problems – i.e., it can compensate any skipped exercise problem.

a. Write analytically the 2 x 2 *transmission* (ABCD) *matrix* for each LC resonator blocks 1-3 of the circuit shown below. Calculate numerically (e.g., with Matlab or Mathematica) the total transmission matrix of a cascaded connection of the three-resonator block. Next, convert the total transmission matrix into the S-parameter matrix. Finally, plot the scattering parameters $|S_{11}|$ and $|S_{21}|$ in dB as a function of frequency in the frequency range of 0.70 – 1.2 GHz to the same Cartesian coordinate system, for example, with Matlab or Mathematica. What is the functionality of the circuit?

    Hint: The transmission matrix and the conversions between the network parameters are introduced in the course book chapter 4.4 "The transmission (ABCD) matrix".

b. Calculate the resonant frequency of each resonator blocks 1-3. Compare them with the resonant frequencies of the frequency response of $|S_{11}|$ plotted in a. part. What interesting (also surprising?) discovery can you make?

c. Simulate the filter with AWRDE and compare with the a. part curves.
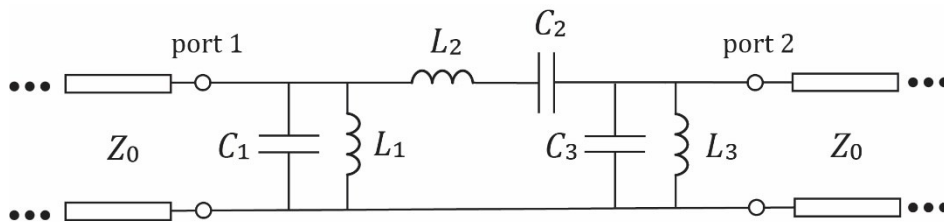
$C_1 = C_3 = 40$ pF, $L_1 = L_3 = 0.72$ nH, $C_2 = 0.40$ pF, $L_2 = 72$ nH