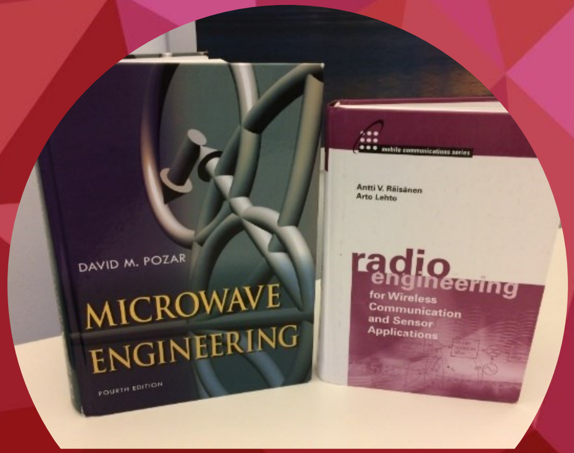
Microwave engineering I (MiWE I)

Interactive lecture 1 of Topic 3

Resonators

February 10, 2022

The main learning outcome of the course is to create readiness to work in microwave engineering related tasks and projects and enable further studies and continuous learning in microwave engineering.



Topic 3: Learning outcomes and content

- The student can
  - **analyze** the operation of basic microwave circuits and resonators based on calculations and simulations (AWRDE).
  - **model** and **analyze** the operation of microwave circuits and resonators with suitable circuit parameters, especially the scattering parameters (S-parameters).

- Series and parallel resonant circuits (Pozar chapter 6.1)
- The scattering matrix (Pozar chapter 4.3)
- The transmission (ABCD) matrix (Pozar chapter 4.4

These lecture slides and notes are not designed for self-study.
Please, use the course book chapters 4 and 6 for self-study.

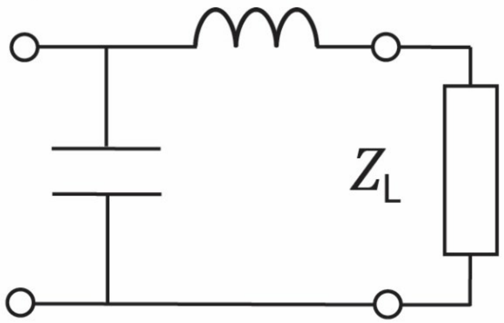
AWR demo on last week's pre and in-class tasks

Pre task: $Z_L = (15 + j15)\ \Omega$

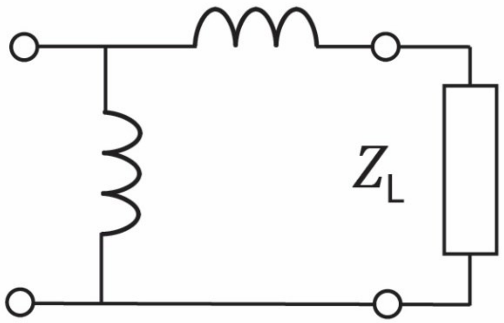
In-class task: $Z_L = (100.0 + j100.0)\ \Omega$

at $f = 1.5$ GHz

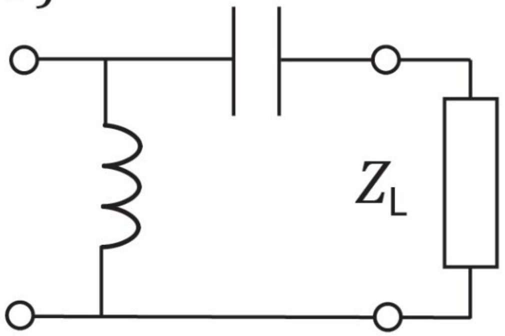
1)



2)



3)



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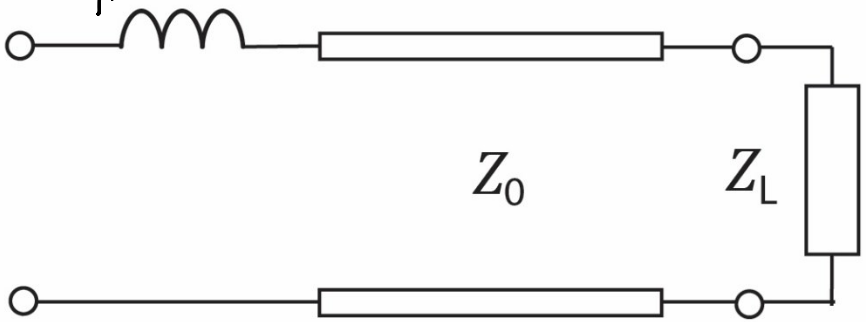
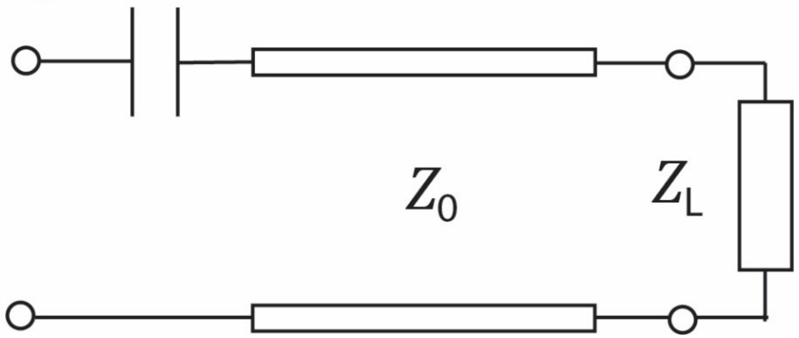


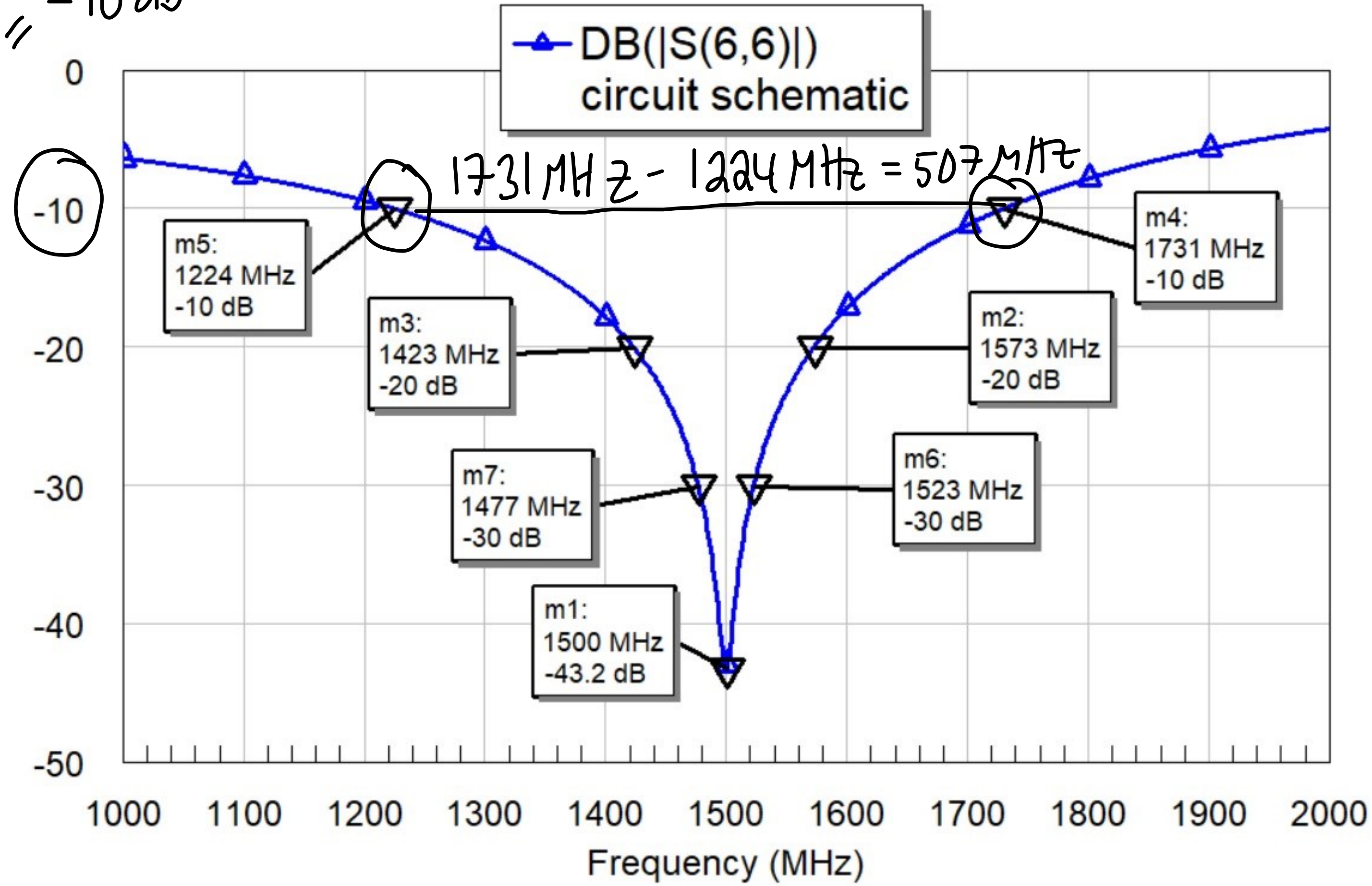
5)



Q1: The figure shows the frequency response of a resonant component. What is the approximated **impedance bandwidth (MHz)** where **less than 10 %** of the input power is reflected due to mismatching?

$|\Gamma_m|^2 = 0.10 \rightarrow 10\log_{10}|\Gamma_m|^2 = 10\lg 0.10 = -10\,\text{dB}$

1. 0 MHz
2. 50 MHz — 15%.
3. 150 MHz — 27%.
4. 300 MHz
5. [✓ 5.] 500 MHz — 45%.
6. More than 1000 MHz — 6%.
7. I don't know — 6%.



1731 MHz - 1224 MHz = 507 MHz

dB $10\log_{10}\frac{P_{refl}}{P_{in}} = 10\log_{10}\frac{\frac{|V^-|^2}{\cancel{2Z_0}}}{\frac{|V^+|^2}{\cancel{2Z_0}}} = 10\log\left|\frac{V^-}{V^+}\right|^2 = 10\log|\Gamma|^2 = 20\log|\Gamma| = 20\log 0.316$

$\simeq \underline{\underline{-10\text{ dB}}}$

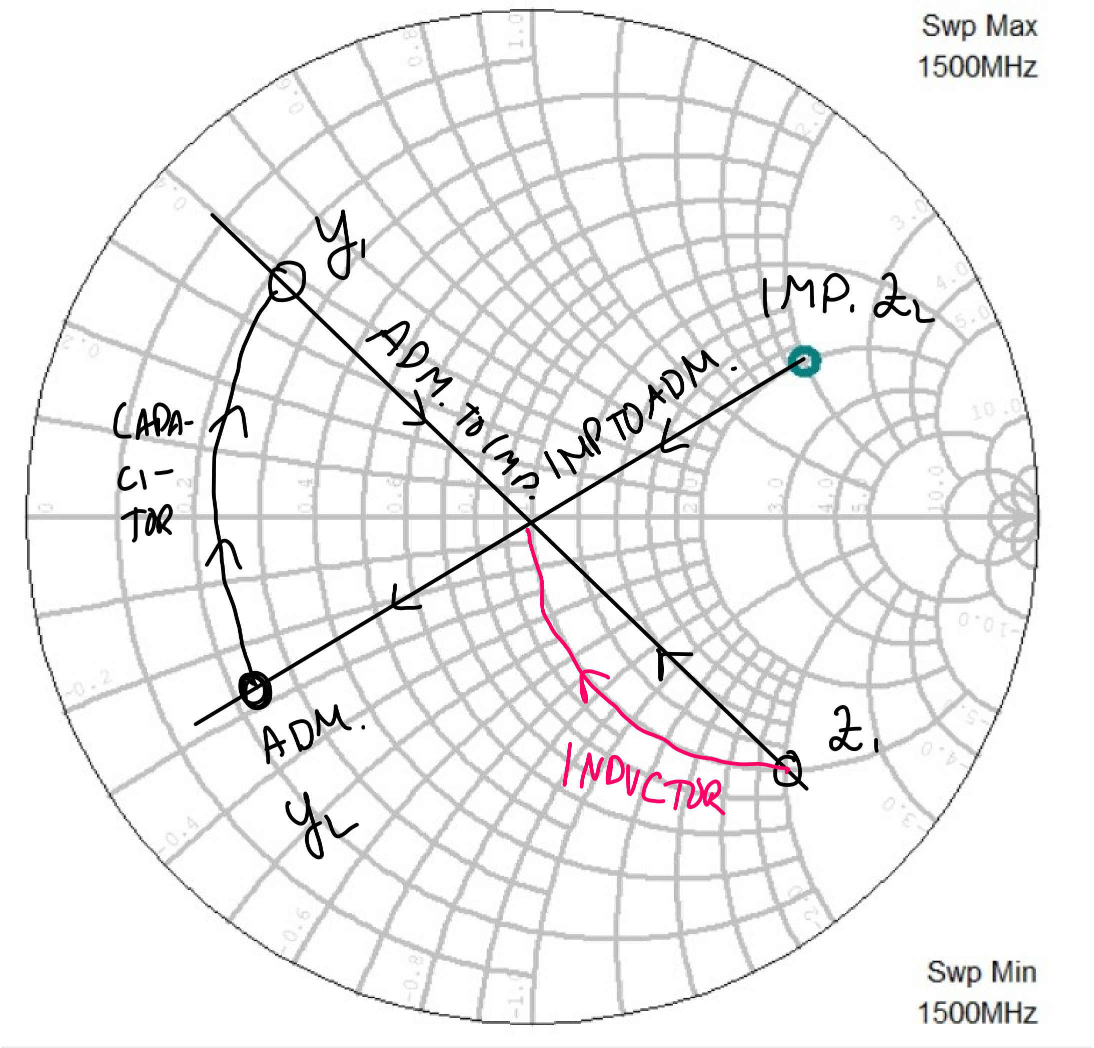
$|\Gamma|^2 = 0.10 \qquad |\Gamma| = 0.316$

POLLING AT 10:25

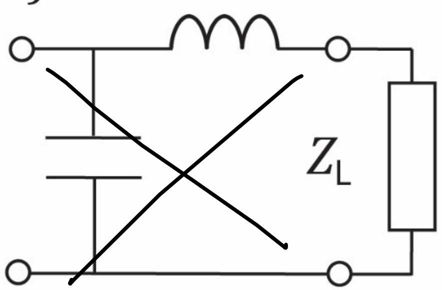
TEACHING CONTINUES 10:30

– enjoy the break now

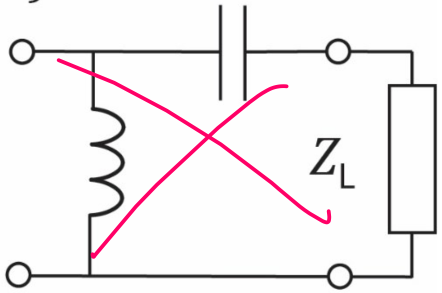
Q2: Which of the topologies can be used for perfectly matching the given load impedance?



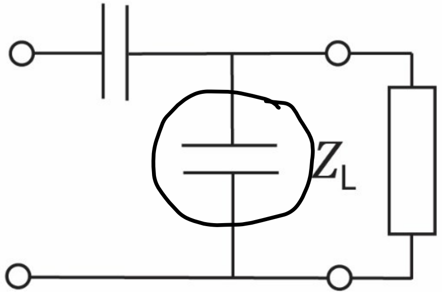
1) 10%



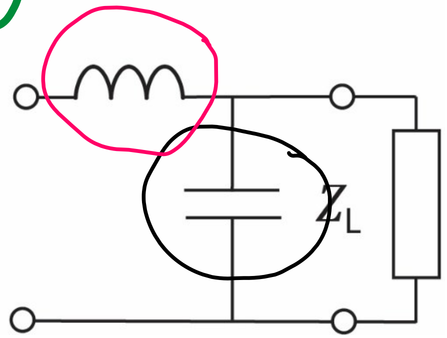
2) 27%



3) 13%



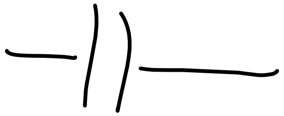
4) 37%



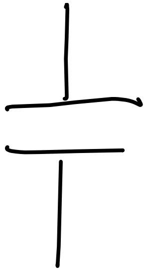
5) I don't know. 13%

$s = j\omega$

$Z = \overbrace{j\omega L}^{SL}$ $\omega L = X > 0$ → upper side of Smith chart



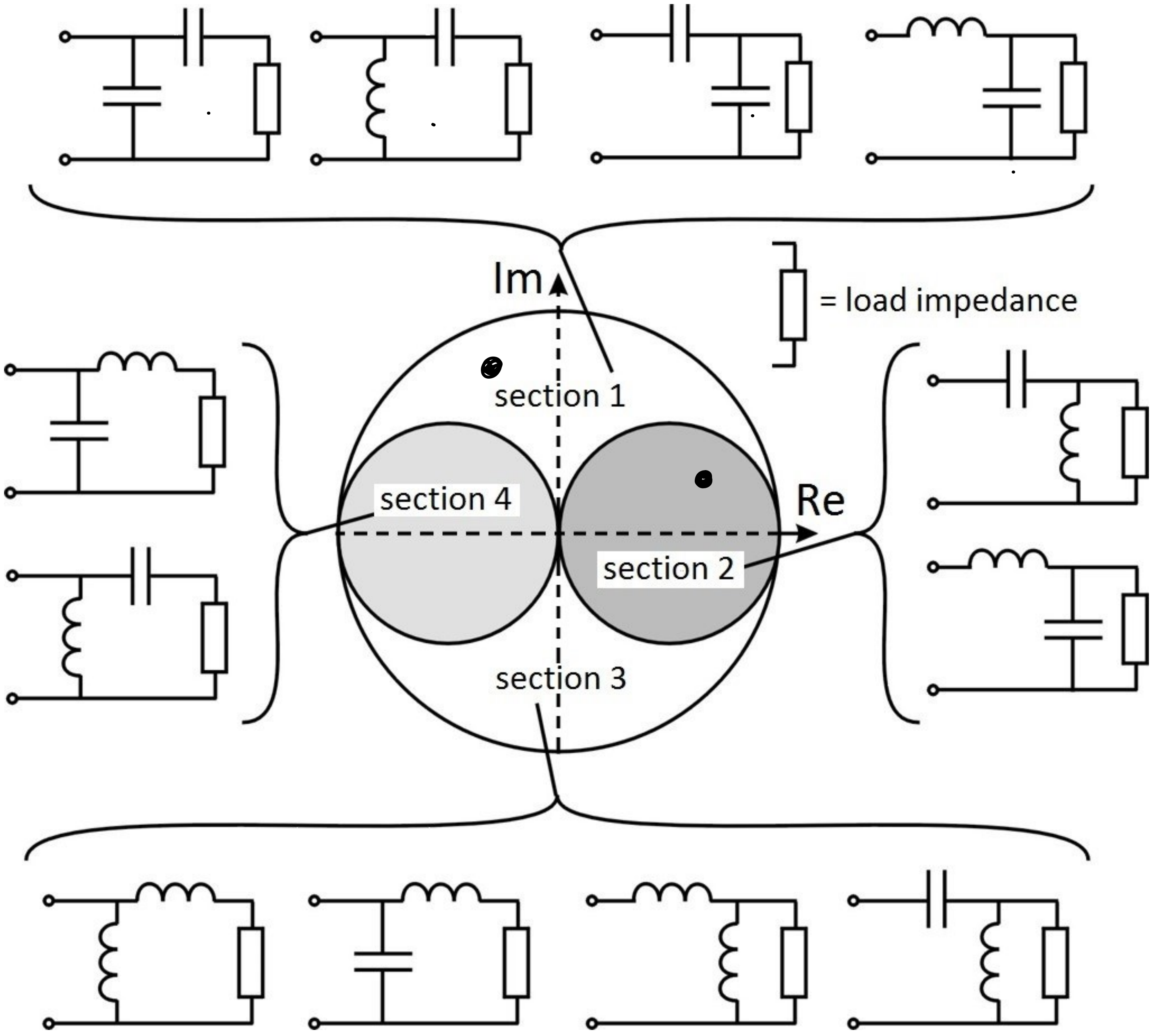
$Z = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$ $X = -\frac{1}{\omega C} < 0$ → lower side of SC



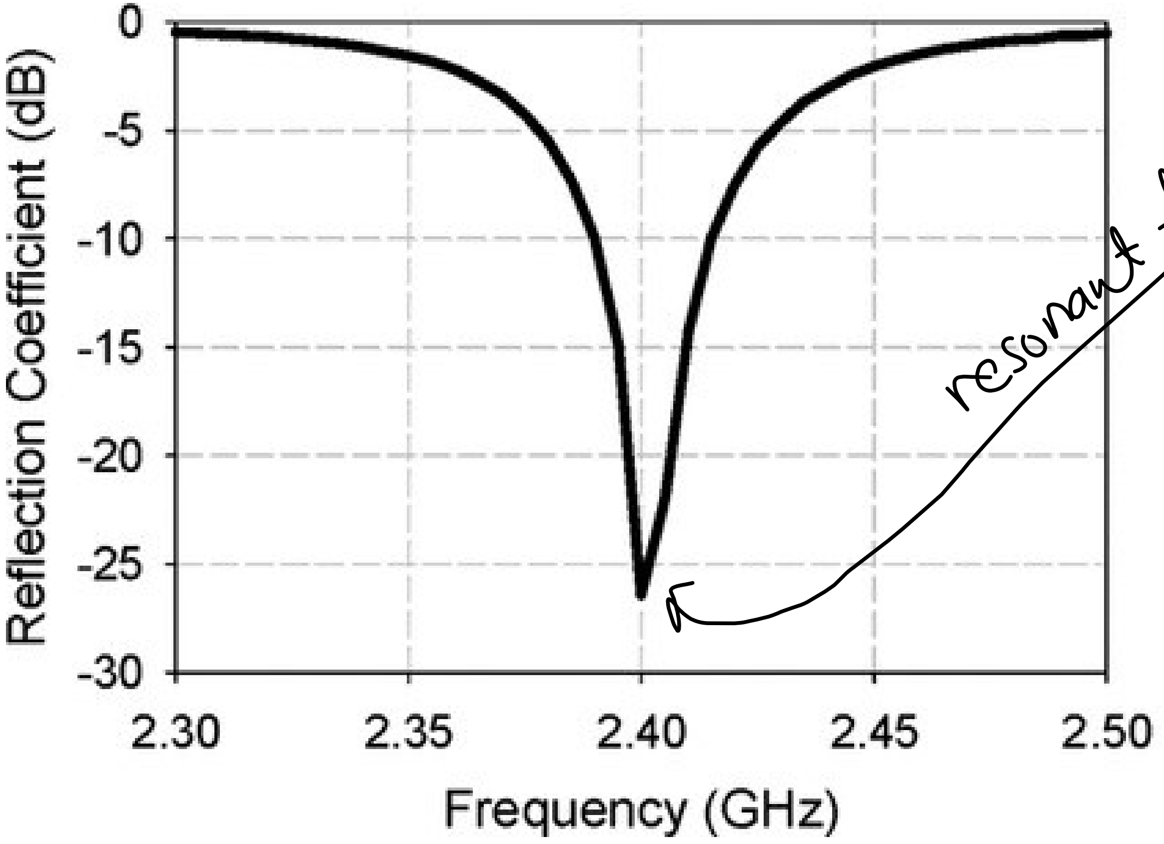
$Y = j\omega C$ $\omega C = B > 0$ → upper side of SC

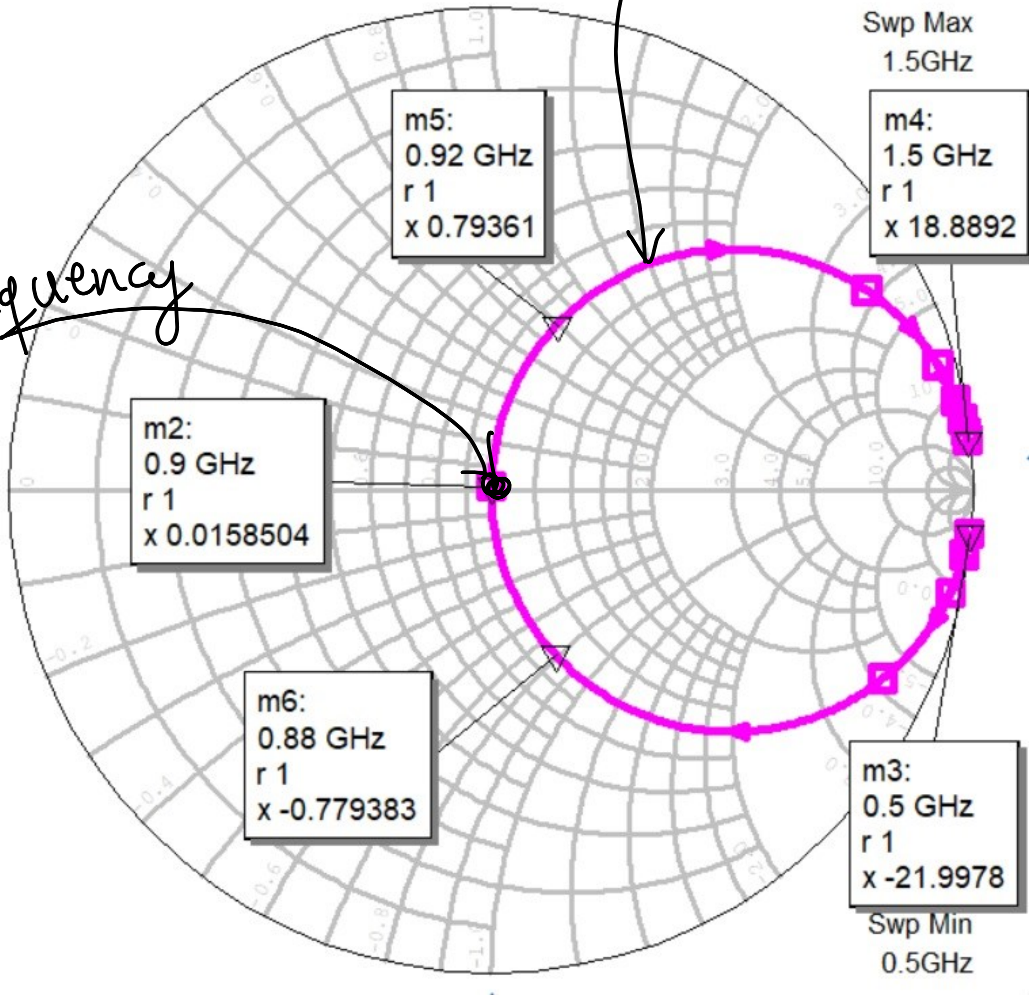
$Y = \frac{1}{j\omega L} = -j\frac{1}{\omega L}$ $B = -\frac{1}{\omega L} < 0$ → lower side of SC

The useable matching circuit topology depends on the load impedance



Resonant behaviour can be observed from the frequency response





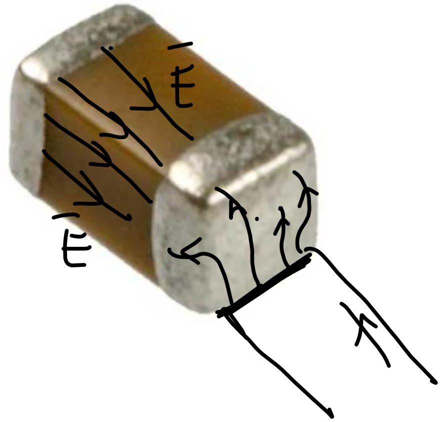
resonant frequency

series RLC
resonator

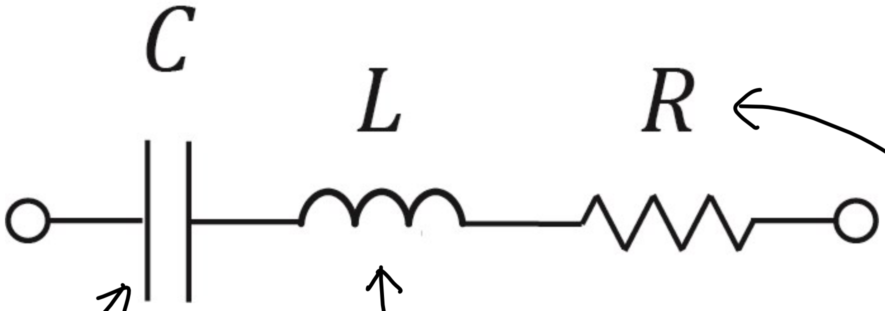
Lumped elements are resonators in practice

3.1 exercise problem

surface-mounted capacitor



≈

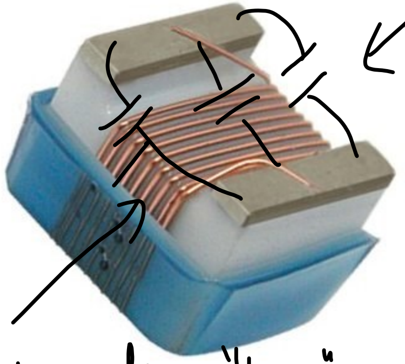


parasitic inductor

wanted

resistive losses

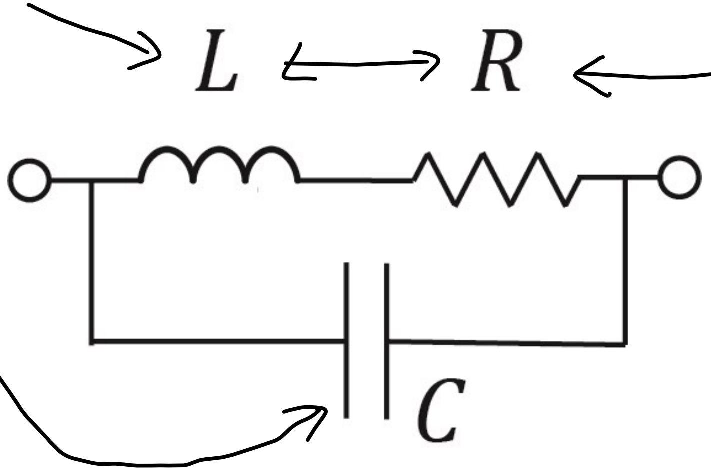
surface-mounted inductor



many rounds → "large" inductance

≈

parasitic capacitor

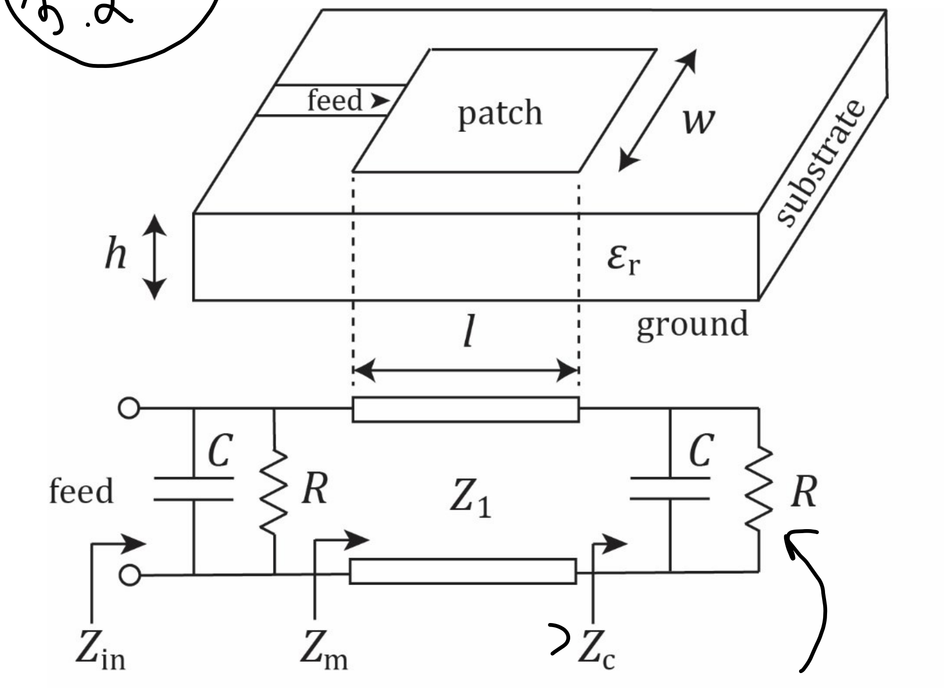


the effect is small at "low" freq.

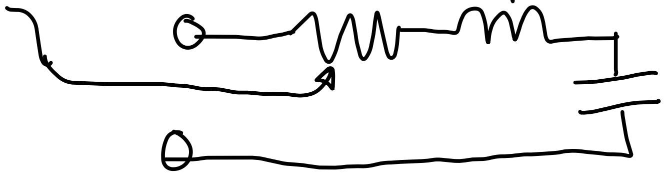
Why don't we "care" about the parasitic effects at "low" frequencies?

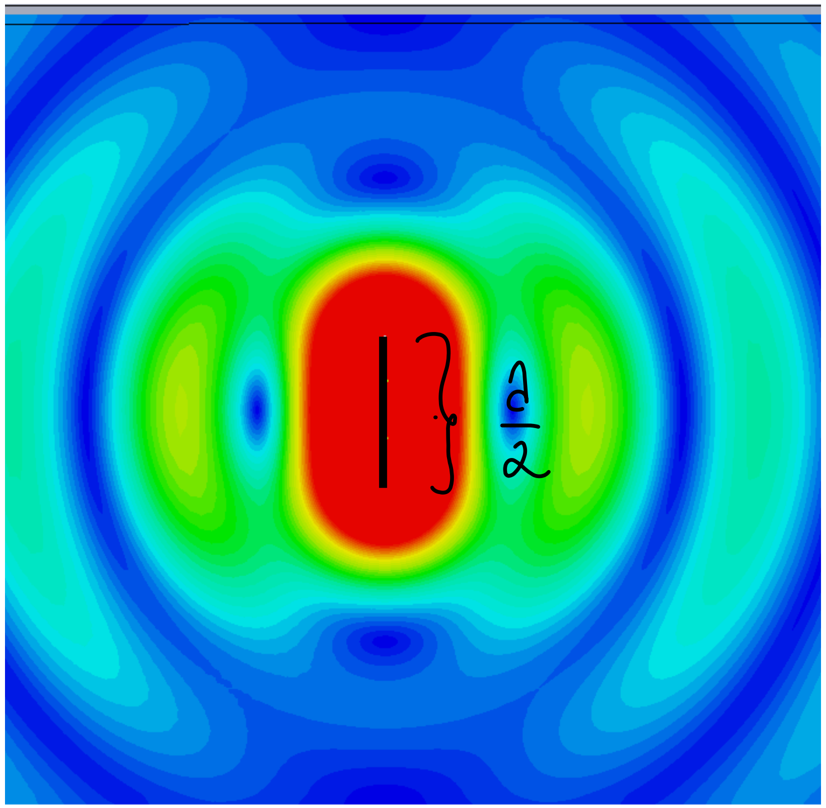
Operation of many antennas is based on the resonance phenomenon

problem 3.2

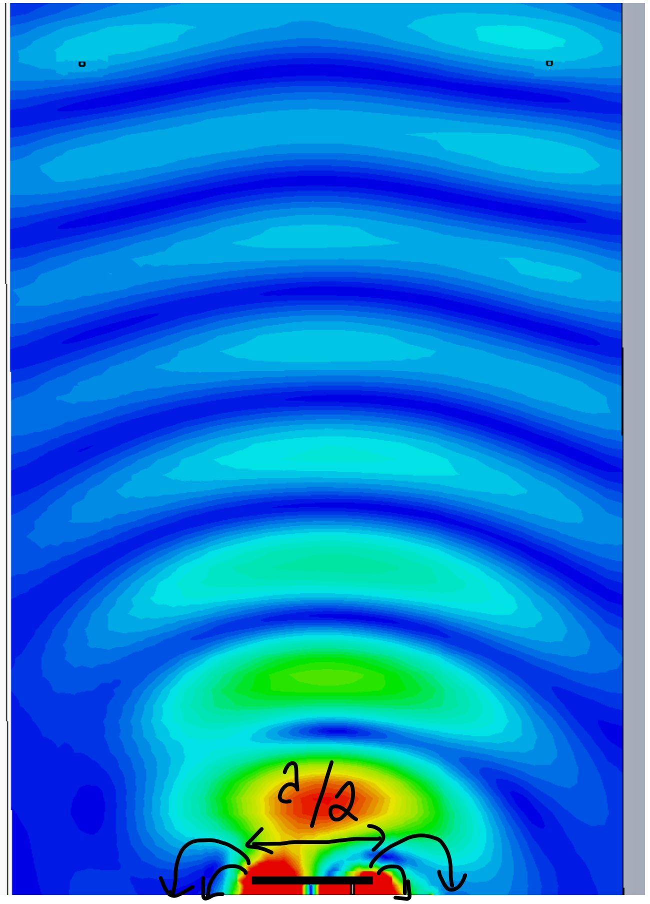


resistive losses are radiation



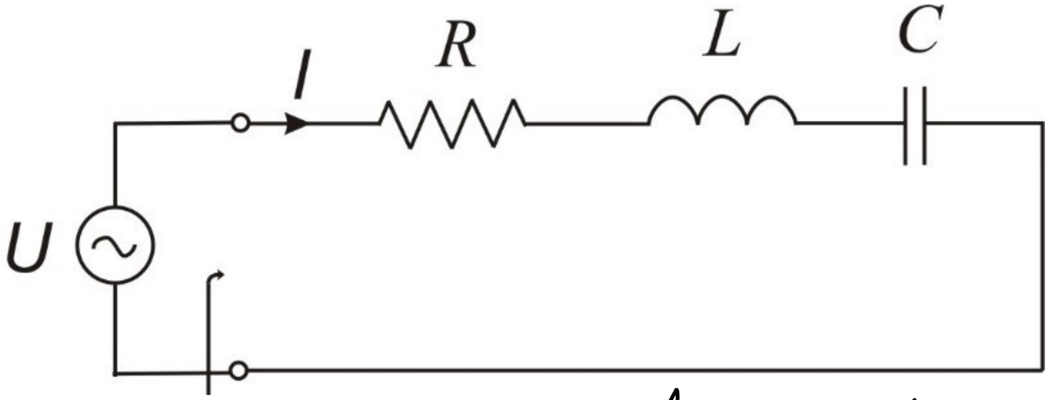


half-wave dipole antenna



half-wave microstrip patch antenna

Series (or parallel) RLC circuit is the simplest form of resonator circuit



$$Z_{in} = R + j\omega L + \frac{1}{j\omega C} = R + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{X}$$

$$X = \omega L - \frac{1}{\omega C}$$

when $X = 0$ -> current I "peaks"

$I = \frac{U}{R}$ in resonance

$$X = 0 \Rightarrow \quad \omega = \frac{1}{\sqrt{LC}}$$

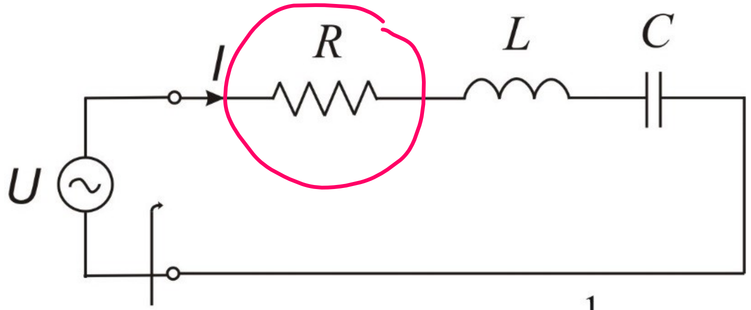
RMS power:
complex power

$$P_{in} = \frac{1}{2} U I^* = \frac{1}{2}(Z I) I^* = \frac{1}{2} Z \overbrace{I \cdot I^*}^{|I|^2}$$

$$= \frac{1}{2}\left[R + j\omega\left(L - \frac{1}{\omega^2 C}\right)\right]|I|^2$$

$$P_{in} = \underbrace{\frac{1}{2} R|I|^2}_{\text{resistive power loss (radiation in antennas)}} + \underbrace{j\omega}_{\frac{d}{dt}} \underbrace{\frac{1}{2} L|I|^2}_{\text{energy stored in L (magnetic fields)}} \overset{\text{minus!}}{\boxed{-}} \underbrace{j\omega}_{\frac{d}{dt}} \underbrace{\frac{|I|^2}{\omega^2 C}}_{\text{energy stored in C (electric fields)}}$$

Series RLC circuit is the simplest form of resonator circuit



$$Z_{in} = R + jX = R + j\omega L + \frac{1}{j\omega C}$$

$$P_{in} = \frac{1}{2}UI^* = \underbrace{\frac{1}{2}R|I|^2}_{P_{loss}} + j\omega \underbrace{\left[\frac{1}{2}L|I|^2\right]}_{W_L} \overset{\text{minus!}}{\boxed{-}} j\omega \underbrace{\left[\frac{1}{2}\frac{|I|^2}{\omega^2 C}\right]}_{W_C}$$

$$P_{in} = P_{loss} + \underbrace{\frac{d}{dt}W_L} \boxed{-} \underbrace{\frac{d}{dt}W_C}$$

when energy increases

(decreases)

maximum

(minimum)

energy decreases

(increases)

minumun

(maximum)

ENERGY OSCILLATES BETWEEN THE TWO MODES

in resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad I = \frac{U}{R}$$

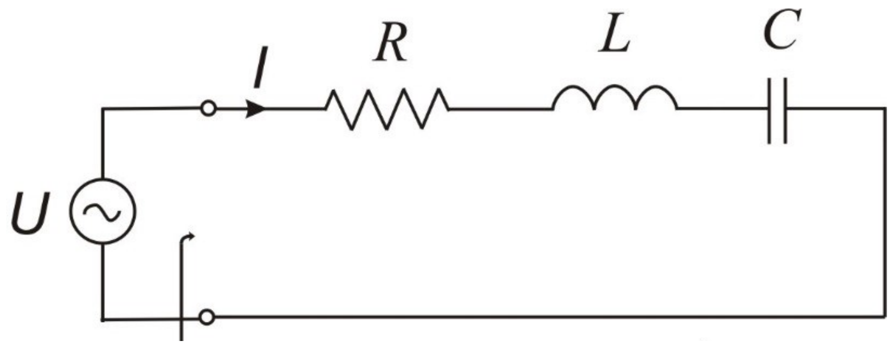
$$W_{L,max} = \frac{1}{2}L|I|^2$$

$$W_{C,max} = \frac{1}{2}\frac{|I|^2}{\omega_0^2 C} = \frac{1}{2}\frac{|I|^2}{\frac{1}{LC}\cdot C}$$

$$W_{C,max} = \frac{1}{2}L|I|^2 = W_{L,max}$$

in resonance

Q3: Which of statements are **correct** when the circuit resonates in steady state (select one or more)



$$P_{in} = \frac{1}{2}UI^* = \frac{1}{2}R|I|^2 + \mathrm{j}\omega \overbrace{\left[\frac{1}{2}L|I|^2\right]}^{W_L} - \mathrm{j}\omega \overbrace{\left[\frac{1}{2}\frac{|I|^2}{\omega^2 C}\right]}^{W_C}$$

In resonance…

out-of-phase

36% 1. the inductive and capacitive energy modes are **in-phase**.

45% 2. the stored energy in the resonator is **zero**: $W = W_L - W_C \neq 0$

45% [✓ 3.] the stored energy in the resonator is $W = W_L + W_C = \frac{1}{2}L|I|^2$

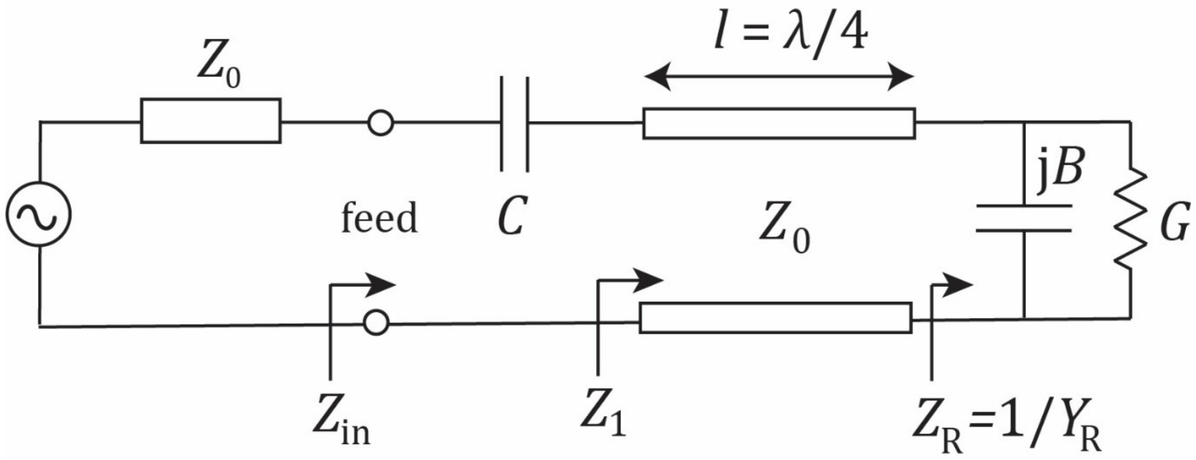
stored energy in resonator in resonance

9% 4. the active power is **zero:** $P_{loss} = \frac{1}{2}R|I|^2 \neq 0$

85% [✓ 5.] the reactive power is **zero:** $P_{reac} = \mathrm{j}\omega\underbrace{(W_L - W_C)}_{=0 \quad \text{in resonance}} = 0$

6. I don't know

In-class task in Breakout rooms



$$G = \frac{1}{50}\,\mathrm{S}$$
$$B = \frac{1}{100}\,\mathrm{S}$$
$$C = 6.3\ \mathrm{pF}$$
$$Z_0 = 50\ \Omega$$

$$Z_1 = Z_0 \frac{Z_R + \mathrm{j}Z_0 \tan(\beta l)}{Z_0 + \mathrm{j}Z_R \tan(\beta l)}$$

a. Calculate analytically or graphically using the Smith chart (see the next page), at which frequency the circuit is in resonance – i.e., calculate at which frequency $Z_{in} = Z_0$.

b. Explain, how is it possible that the circuit is in resonance even though there are no inductive components in the circuit.

c. If you have time, simulate the input reflection coefficient with AWRDE in the frequency range of 0.5-1.5 GHz.

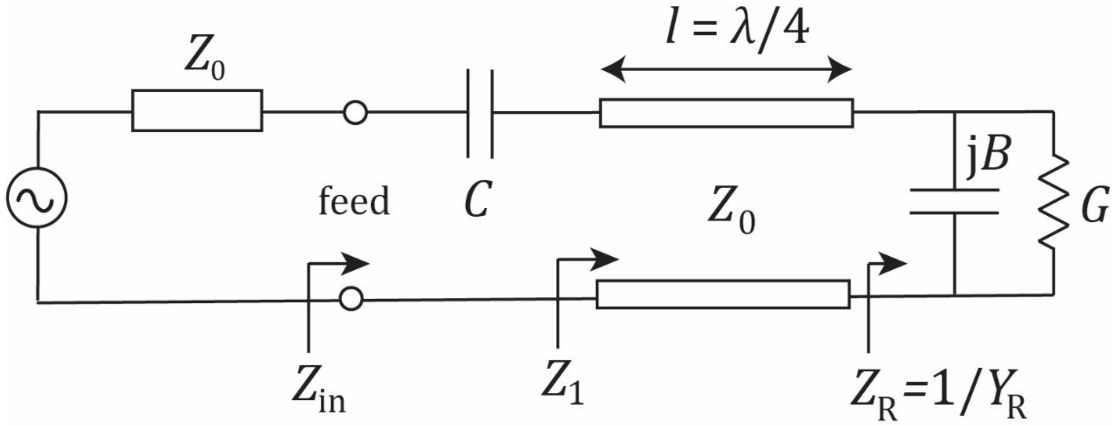
d. Return your effort (e.g., analytic calculation) in MyCourses latest at 12:30.

In-class task

$G = \frac{1}{50}\,\mathrm{S}$; $B = \frac{1}{100}\,\mathrm{S}$

$C = 6.3\,\mathrm{pF}$; $Z_0 = 50\,\Omega$

$$Z_1 = Z_0 \frac{Z_R + \mathrm{j}Z_0 \tan(\beta l)}{Z_0 + \mathrm{j}Z_R \tan(\beta l)}$$



$Y_R = G + jB = \left(\frac{1}{50} + j\frac{1}{100}\right)S$; $Z_R = \frac{1}{Y_R}$

$\tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \tan\left(\frac{\pi}{2}\right) = \infty$

$\tan(\beta l)$

a) $Z_1 = Z_0 \cdot \frac{Z_R + jZ_0\tan(\beta l)}{Z_0 + jZ_R\tan(\beta l)} = Z_0 \frac{\frac{Z_R}{\tan(\,)} + jZ_0}{\frac{Z_0}{\tan(\,)} + jZ_R} = Z_0 \cdot \frac{\cancel{j}Z_0}{\cancel{j}Z_R} = \frac{Z_0^2}{Z_R} = Z_0^2 \cdot Y_R = (50\Omega)^2 \cdot \left(\frac{1}{50} + j\frac{1}{100}\right)S$

$= 50 + j25\,\Omega$

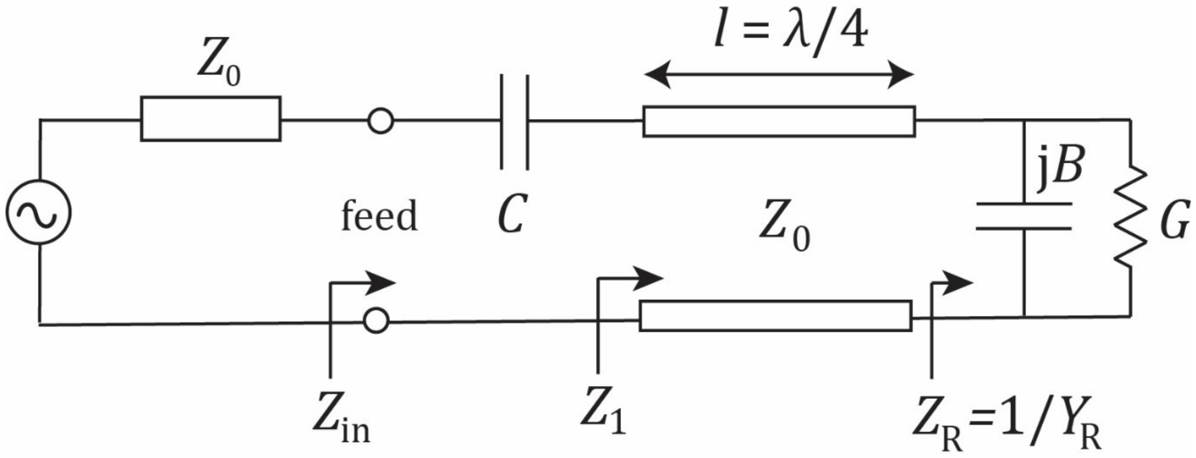
The circuit is in resonance if $-\frac{1}{\omega C} = -25\,\Omega \Leftrightarrow f = \frac{-1}{-25\Omega \cdot 2\pi \cdot 6{,}3 \cdot 10^{-12}F} = 1.0$ GHz

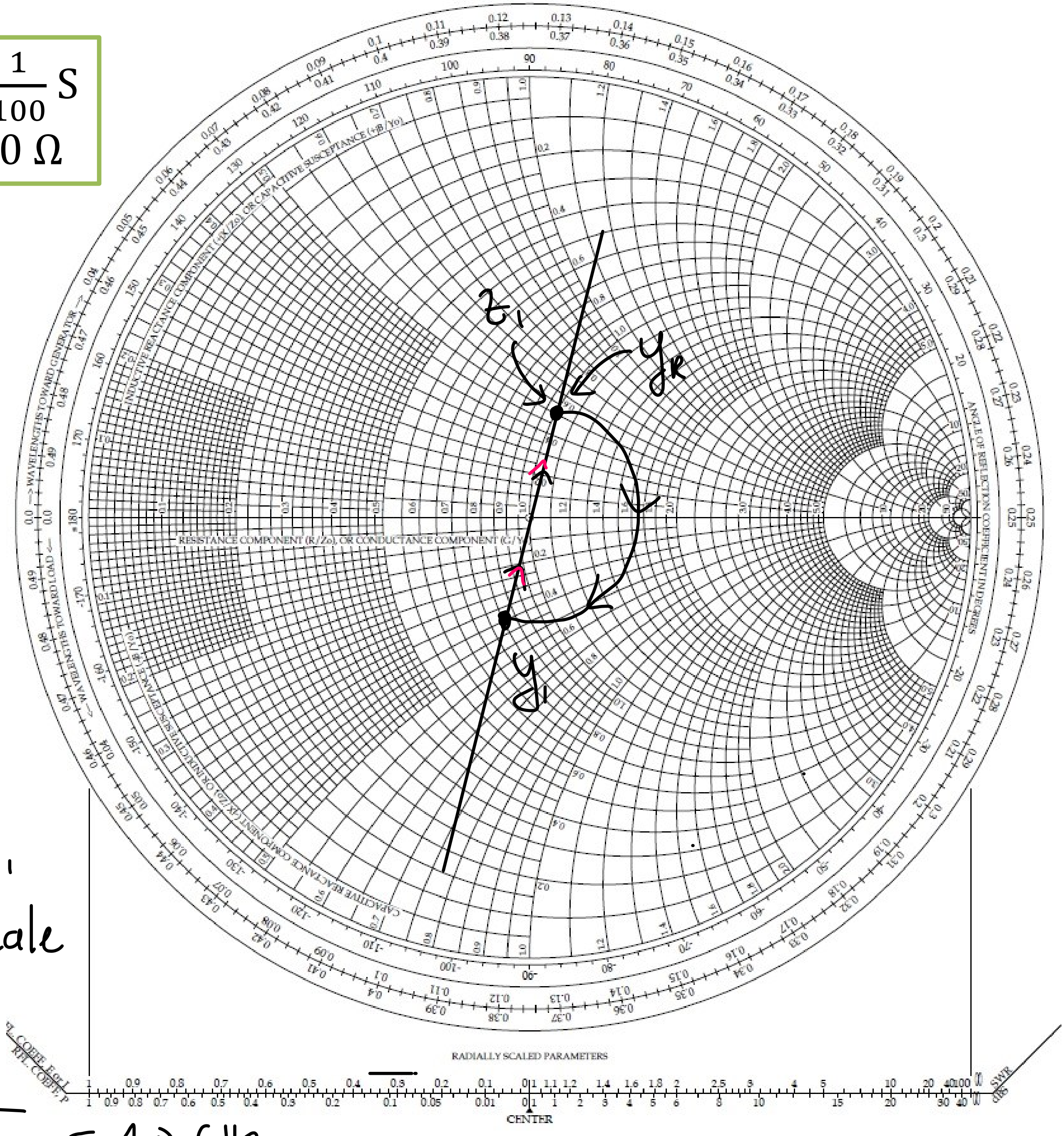
b) $\lambda/4$ long transmission line moves a capacitive impedance Z_R into an inductive impedance Z_1. Inductance comes through the phasing of the transmission line.

In-class task

$G = \frac{1}{50}$ S ; $B = \frac{1}{100}$ S

$C = 6.3$ pF ; $Z_0 = 50\ \Omega$





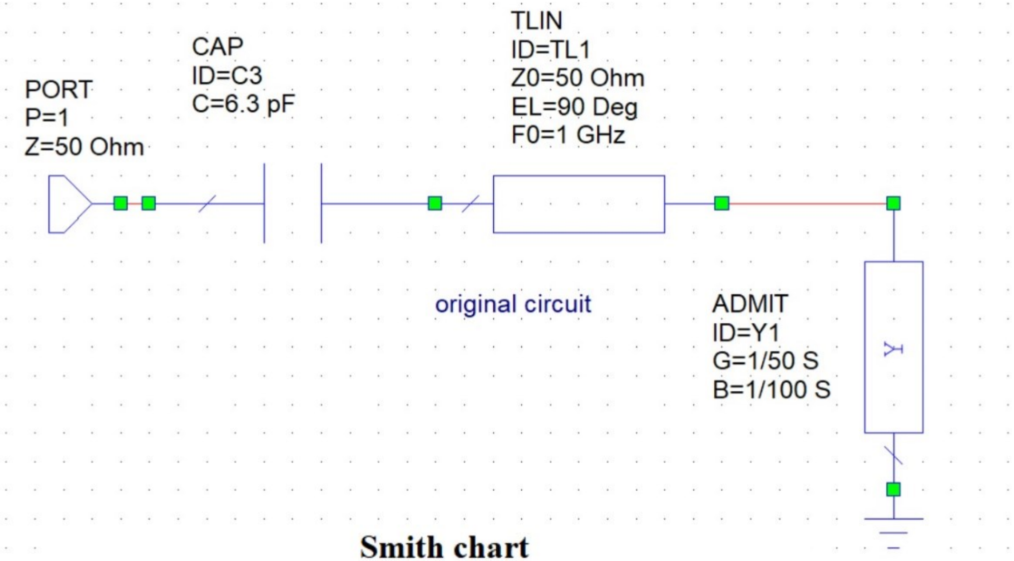
$Y_R = G + jB$; $y_R = \frac{Y_R}{Y_0} = \frac{\frac{1}{50} + j\frac{1}{100}\ S}{\frac{1}{50}\ S} = 1 + j0.5$

rotate λ/4 towards the load in position y_1

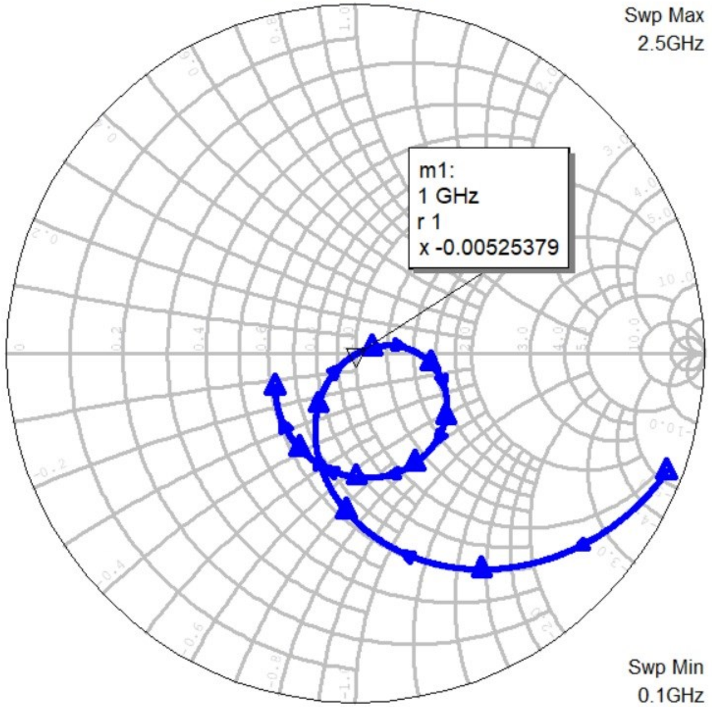
then rotate another λ/4 to the impedance scale

$z_1 = 1 + j0.5 \Rightarrow Z_1 = z_1 \cdot Z_0 = 50 + j25\Omega$

Hence $-\frac{1}{\omega C} = -25\Omega \Rightarrow f = -\frac{1}{(-25\Omega)\cdot 2\pi \cdot 6.3 10^{-12} F} = 1{,}0$ GHz



Smith chart



magnitude of the reflection coefficient in dB