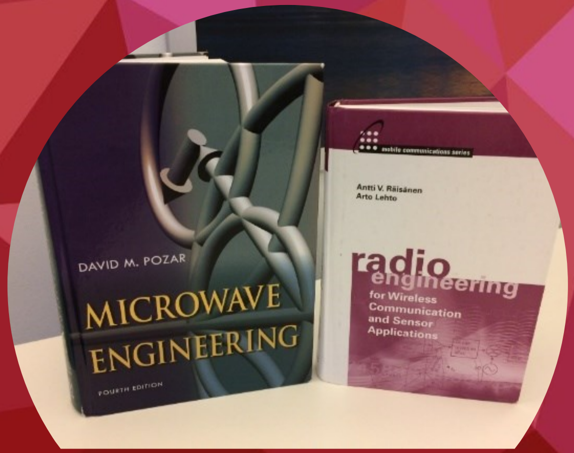
Microwave engineering I (MiWE I)

Interactive lecture 2 of Topic 3
Scattering parameters

February 17, 2022

The main learning outcome of the course is to create readiness to work in microwave engineering related tasks and projects and enable further studies and continuous learning in microwave engineering.



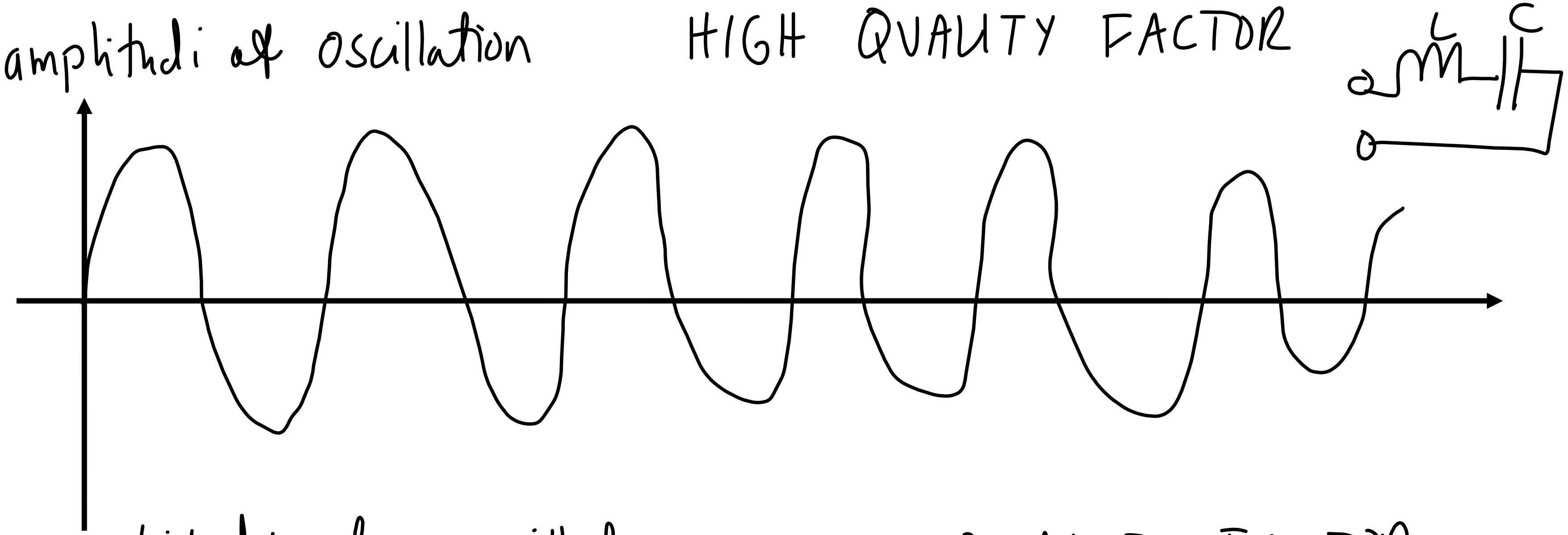
Topic 3: Learning outcomes and content

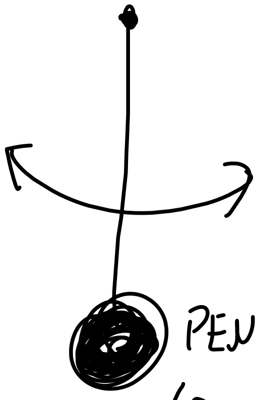
- The student can
  - **analyze** the operation of basic microwave circuits and resonators based on calculations and simulations (AWRDE).
  - **model** and **analyze** the operation of microwave circuits and resonators with suitable circuit parameters, especially the scattering parameters (S-parameters).

- Series and parallel resonant circuits (Pozar chapter 6.1)
- The scattering matrix (Pozar chapter 4.3)
- The transmission (ABCD) matrix (Pozar chapter 4.4

These lecture slides and notes are not designed for self-study.
Please, use the course book chapters 4 and 6 for self-study.

amplitudi of oscillation HIGH QUALITY FACTOR

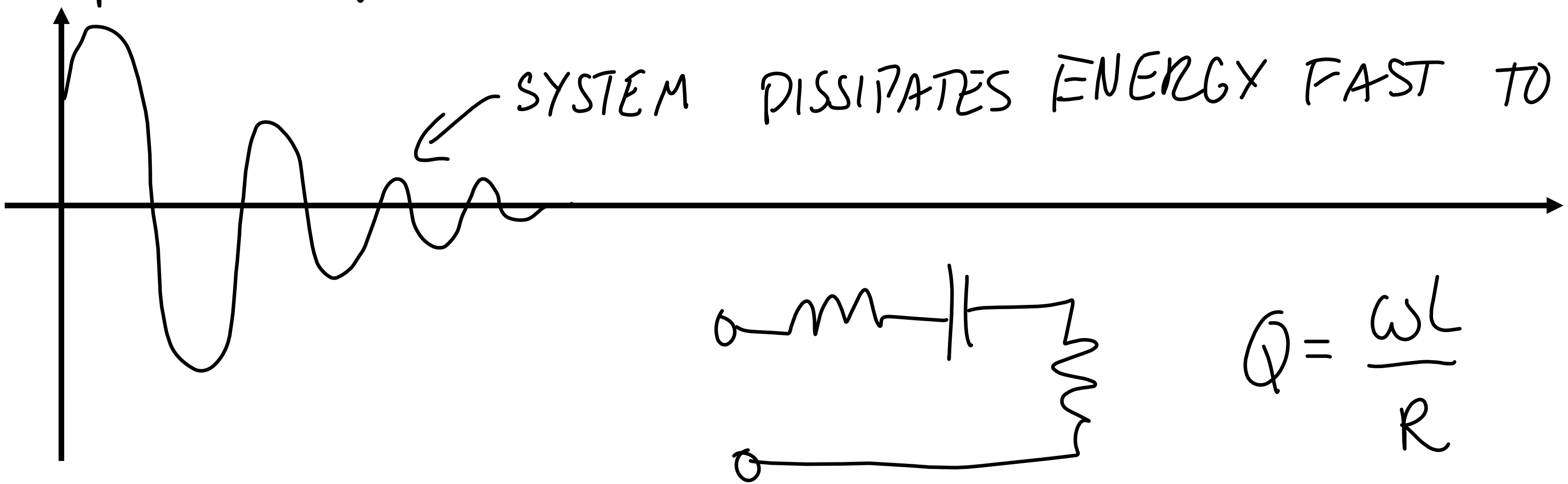




PENDULUM

(SMALL FRICTION → HIGH Q)

amplitude of oscillation LOW QUALITY FACTOR



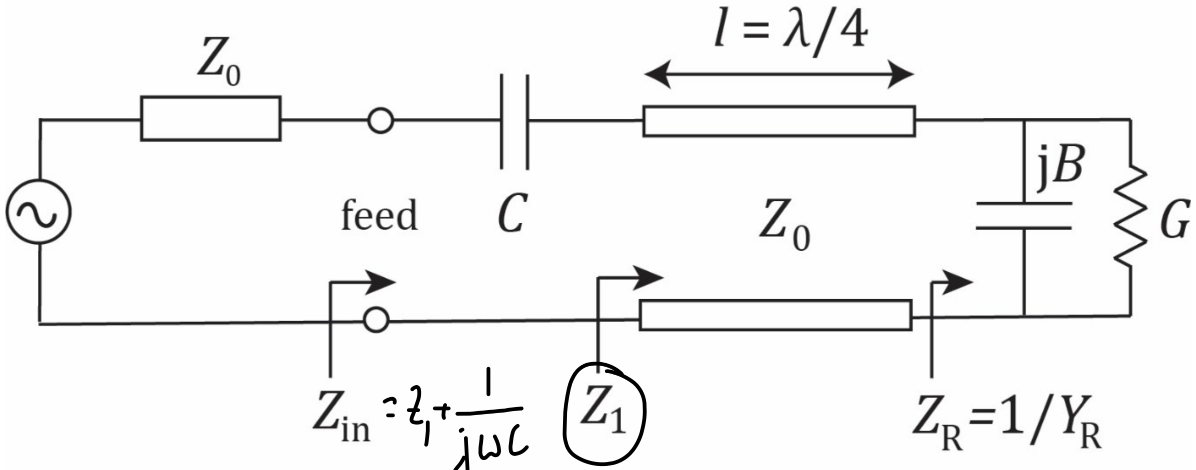
SYSTEM DISSIPATES ENERGY FAST TO THE INTERNAL LOSSES

$$Q = \frac{\omega L}{R}$$

↑

if R large → Q is small

In-class task in Breakout rooms



$Z_{in} = Z_1 + \frac{1}{j\omega C}$

$= R_{in} + jX_{in}$, resonance when $X_{in} = 0$

$$G = \frac{1}{50}\,\text{S}$$
$$B = \frac{1}{100}\,\text{S}$$
$$C = 6.3\text{ pF}$$
$$Z_0 = 50\ \Omega$$

"tangent formula"

$$Z_1 = Z_0 \frac{Z_R + jZ_0 \tan(\beta l)}{Z_0 + jZ_R \tan(\beta l)}$$

$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$ $\tan\left(\frac{\pi}{2}\right) = \infty$

$Z_1 = Z_0 \frac{jZ_0}{jZ_R} = \frac{Z_0^2}{Z_R} = Z_0^2 Y_R$

a. Calculate analytically or graphically using the Smith chart (see the next page), at which frequency the circuit is in resonance – i.e., calculate at which frequency $Z_{in} = Z_0$.

b. Explain, how is it possible that the circuit is in resonance even though there are no inductive components in the circuit.

c. If you have time, simulate the input reflection coefficient with AWRDE in the frequency range of 0.5-1.5 GHz.

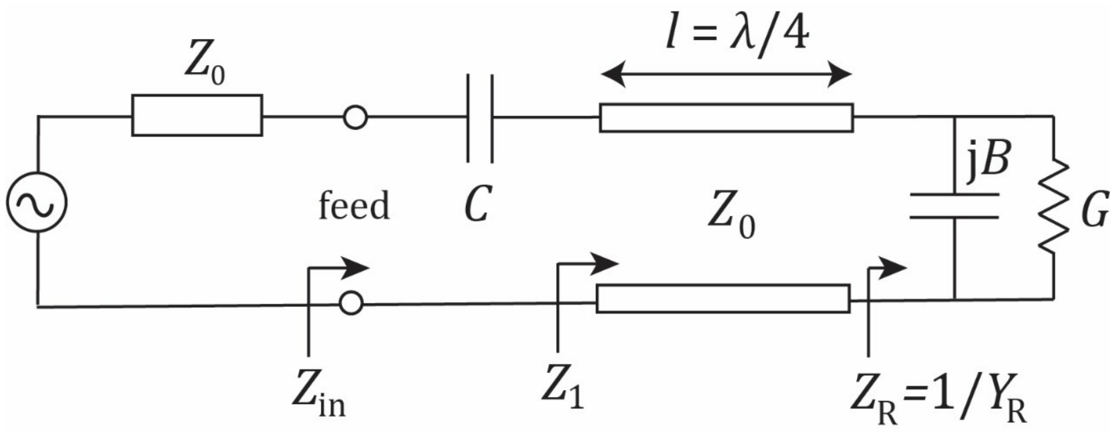
d. Return your effort (e.g., analytic calculation) in MyCourses latest at 12:30.

In-class task

$G = \frac{1}{50}\,\mathrm{S}$; $B = \frac{1}{100}\,\mathrm{S}$

$C = 6.3\,\mathrm{pF}$; $Z_0 = 50\,\Omega$

$$Z_1 = Z_0 \frac{Z_R + \mathrm{j}Z_0 \tan(\beta l)}{Z_0 + \mathrm{j}Z_R \tan(\beta l)}$$



$Y_R = G + jB = \left(\frac{1}{50} + j\frac{1}{100}\right) S$; $Z_R = \frac{1}{Y_R}$

$\tan\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\right) = \tan\left(\frac{\pi}{2}\right) = \infty$

$\tan(\beta l)$

a) $$Z_1 = Z_0 \cdot \frac{Z_R + jZ_0 \tan(\beta l)}{Z_0 + jZ_R \tan(\beta l)} = Z_0 \frac{\frac{Z_R}{\tan(\;)} + jZ_0}{\frac{Z_0}{\tan(\;)} + jZ_R} = Z_0 \cdot \frac{jZ_0}{jZ_R} = \frac{Z_0^2}{Z_R} = Z_0^2 \cdot Y_R = (50\,\Omega)^2 \cdot \left(\frac{1}{50} + j\frac{1}{100}\right) S$$

$$= 50 + j25\,\Omega$$

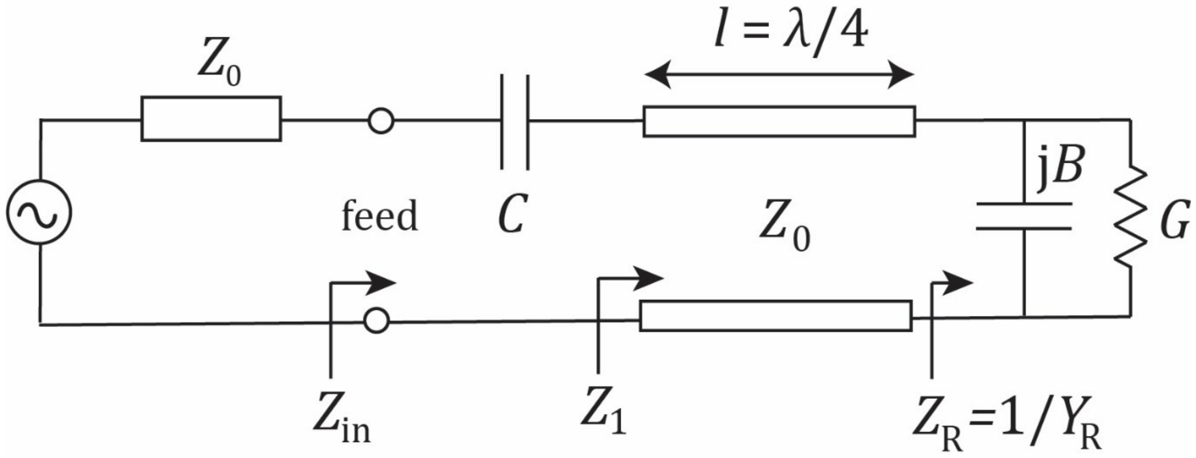
The circuit is in resonance if $-\frac{1}{\omega C} = -25\,\Omega \Leftrightarrow f = \frac{-1}{-25\,\Omega \cdot 2\pi \cdot 6.3 \cdot 10^{-12} F} = 1.0$ GHz

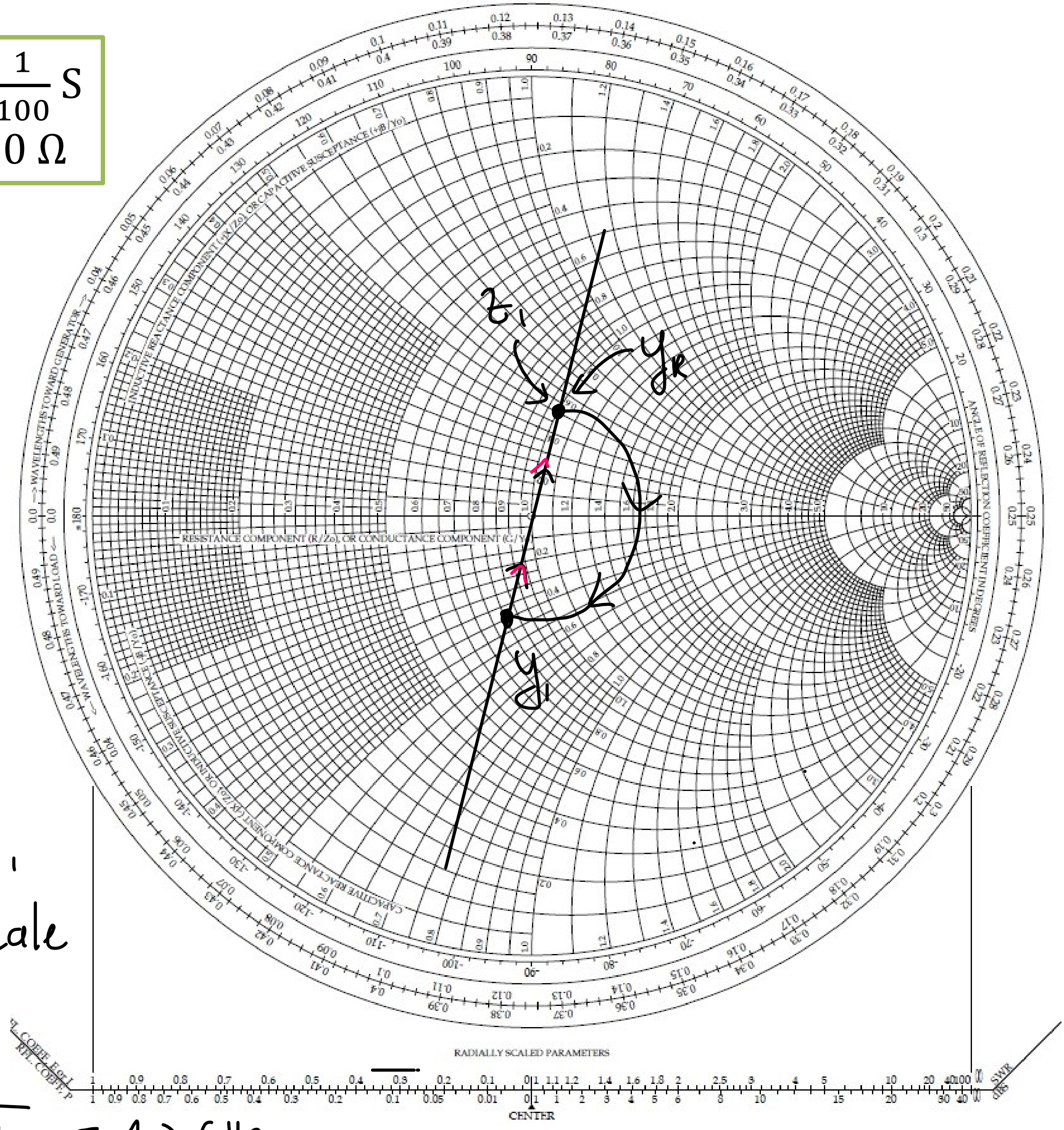
b) $\lambda/4$ long transmission line moves a capacitive impedance Z_R into an inductive impedance Z_1. Inductance comes through the phasing of the transmission line.

In-class task

$G = \frac{1}{50}\,\mathrm{S}$; $B = \frac{1}{100}\,\mathrm{S}$

$C = 6.3\ \mathrm{pF}$; $Z_0 = 50\ \Omega$





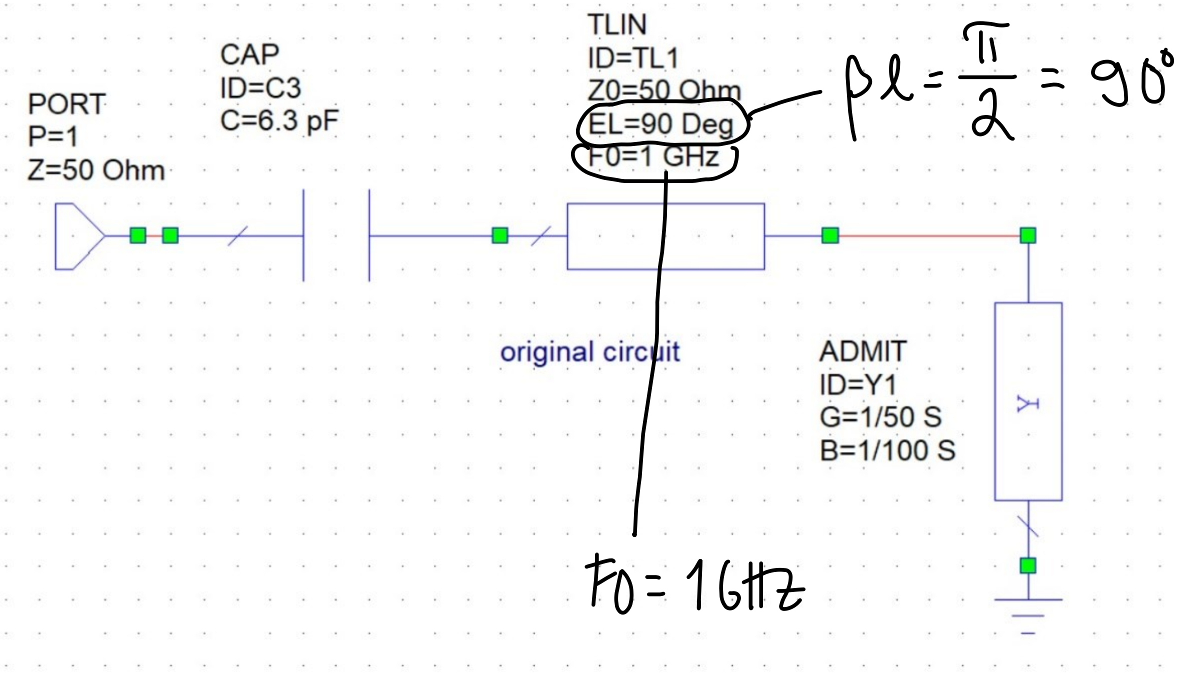
$Y_R = G + jB$; $y_R = \frac{Y_R}{Y_0} = \frac{\frac{1}{50} + j\frac{1}{100}\ S}{\frac{1}{50}\ S} = 1 + j0.5$

rotate λ/4 towards the load in position y_1

then rotate another λ/4 to the impedance scale

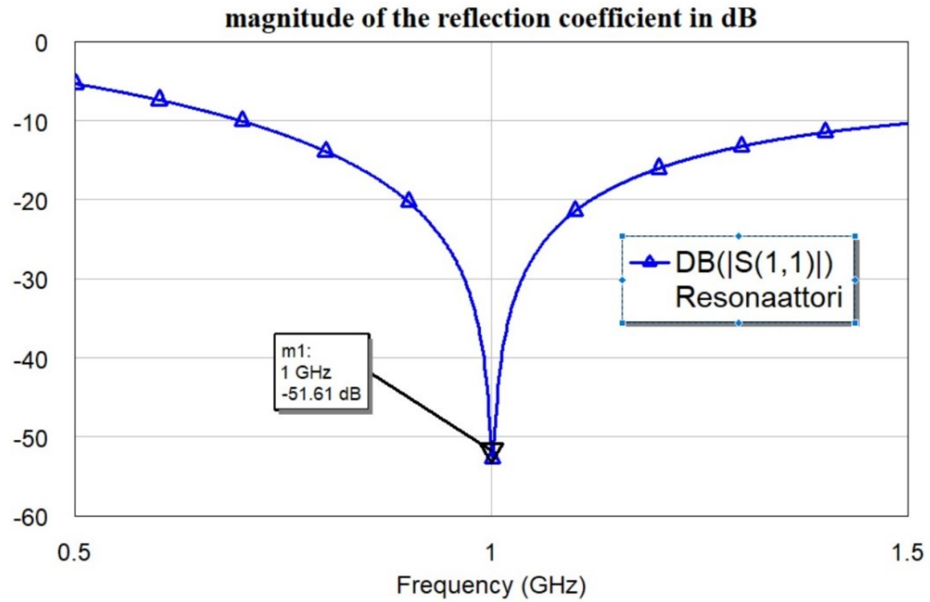
$z_1 = 1 + j0.5 \Rightarrow Z_1 = z_1 \cdot Z_0 = 50 + j25\Omega$

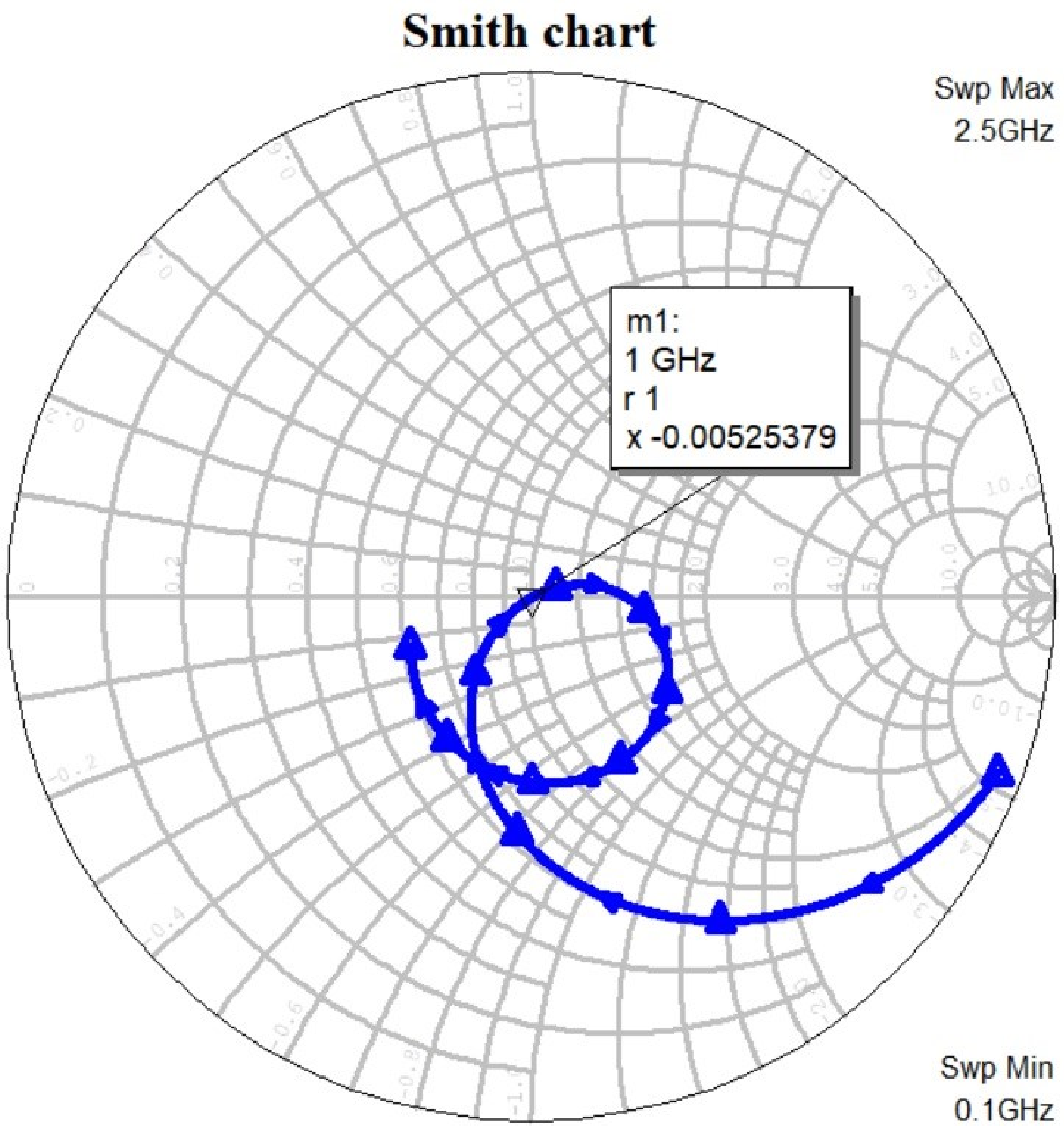
Hence $-\frac{1}{\omega C} = -25\Omega \Rightarrow f = -\frac{1}{(-25\Omega)\cdot 2\pi \cdot 6.3 10^{-12}F} = 1{,}0\ \mathrm{GHz}$



$\beta l = \frac{\pi}{2} = 90°$

F0 = 1GHz


magnitude of the reflection coefficient in dB


Smith chart

Q1: A 2-port with lumped impedance Z is connected between two transmission lines (with Z_0). What is the input impedance Z_{in} seen in Port 1 to the **right** (towards Port 2)? Assume the transmission lines semi-infinite long

FIRST VOTE ↓ SECOND VOTE ↓

21% 1. $Z_{in} = Z$ 0%

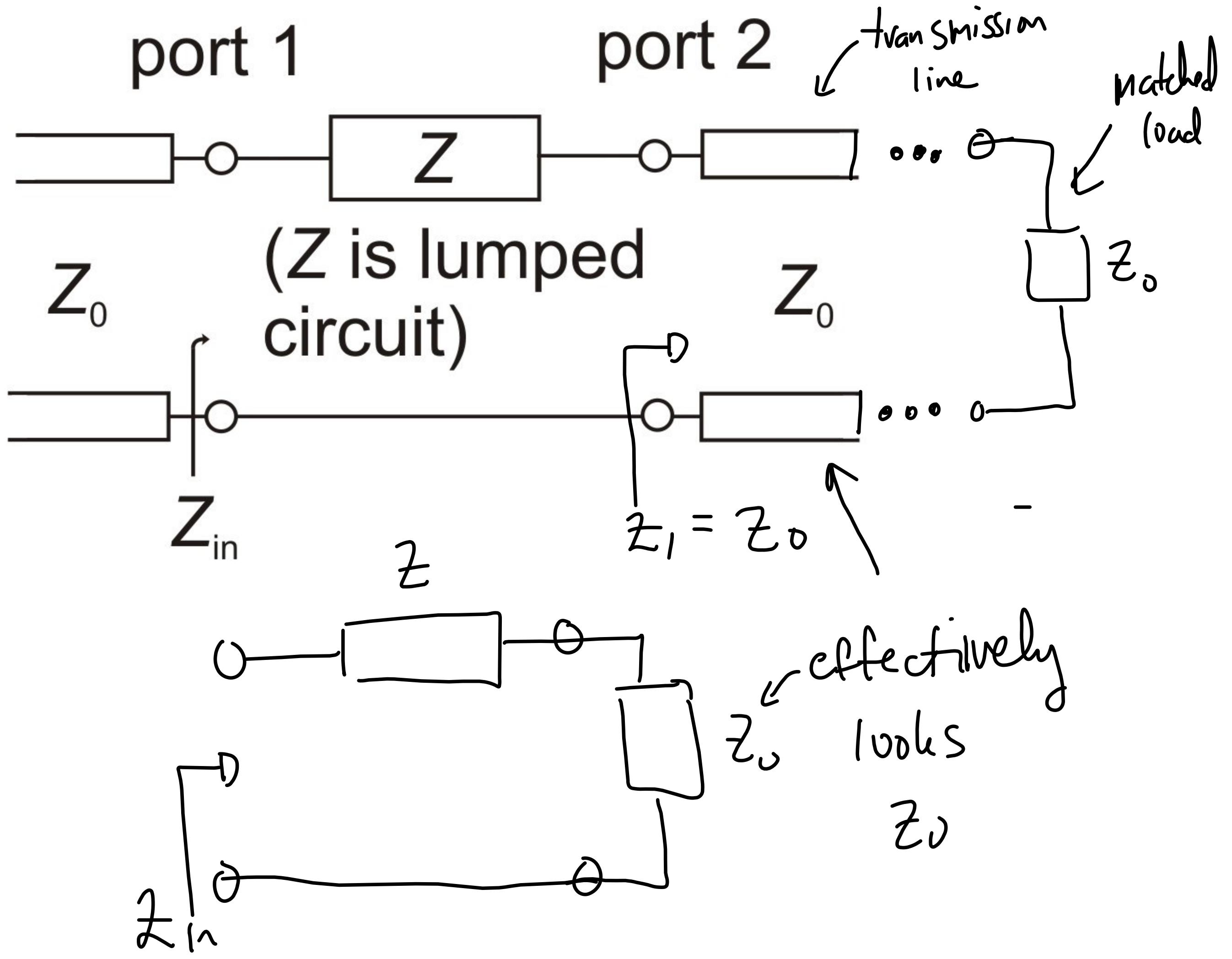
4% 2. $Z_{in} = Z_0$ 0%

46% [✓ 3.] $Z_{in} = Z + Z_0$ 90% CORRECT ANSWER

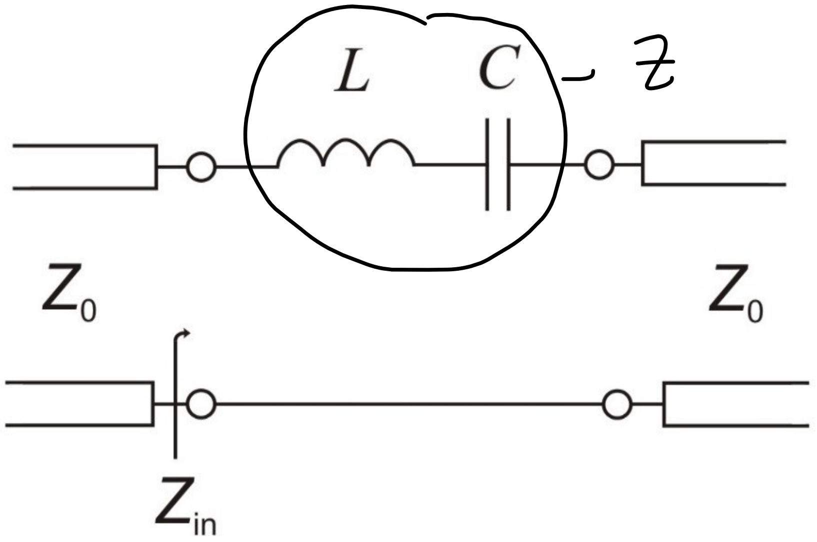
25% 4. $Z_{in} = \frac{Z \cdot Z_0}{Z + Z_0}$ 10%

0% 5. $Z_{in} = \infty$ (open circuit) 0%

4% 6. I don't know



Q2: This LC series circuit topology might work at $\omega_0 = \frac{1}{\sqrt{LC}}$ as ...



$Z = j\omega L + \frac{1}{j\omega C}$

at $\omega_0 = \frac{1}{\sqrt{LC}}$; $Z = 0$; $Z_{in} = Z_0$

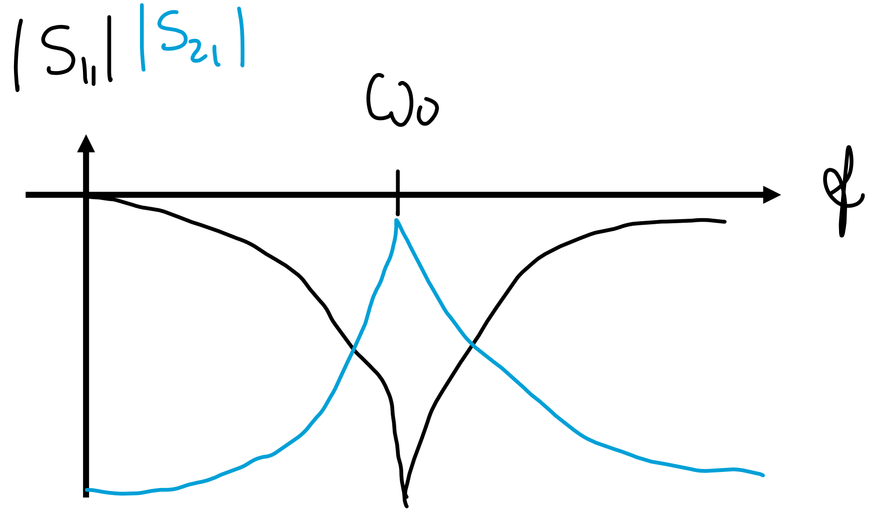
0% 1. Low-pass filter below ω_0

10% 2. High-pass filter above ω_0

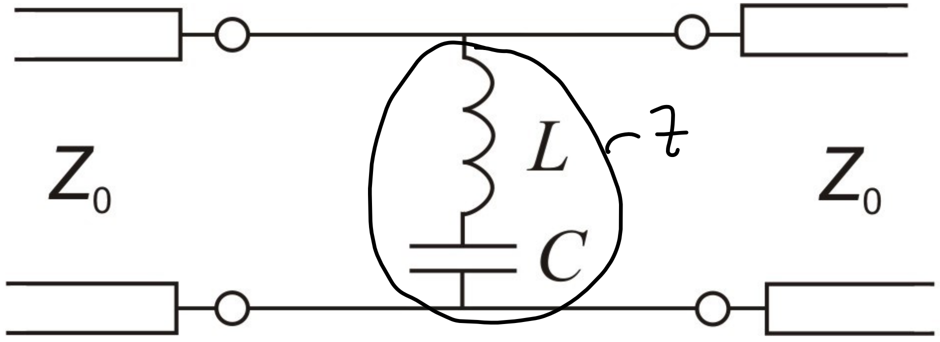
69% [✓ 3.] Band-pass filter around ω_0

17% 4. Band-stop filter around ω_0

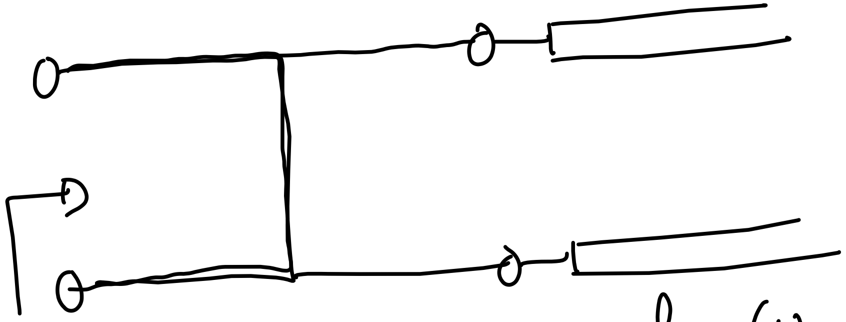
3% 5. I don't know



Q3: This LC series circuit topology might work at $\omega_0 = \frac{1}{\sqrt{LC}}$ as ...



$z = 0$ at $\omega_0 = \frac{1}{\sqrt{LC}}$



$Z_{in} = 0 \Rightarrow \Gamma_{in} = -1$ at ω_0

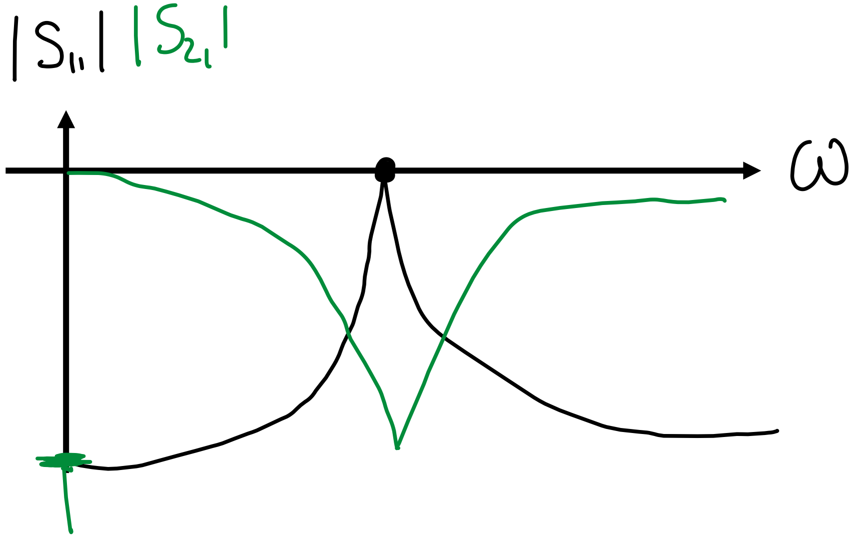
3 % 1. Low-pass filter below ω_0

0 % 2. High-pass filter above ω_0

10 % 3. Band-pass filter around ω_0

77 % [✓ 4.] Band-stop filter around ω_0

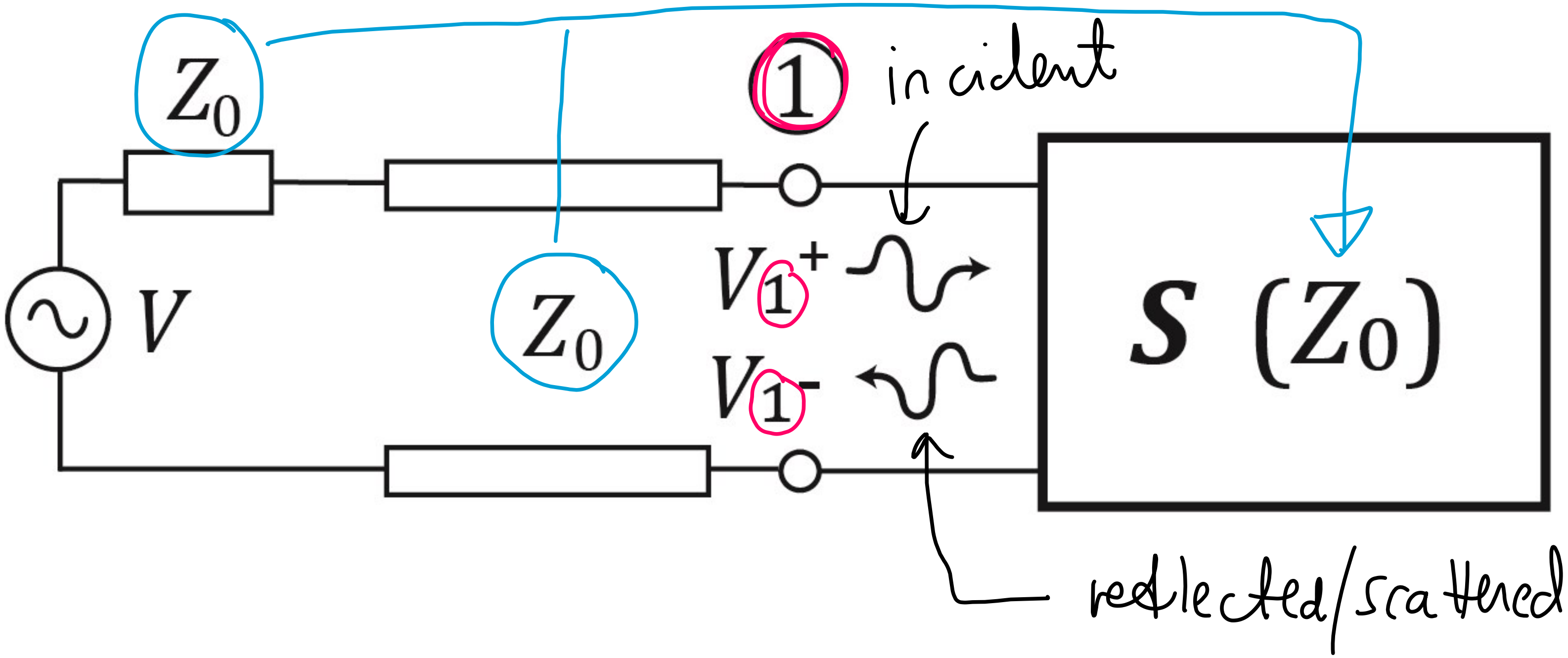
10 % 5. I don't know



The scattering parameters deal with voltage waves that incident (V^+) and scatter (V^-) in the designated ports

V_1^+ = voltage waves IN to the port

V_1^- = voltage waves OUT of the port



incident

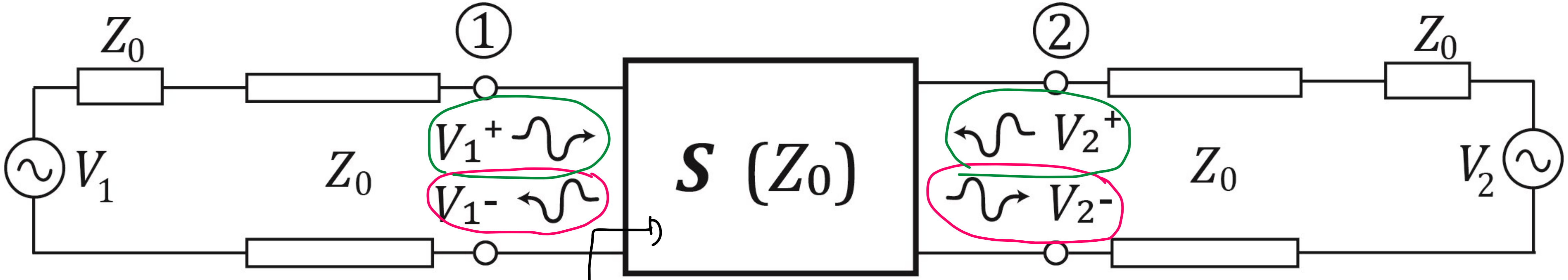
reflected/scattered

$$S_{11} = \frac{\text{scattered (reflected) voltage}}{\text{incident voltage}} = \frac{V_1^-}{V_1^+} = \Gamma_{in} = \frac{Z_{in}-Z_0}{Z_{in}+Z_0}$$

S_{11} is traditional reflection coef.

SINGLE PORT → ONE SINGLE VALUE S_{11} (complex number)

The S matrix describes the full relationship between designated ports in terms of voltage waves



$S_{11} =$

Γ_{in}

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}}_{\text{2 × 2 matrix 2 ports}} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

voltage wave OUT

2 × 2 matrix 2 ports

Voltage wave IN

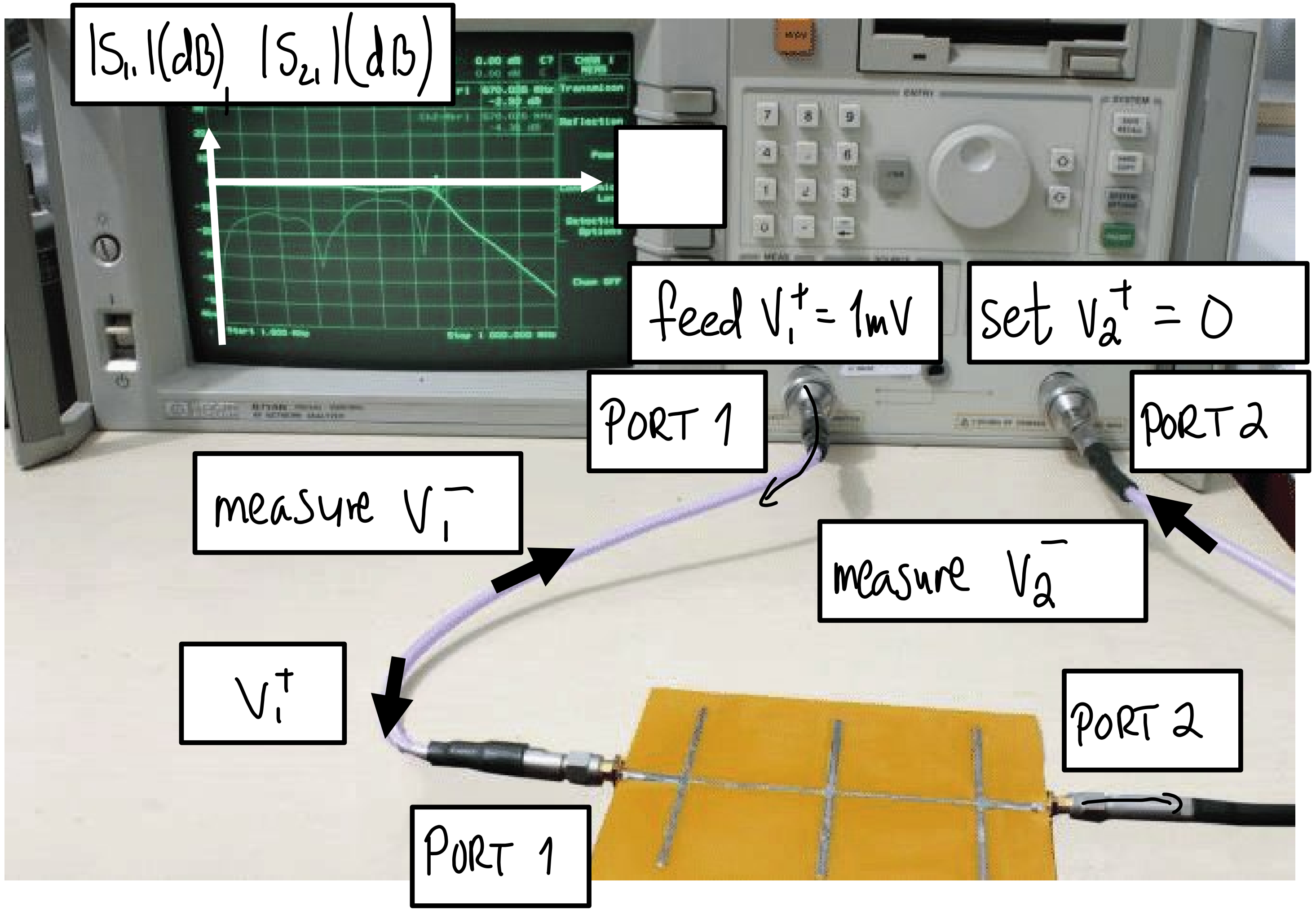
$$\begin{cases} v_1^- = S_{11} \cdot v_1^+ + S_{12} \cdot V_2^+ \\ v_2^- = S_{21} \cdot v_1^+ + S_{22} V_2^+ \end{cases}$$

S_{nm}

port m signal IN

port n signal OUT

The scattering parameters can be measured with vector network analyzer (VNA), simulated with circuit or EM simulator or calculated pen & paper method

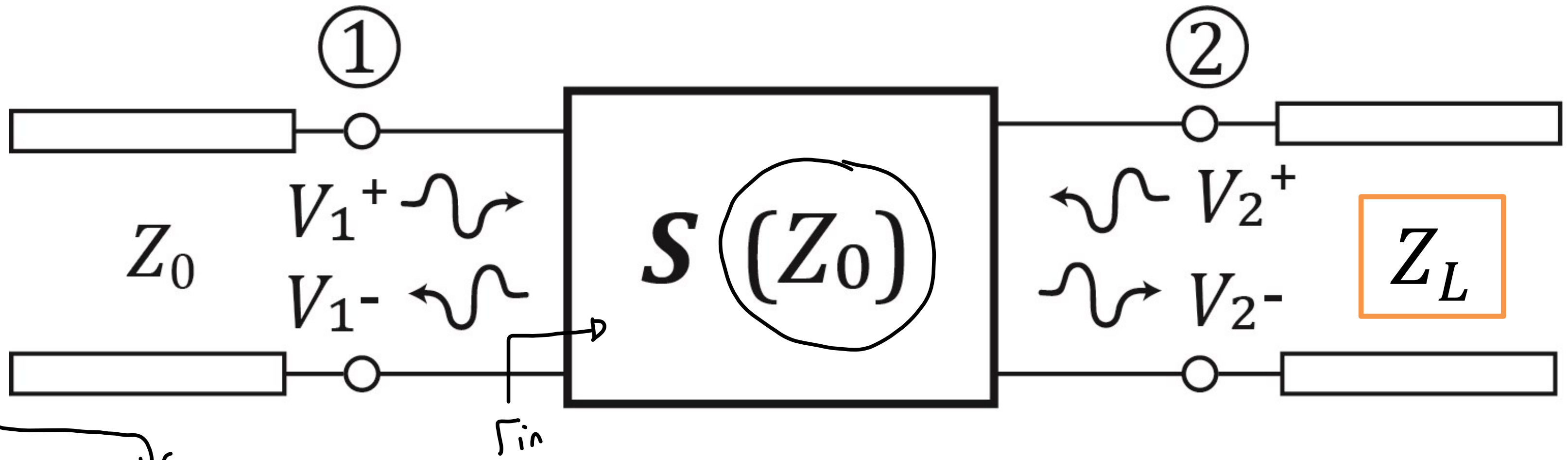


$$\begin{bmatrix}V_1^-\\V_2^-\end{bmatrix}=\begin{bmatrix}S_{11}&S_{12}\\S_{21}&S_{22}\end{bmatrix}\begin{bmatrix}V_1^+\\\boxed{V_2^+}=0\end{bmatrix}$$

$$\begin{cases} v_1^- = S_{11}\cdot v_1^+ + 0 \\ v_2^- = S_{21}\cdot v_1^+ + 0 \end{cases}$$

$$\Rightarrow \begin{cases} S_{11} = \frac{v_1^-}{v_1^+} \text{ when } V_2^+ = 0 \\ S_{21} = \frac{v_2^-}{v_1^+}, \text{ when } V_2^+ = 0 \end{cases}$$

If $Z_L \neq Z_0$ in port 2, the input reflection $\Gamma_{in} \neq S_{11}$



$v_2^+ = \Gamma_L \cdot V_2^-$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{\text{refl. volt.}}{\text{incid. volt.}}$$

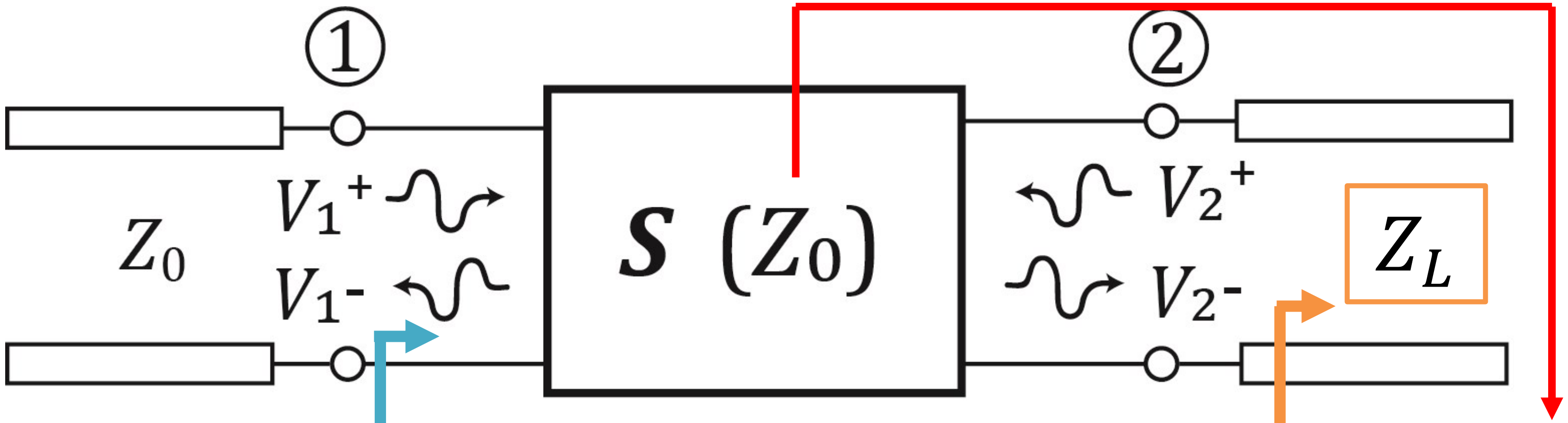
$$\Gamma_L = \frac{v_2^+}{v_2^-}$$

$$\begin{cases} v_1^- = S_{11} v_1^+ + S_{12} \cdot v_2^+ \\ v_2^- = S_{21} v_1^+ + S_{22} v_2^+ \end{cases} \Leftrightarrow \begin{cases} v_1^- = S_{11} v_1^+ + S_{12} \Gamma_L \cdot v_2^- \\ v_2^- = S_{21} v_1^+ + S_{22} \Gamma_L \underline{v_2^-} \end{cases}$$

$$v_2^- (1 - S_{22} \Gamma_L) = S_{21} \cdot v_1^+ \Rightarrow v_2^- = \frac{S_{21}}{1 - S_{22} \cdot \Gamma_L} \cdot v_1^+$$

$$v_1^- = S_{11} \cdot v_1^+ + \frac{S_{12} \cdot \Gamma_L \cdot S_{21}}{1 - S_{22} \cdot \Gamma_L} \cdot v_1^+ \rightarrow \frac{v_1^-}{v_1^+} = \boxed{\Gamma_{in} = S_{11} + \frac{S_{12} \cdot \Gamma_L \cdot S_{21}}{1 - S_{22} \cdot \Gamma_L}} \neq S_{11}$$

If $Z_L \neq Z_0$ in port 2, the input reflection $\Gamma_{in} \neq S_{11}$



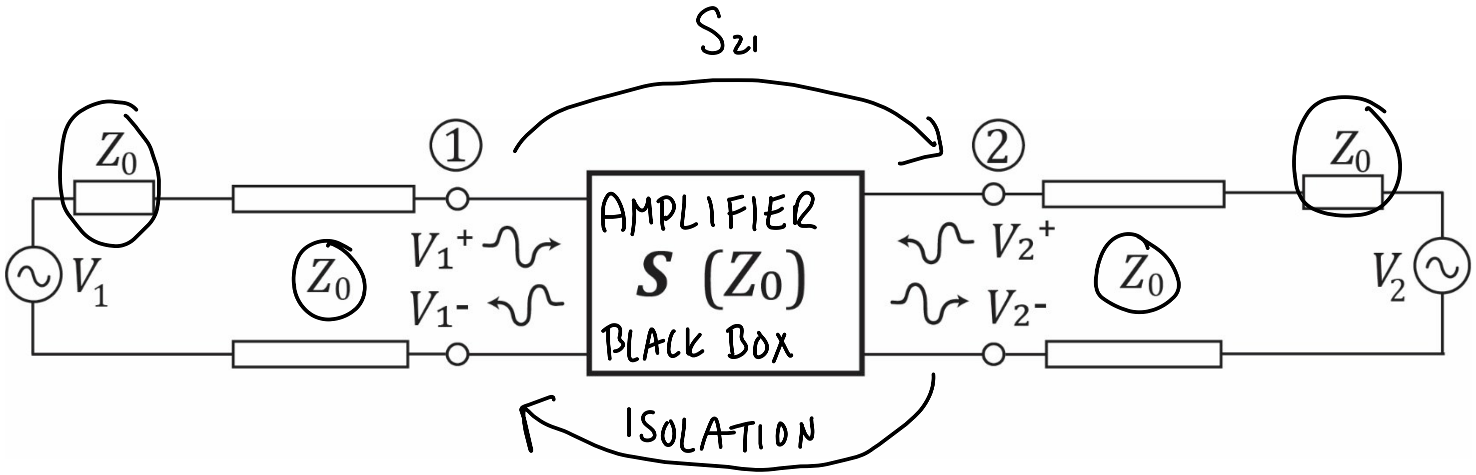
$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}\Gamma_L S_{21}}{1 - S_{22}\Gamma_L}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

reflection caused by
the **circuit itself**
seen in port 1

infinite number of reflections between
the **circuit** and the **load Z_L**
(a form of a convergent geometric series)

The scattering matrix of active components is non-reciprocal and non-symmetric



E.g., a transistor amplifier at *f*:

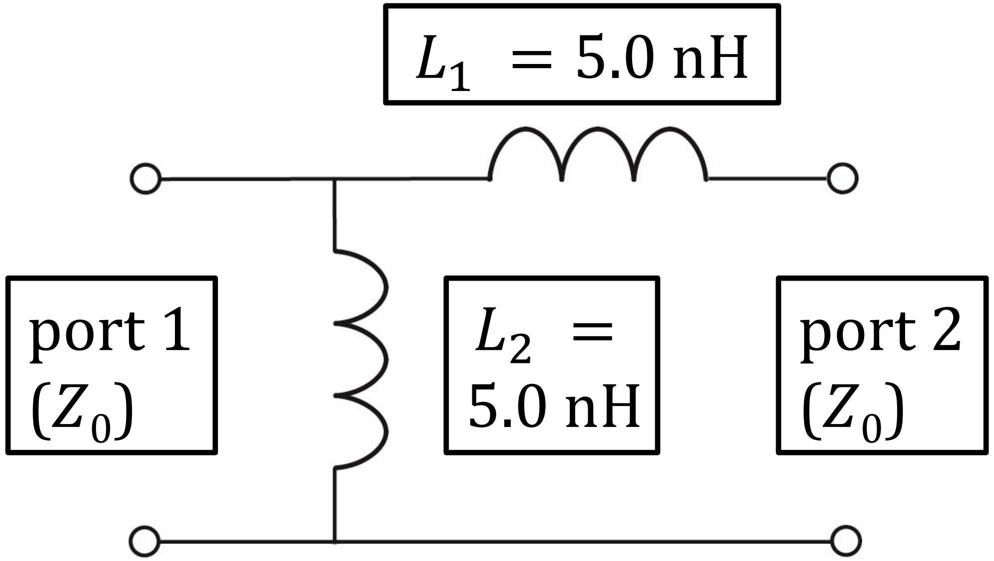
$$S = \begin{bmatrix} 0.10\angle -137^o & 0.010\angle 27^o \\ 4.0\angle -36{,}18^o & 0.10\angle -88^o \end{bmatrix}$$

Matching: $|S_{11}| = |S_{22}| = 0.10 = 10\log_{10}|S_{11}|^2 = 20\log 0.10 = 20\log 10^{-1} = \underline{\underline{-20\,dB}}$

Power gain (1 → 2): $G_{21} = \dfrac{\dfrac{|V_2^-|^2}{2Z_0}}{\dfrac{|V_1^+|^2}{2Z_0}} = \left|\dfrac{V_2^-}{V_1^+}\right|^2 = |S_{21}|^2 = 10\log|S_{21}|^2 = \underline{\underline{12\,dB}}$

Isolation (2 → 1): $I_{12} = -10\log|S_{12}|^2 = -20\log 10^{-2} = \underline{\underline{+40\,dB}}$

Q4: Which of the alternatives is/are **false** concerning the two-port in the figure? (select one or more)



$Z_0 = 50\ \Omega$

24% 1. When inductors are ideal, the circuit is lossless, and the S matrix satisfies $[S]^H[S] = [I]$ TRUE

34% 2. The circuit is **not** perfectly matched; $S_{11} \neq 0$ and $S_{22} \neq 0$ TRUE

48% [✓ 3.] The circuit is symmetric - i.e., it "looks" the same from each of its ports: $S_{11} = S_{22}$ FALSE

34% 4. The circuit is reciprocal; the S matrix is symmetric $[S] = [S]^T$ TRUE $S_{21} = S_{12}$

31% 5. Power **into** port i given by → $P_i^+ = \frac{|V_i^+|^2}{2Z_0}$

31% 6. Power **out** from port i is given by → $P_i^- = \frac{|V_i^-|^2}{2Z_0}$

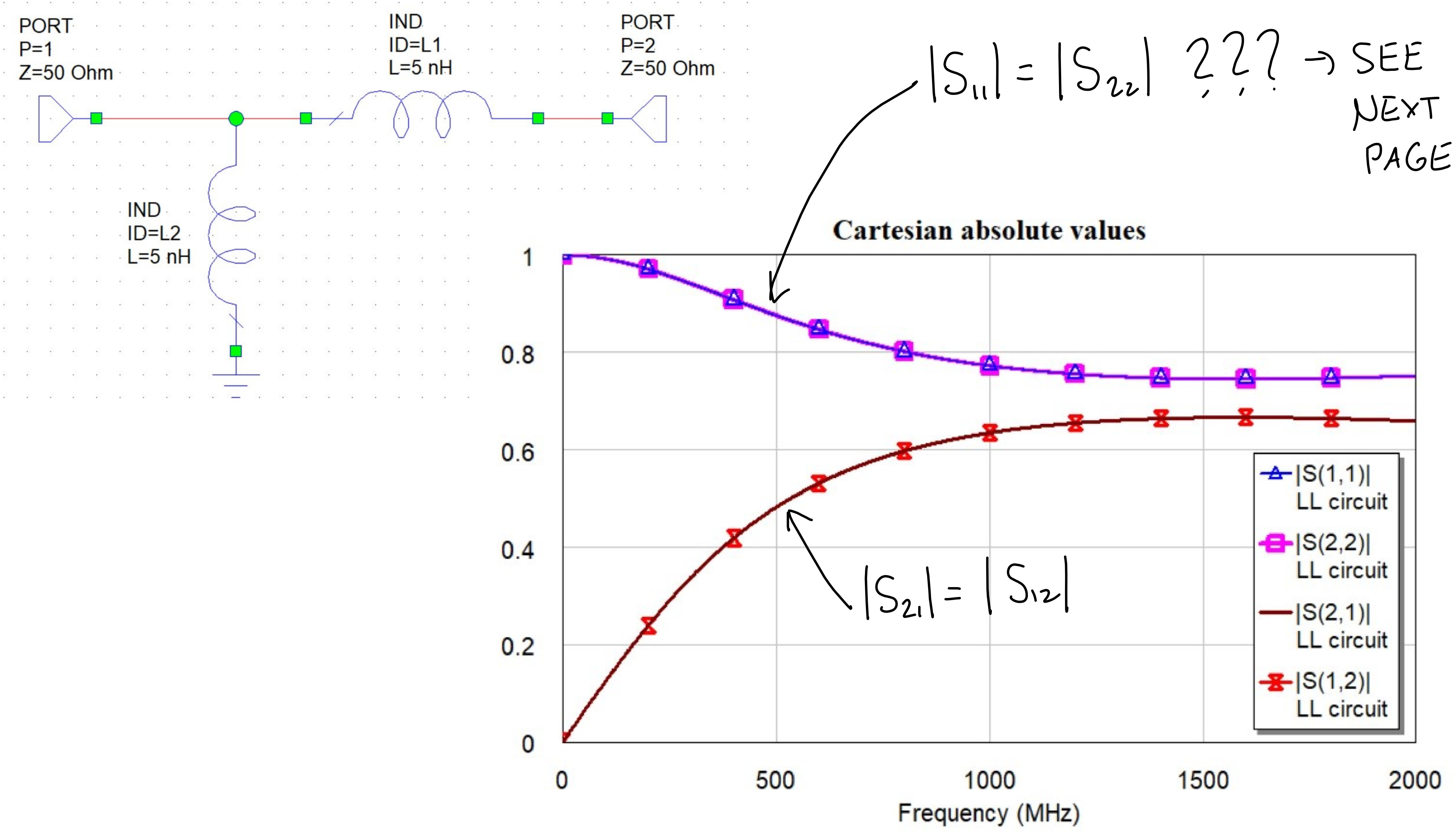
TRUE

10% 7. I don't know

$+$ = signal IN

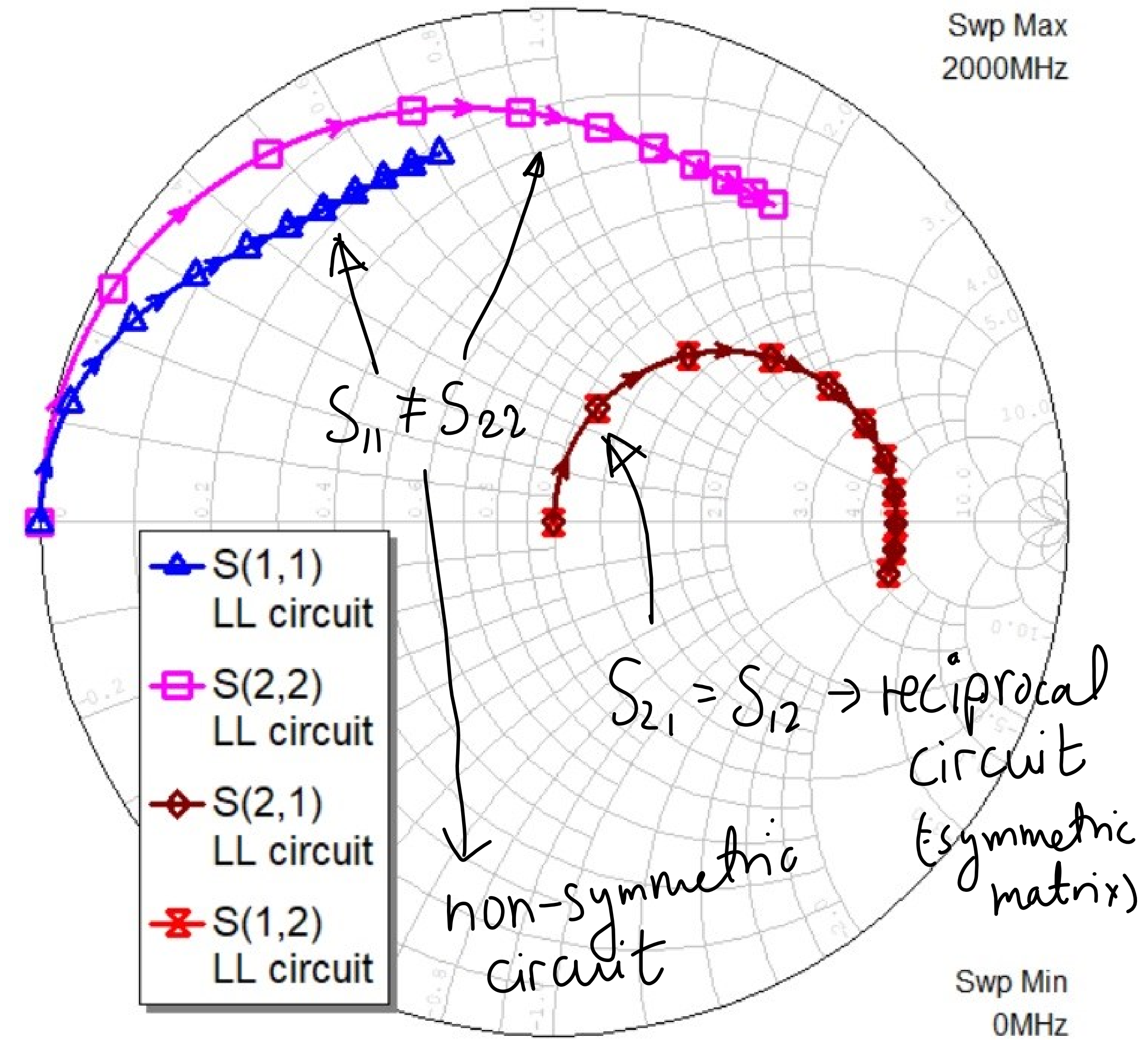
$-$ = signal OUT

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$



$|S_{11}| = |S_{22}|$??? → SEE NEXT PAGE

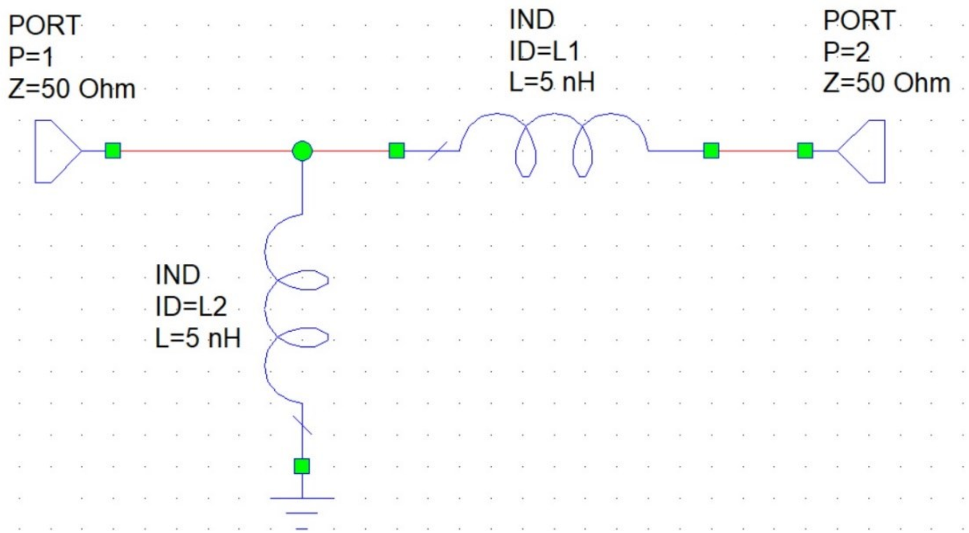
$|S_{21}| = |S_{12}|$

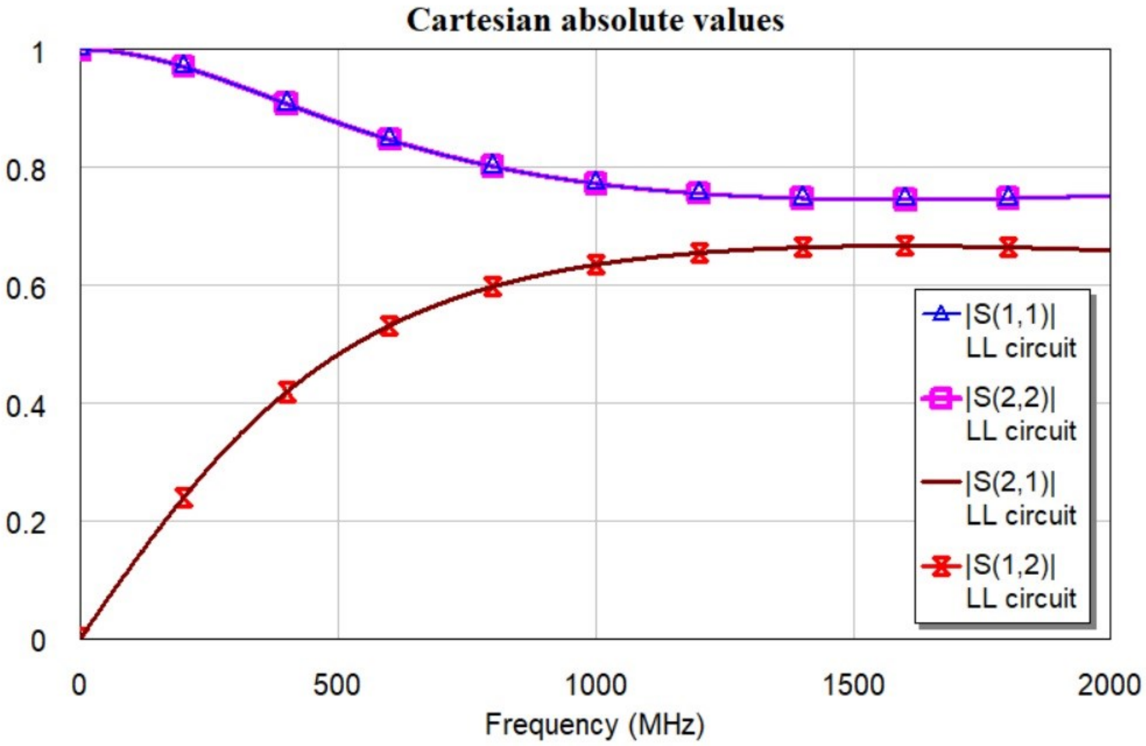


$S_{11} \neq S_{22}$

non-symmetric circuit

$S_{21} = S_{12}$ → reciprocal circuit (symmetric matrix)





In-class task: combine pairs!

Consider all circuits consist of ideal components and lines!

I. Does any of the circuits (1.-3.) have **resistive losses**? If yes, which? Which (a.-d.) is the corresponding S matrix?

II. Is any of the circuits (1.-3.) **non-symmetric**? If yes, which? Which (a.-d.) is the corresponding S matrix?

III. Is any of the circuits (1.-3.) **non-reciprocal**? If yes, which? Which (a.-d.) is the corresponding S matrix?

IV. Is any of the circuits (1.-3.) **lossless and symmetric**? If yes, which? Which (a.-d.) is the corresponding S matrix?

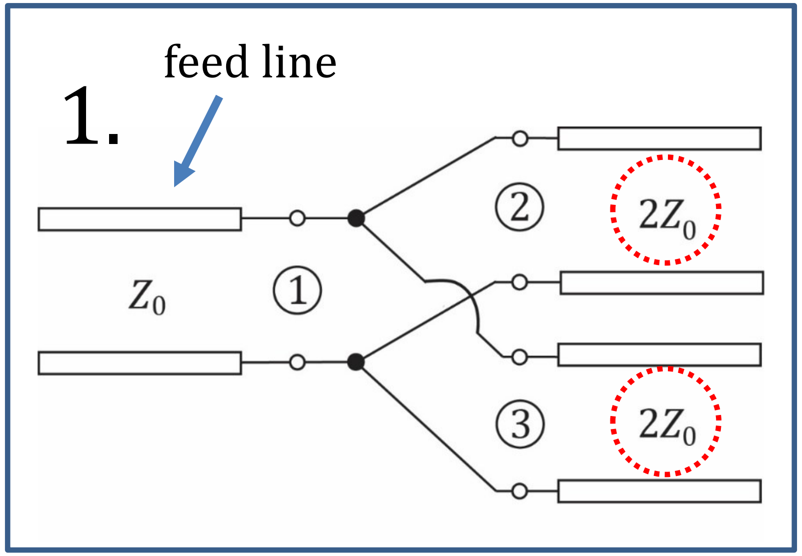
Extra: is any circuit simultaneously **lossless, matched and reciprocal**?

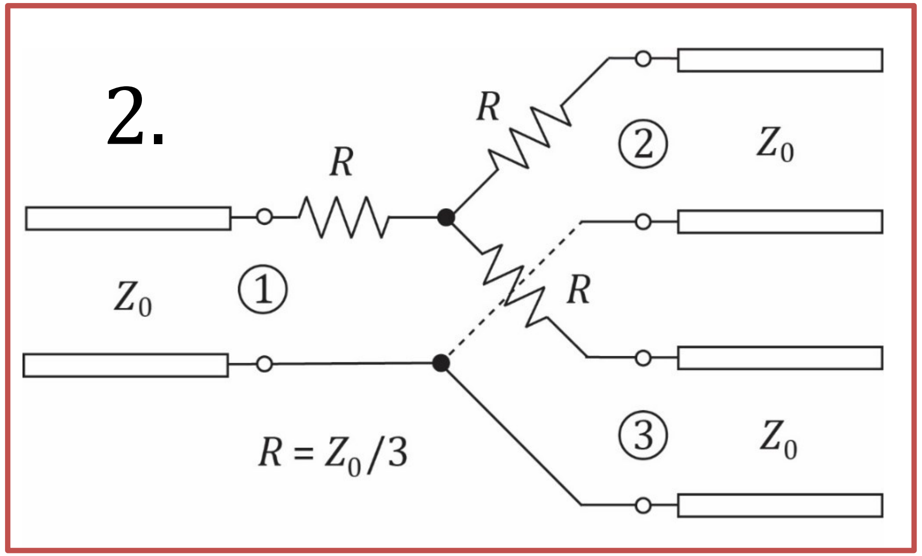
a.
$$S_a = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

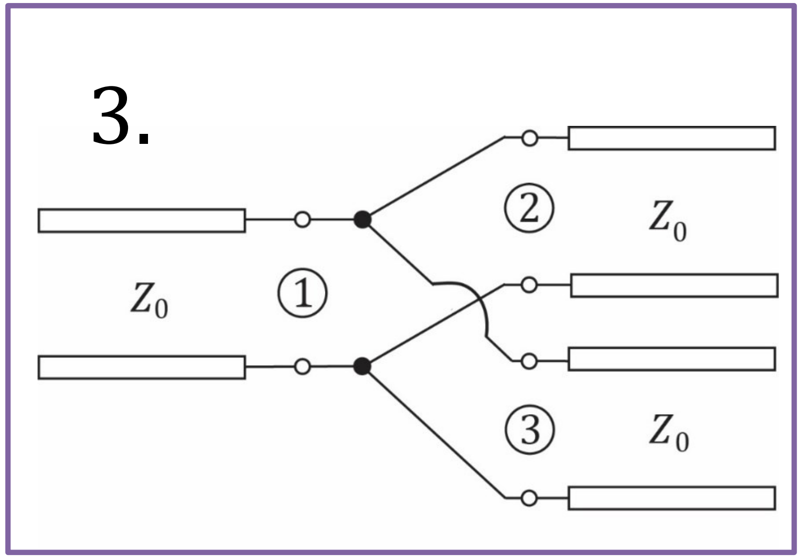
b.
$$S_b = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

c.
$$S_c = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

d.
$$S_d = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$







In-class task: combine pairs!

Consider all circuits consist of ideal components and lines!

- I. **resistive losses**? Circuit 2. contains resistors – i.e., is lossy. So is matrix d: $|S_{1i}|^2 + |S_{2i}|^2 + |S_{3i}|^2 = \frac{1}{2} \neq 1$
- II. **non-symmetric**? Circuit 1. is non symmetric. So is matrix a: $S_{11} \neq S_{22}$. (The circuit is lossless and so is its S matrix.)
- III. **non-reciprocal**? All circuits contain only passive, non-isotropic components → none of the circuits are non-reciprocal. Matrix b. is non-reciprocal.
- IV. **lossless and symmetric**? Circuit 3. is lossless and symmetric. Its matrix is c.

Extra: **lossless, matched and reciprocal**? There is no three-port which is lossless, matched and reciprocal. See Pozar C. 7.1.

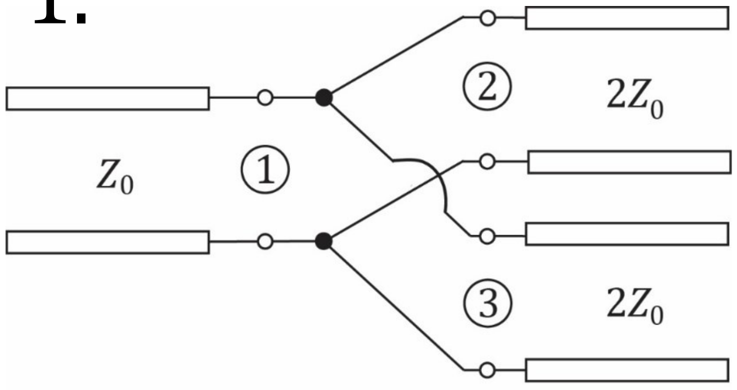
a. $$S_a = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

b. $$S_b = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

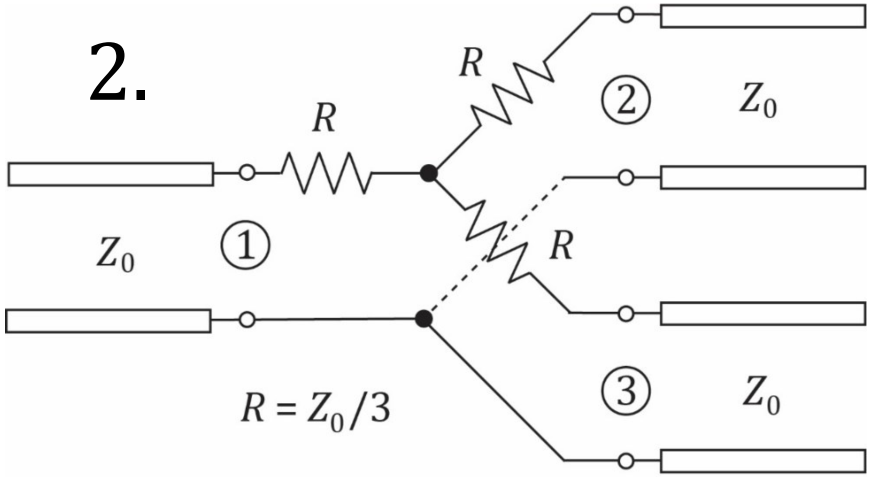
c. $$S_c = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

d. $$S_d = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

1.



2.



3.