Topic 4: Radio systems

Problem 4.1: Noise

A real, noisy resistor at the physical temperature T_{phys} can be modelled as a noiseless resistor and a (thermal) noise voltage source v_{rms} in series as shown Fig. 1, where $\mathrm{d}f$ is infinitesimally small bandwidth, f is the frequency, h is the Planck's constant and k is the Boltzmann's constant.

a) Show that up to the millimetre wave frequencies $hf \ll kT_{\mathrm{phys}}$ in the room temperature.

b) For the Taylor's polynomial $e^x \approx 1 + x$ holds when x is small. Using the approximation, show that the formula for the available power P_{n} from a resistor at the noise bandwidth B is

$$P_{\mathrm{n}} = kT_{\mathrm{phys}}B. \tag{1.1}$$

c) The thermal noise of a resistor is called white noise. Describe a definition of white noise, and justify why the noise (1.1) is white.

d) From (1.1), identify two effective methods to reduce the noise level at the wireless receiver. Do you know where the identified technique is used in practice, e.g., which application and why it is critical to reduce the noise level?

e) Assume an antenna of 150 K equivalent noise temperature. Explain briefly, what the equivalent noise temperature of an antenna means. Calculate the noise power in dBm for a radio system of 60 MHz bandwidth that appears at the output of the antenna. The impedance of the antenna is 50 Ω.

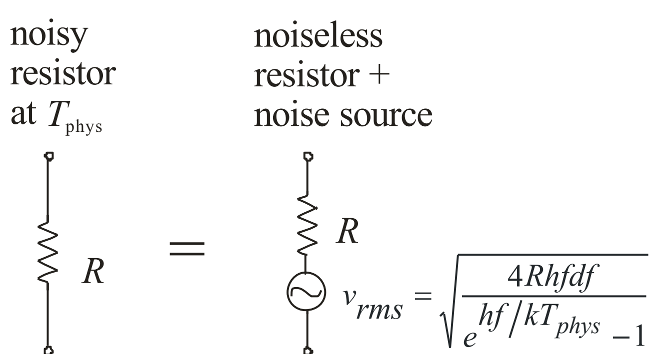

Figure 1: Equivalent circuit model of a noisy resistor consisting of a voltage source and a noiseless resistor.

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Problem 4.2: Noise temperature and noise figure

A horn antenna and a single side-band receiver front-end shown in Fig. 2 are used to receive a signal in a radio frequency signal at 60 GHz. The first stage of the receiver is a low-noise amplifier (LNA) connected to the antenna. The LNA forwards signals to a small cable to a mixer etc. as seen in the figure. The bandwidth of the receiver is defined at an intermediate (IF) frequency output by using a bandpass filter which has negligible loss. The signal received by the antenna is $S_\mathrm{i} = 1$ nW.

a) In problem 4.1, the antenna noise temperature was 150 K, while the same is 290 K in Fig. 2. Describe, what causes the difference of antenna noise temperature T_A.

b) Calculate the equivalent noise temperature T_R of the receiver (defined with respect to the input of the LNA) and the system noise temperature T_S.

c) Calculate the noise figure F of the receiver.

d) Calculate the signal-to-noise ratio (SNR) at the IF output, when the SNR at the LNA input is 30 dB.

e) Calculate the noise bandwidth B_n of the IF filter, given the other parameters specified above.

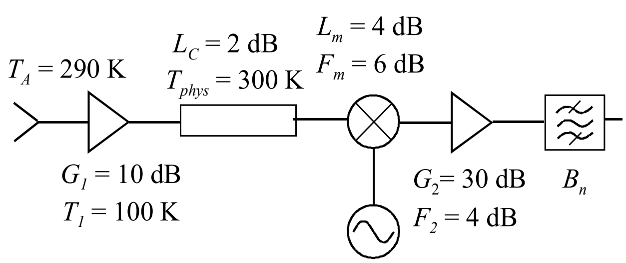

Figure 2: A single side-band receiver frontend.

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Problem 4.3: Antenna gains

The gain of an antenna to a given direction (typically in the main lobe) can be defined by measuring the radiated power density ($\mathrm{W/m^2}$) in the far-field region. The power density radiated by a horn antenna is 4.0 $\mathrm{mW/m^2}$ in the main lobe at the distance of 2.0 m. The feed power of the antenna is 3.5 mW. The measurement was performed at 7.2 GHz. The aperture size of the horn is $10 \times 9\ \mathrm{cm^2}$ (width and height) but the aperture efficiency is unknown. Assume that the impedance matching is "very good".

a) Explain using your own words, what does the gain (unit dBi) of an antenna mean. Does an antenna with a higher gain radiate more power (unit W) than another antenna with a smaller gain (assume the resistive losses are the same).

b) In the above setting, is the power density measured in the far-field region or not? Justify your answer.

c) Estimate the the gain of the antenna in dBi.

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Problem 4.4: Friis formula

a) When there is an isotropic transmit antenna that radiates energy to the entire solid angle, i.e., to all the direction in space uniformly, derive the power density [W/m^2] of the field in vacuum at a distance R from the antenna; the total power input to the antenna is P_t and assume perfect matching and radiation efficiency. The antenna has a uniform gain of $G_\mathrm{Tx} = 0$ dBi.

b) Calculate the total power available at distance R and compare it with the radiated power from the antenna.

c) At distance R, an antenna with effective aperture A_e receives the field from the transmit antenna. Express the power output at the receive antenna port P_r using P_t, R and A_e assuming that the receive antenna does not have matching losses. Do NOT use the receive antenna gain G_Rx in this part. The formula you get in this problem is called *the Friis formula*.

d) What is a maximal gain G of an antenna with effective aperture size A_e? Use the relationship to express the power output at the receive antenna port P_r using P_t, R and G_Rx assuming that the receive antenna does not have matching losses.

e) Let us think about mobile radio communications exploiting millimeter-wave and Terahertz waves in fifth-generation (5G) and beyond. They use higher frequencies than what the present 5G and legacy systems typically consider. Researchers have discussed if the use of higher carrier frequency is advantageous compared to the legacy systems or not. The following is a quite common understanding among researchers about antenna receiving power of 5G-and-beyond and legacy systems. Is the understand *always* correct, according to the results of c) and d)? If not, mention a case when the understanding is incorrect.

   Understanding: According to the Friis' formula, the millimeter-wave and Terahertz radios for 5G-and-beyond systems are inferior to lower frequency radios in the legacy systems because the received power is lower.