

# Second interactive lecture of Topic 4: Noise temperature, antenna gains, Friis formula and link budget

March 3, 2022

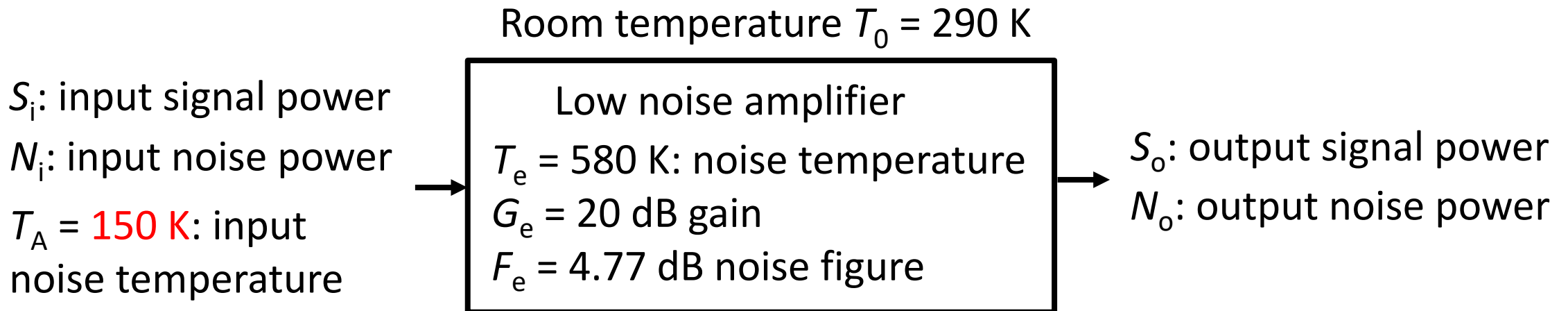
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This lecture covers Pozar Chapters 14.1-3.

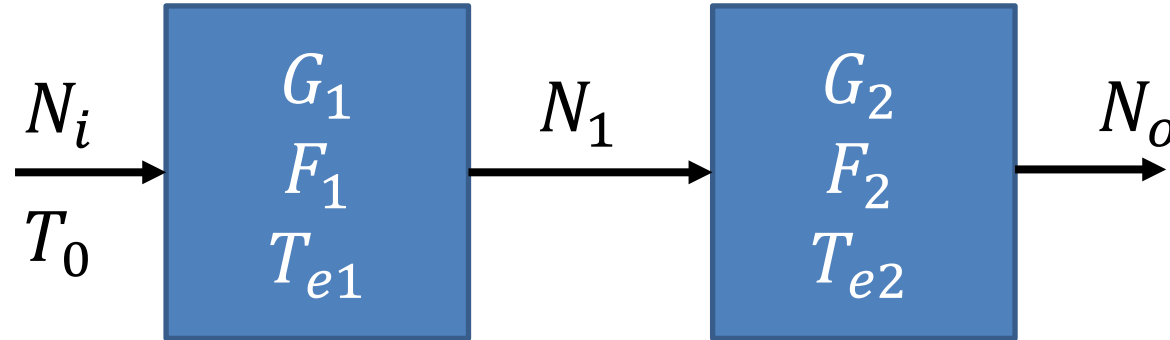
# First in-class exercise of Topic 4

- The input signal-to-noise ratio  $S_i/N_i = 30$  dB. What is the output signal-to-noise ratio  $S_o/N_o$  in dB?



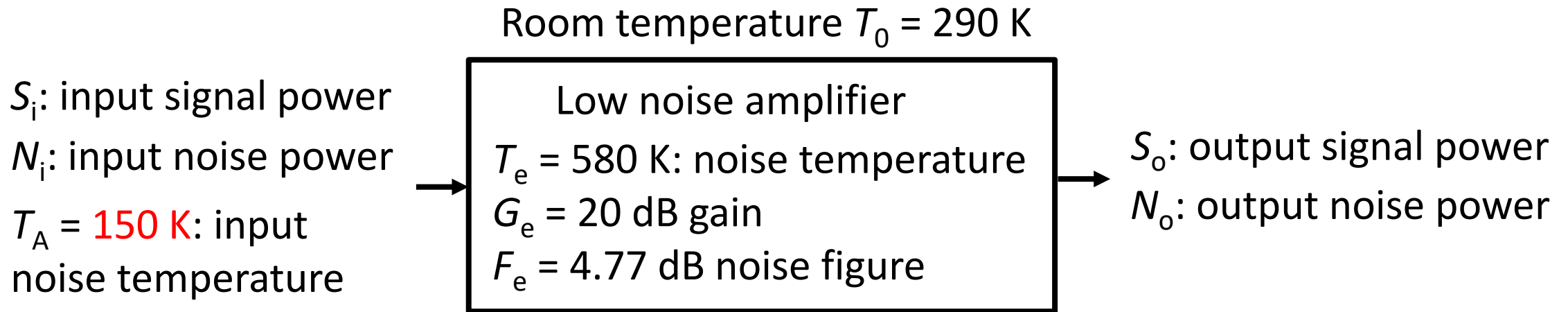
# Noise temperature of a receiver

- The first component at the RF front-end is most influential to total noise of the receiver.
  - We therefore use “low-noise” amplifier in the receiver.



$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1}$$

Q0: There is a low noise amplifier of the following input, output and component parameters. Choose an incorrect formula or explanation. Choose 5. if you do not know what to choose.



1.  $(S_o / N_o) = (S_i / N_i) - F_e$  in dB.
2.  $F_e = 1 + T_e / T_0$ .
3.  $N_o = kT_A B G_e + kT_e B G_e$ .
4.  $(S_o / N_o)$  does not depend on  $G_e$ .
5. I do not know.

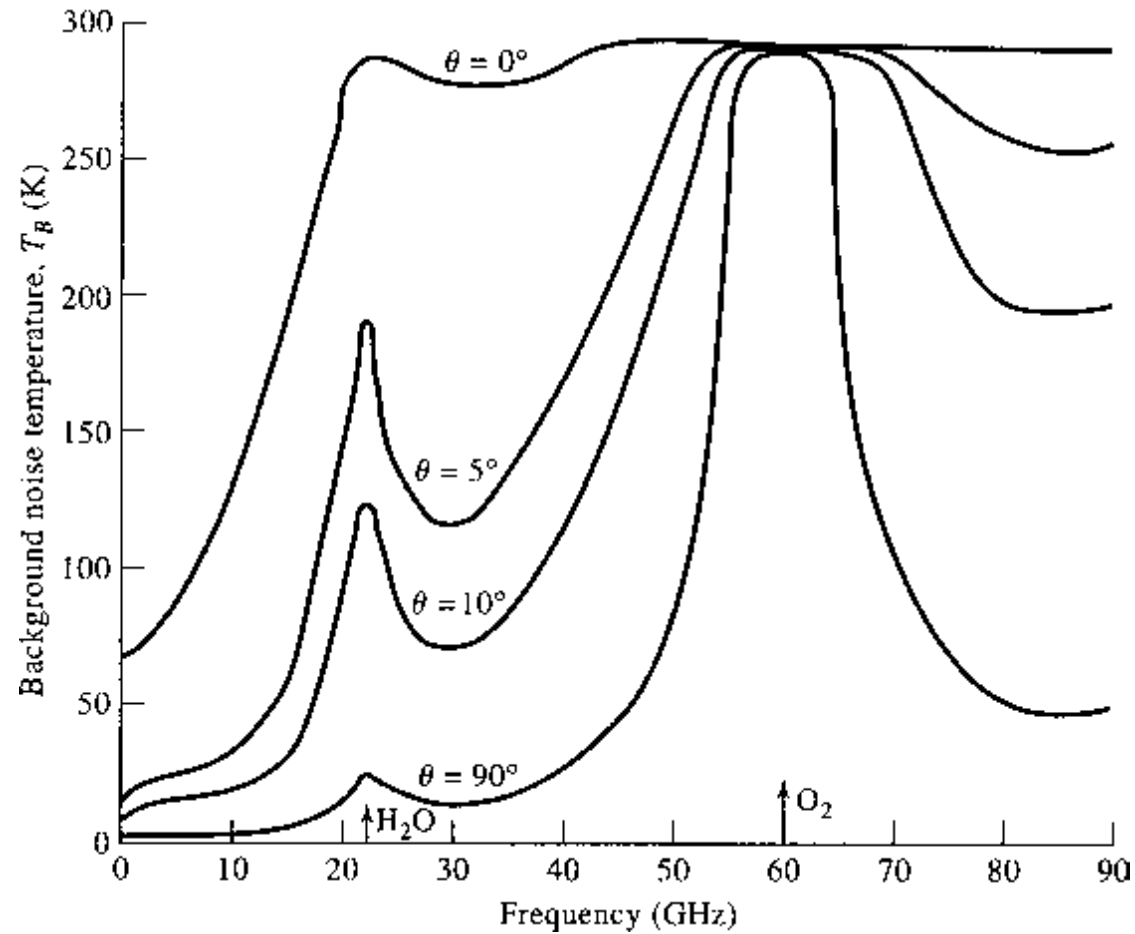
# Second pre-task for Topic 4

Watch a YouTube video (<https://youtu.be/x4yoGGUCZmM>) and read Pozar Chapter 14.1 “System aspects of antennas”, keeping in mind the following questions. After watching the video and reading the chapter, answer the questions.

1. Where does this mentioned noise floor of  $-174$  dBm/Hz come from?
2. Why the noise temperature of the antenna in the video is 150K? What is the corresponding noise floor (dBm/Hz)?
3. What happens to the noise floor if we point the directive antenna to the sky?
4. How would the situation change if the radiation pattern of the antenna was non-directive, i.e., omnidirectional?

# Background noise temperature

Pozar, Ch. 14, pp. 668.



**FIGURE 14.6**

Background noise temperature of sky versus frequency.  $\theta$  is elevation angle measured from the horizon. Data are for sea level, with surface temperature of  $15^\circ\text{C}$  and surface water vapor density of  $7.5\text{ gm/m}^3$ .

Q1: Choose an **incorrect explanation** about noise temperature.  
Choose 5. if you do not know what to choose.

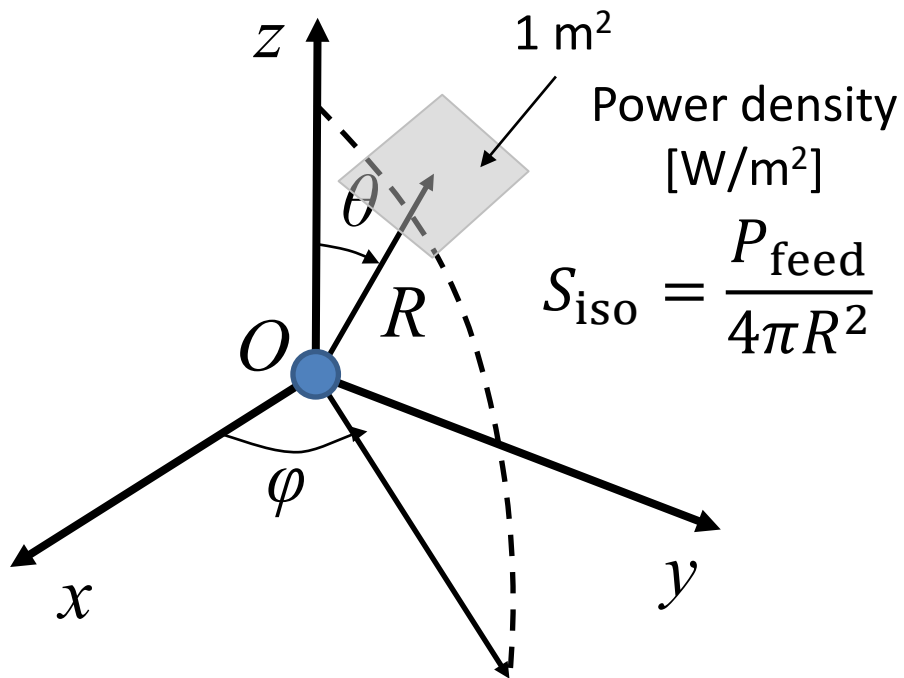
1. When efficiency of the antenna is 1, the antenna noise temperature is averaged background noise temperature.
2. Antenna noise temperature is always around 290 K in a room temperature.
3. When the efficiency of the antenna is 0, antenna noise temperature is temperature of the antenna.
4. Background noise temperature peaks at 22 and 60 GHz, where water and oxygen absorption presents.
5. I do not know which explanation is incorrect.

# Antenna gains

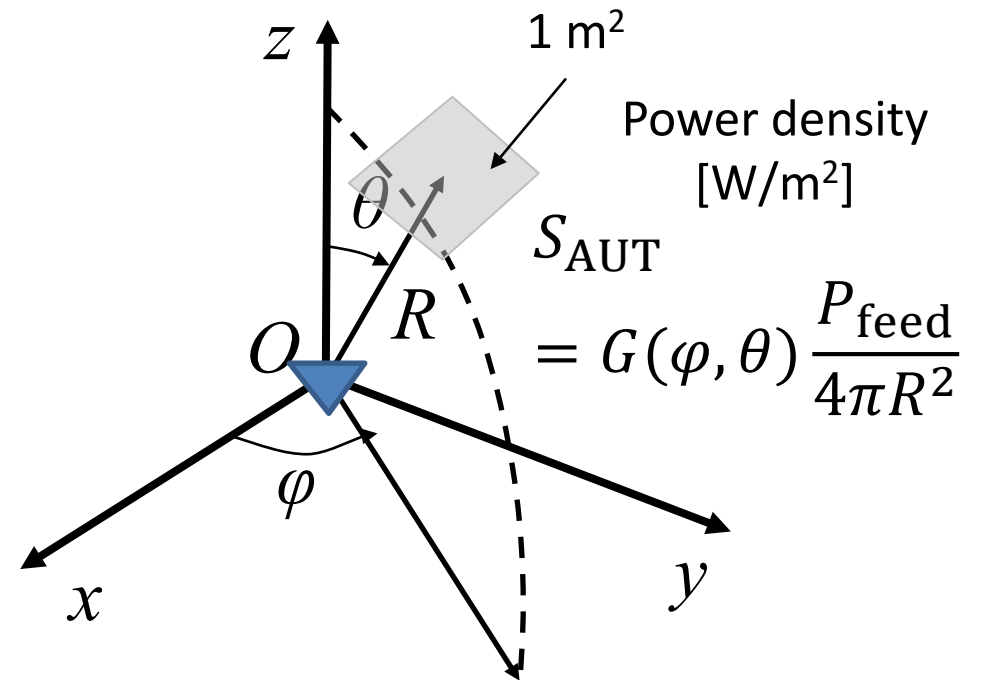
- ... are normalized radiated power density wrt that of an isotropic antenna

$$G(\varphi, \theta) = \frac{S_{\text{AUT}}}{S_{\text{iso}}}$$

Reference: isotropic antenna with the same efficiency as antenna under test



Antenna under test





Q2: Choose a side to the following statement; choose 3. if you do not know which side to choose.

Statement: an antenna amplifies the total signal power (unit in Watt) inputted to a feed port, when it has a gain (unit in dBi).

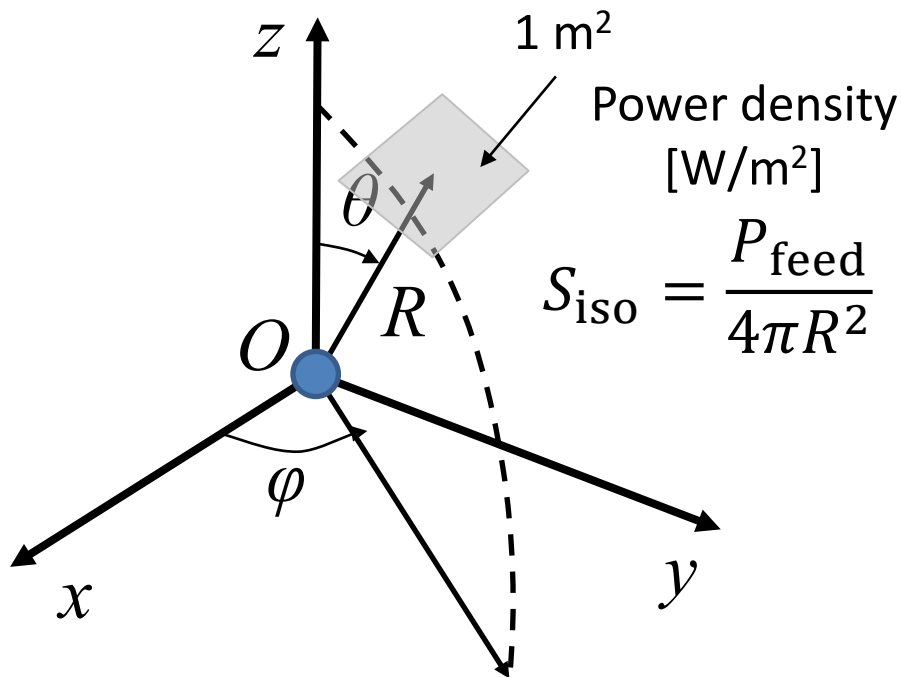
1. The statement is correct.
2. The statement is incorrect.
3. I do not know which side to choose.

# Antenna gains

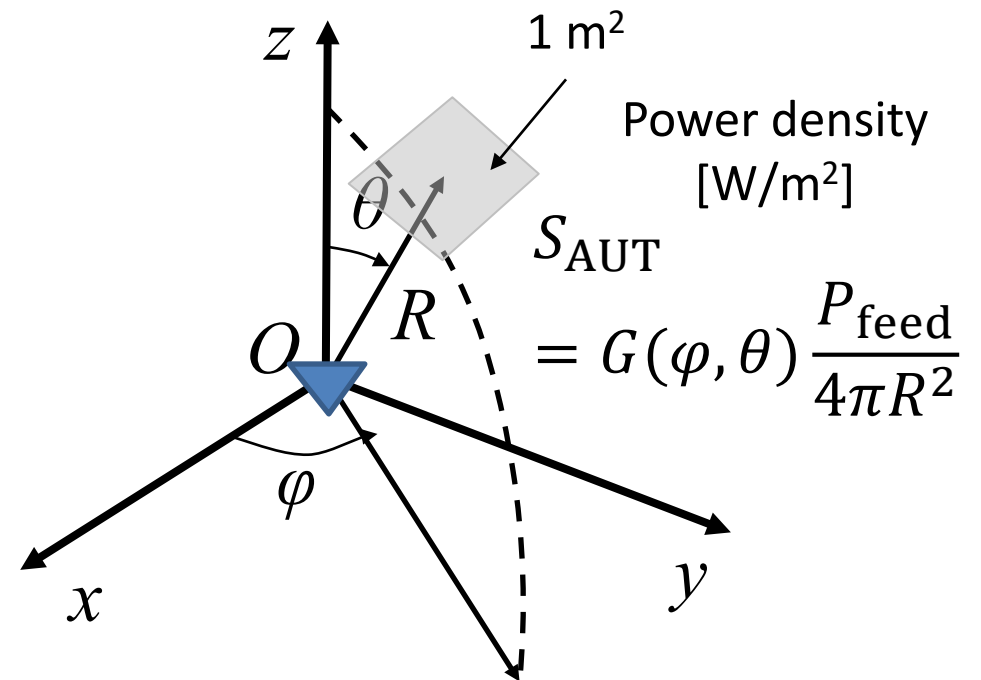
- ... are normalized radiated power density wrt that of an isotropic antenna
  - And hence it is often quantified in dBi scale
  - Gain does NOT mean amplification of total power [W] at an antenna.
    - Total radiated power = power accepted by a lossless antenna.

$$G(\varphi, \theta) = \frac{S_{\text{AUT}}}{S_{\text{iso}}}$$

Reference: isotropic antenna with the same efficiency as antenna under test



Antenna under test



# Antenna gains

- Which distance  $r$  to measure and define the antenna gains?

$$G(\varphi, \theta, r) = \frac{S_{\text{AUT}}}{S_{\text{iso}}} \quad r > \frac{2D^2}{\lambda}$$

The fields observed from the center and edges of the aperture should have a phase difference less than  $22.5^\circ = \pi/8$  as

$$(l' - l)\beta = \left\{ \sqrt{l^2 + \left(\frac{D}{2}\right)^2} - l \right\} \beta = \left[ l \left\{ 1 + \left(\frac{D}{2l}\right)^2 \right\}^{1/2} - l \right] \beta < \frac{\pi}{8},$$

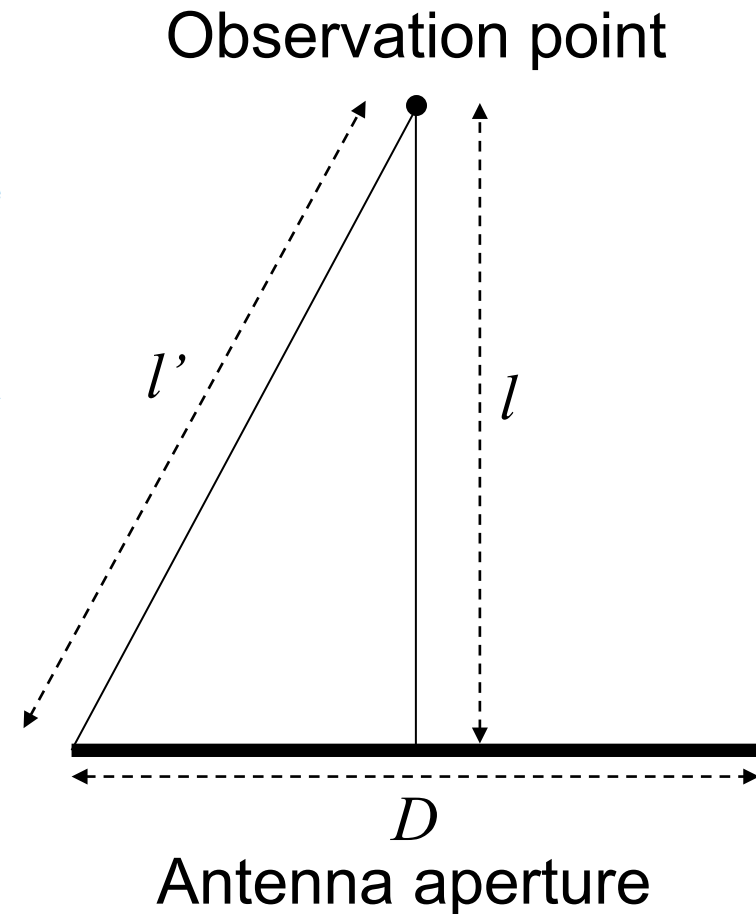
where  $\beta$  is a wavenumber. The above formula can be simplified using the Taylor series expansion  $(1 + x)^n = 1 + nx$  as

$$\begin{aligned} (l' - l)\beta &= \left[ l \left\{ 1 + \frac{1}{2} \left(\frac{D}{2l}\right)^2 \right\} - l \right] \beta, \\ &= \frac{D^2}{8l} \cdot \frac{2\pi}{\lambda} < \frac{\pi}{8}, \end{aligned} \tag{4.7}$$

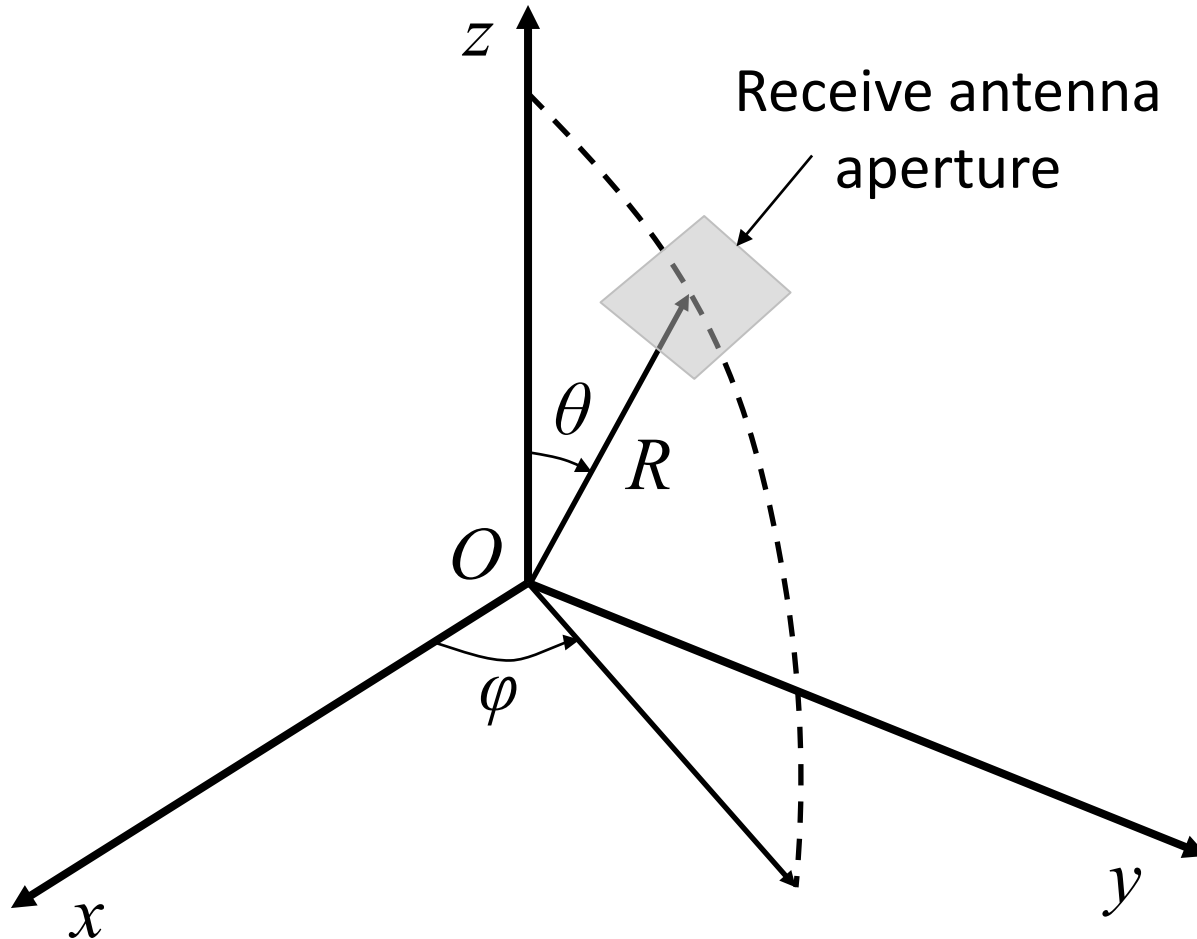
Reformulating (4.7) yields

$$l > \frac{2D^2}{\lambda},$$

for observing far-fields from an antenna aperture.



# Friis formula



$$S_{\text{avg}} = \frac{P_t}{4\pi R^2}$$

$$G = 4\pi \frac{A_e}{\lambda^2}$$

$$P_r = S_{\text{avg}} A_e$$

$$= P_t G \left( \frac{\lambda}{4\pi R} \right)^2$$

Q3: Choose a side to the following statement; choose 3. if you do not know which side to choose.

Statement: when an isotropic antenna radiates fields and a receive antenna has a constant effective aperture size over the frequency such as ideal horn antennas, the receiving power from the antenna port depends on the frequency.

1. The statement is correct.
2. The statement is incorrect.
3. I do not know which side to choose.

$$S_{\text{avg}} = \frac{P_t}{4\pi R^2} \quad P_r = S_{\text{avg}} A_e$$
$$G = 4\pi \frac{A_e}{\lambda^2} \quad = P_t G \left( \frac{\lambda}{4\pi R} \right)^2$$

Q4: Choose a side to the following statement; choose 3. if you do not know which side to choose.

Statement: when an isotropic antenna radiates fields and a receive antenna has a constant gain over the frequency such as dipoles, the receiving power from the antenna port depends on the frequency.

1. The statement is correct.
2. The statement is incorrect.
3. I do not know which side to choose.

$$S_{\text{avg}} = \frac{P_t}{4\pi R^2} \quad P_r = S_{\text{avg}} A_e$$
$$G = 4\pi \frac{A_e}{\lambda^2} \quad = P_t G \left( \frac{\lambda}{4\pi R} \right)^2$$

Q5: Choose a side to the following statement; choose 3. if you do not know which side to choose.

Statement: Radio links operating at 28 GHz carrier frequency is inferior to those at 2 GHz in receiving powers at a mobile, even if line-of-sight exists between base and mobile stations.

1. The statement is correct.
2. The statement is incorrect.
3. I do not know which side to choose.

$$S_{\text{avg}} = \frac{P_t}{4\pi R^2} \quad P_r = S_{\text{avg}} A_e$$
$$G = 4\pi \frac{A_e}{\lambda^2} \quad = P_t G \left( \frac{\lambda}{4\pi R} \right)^2$$

# Friis formula in practice

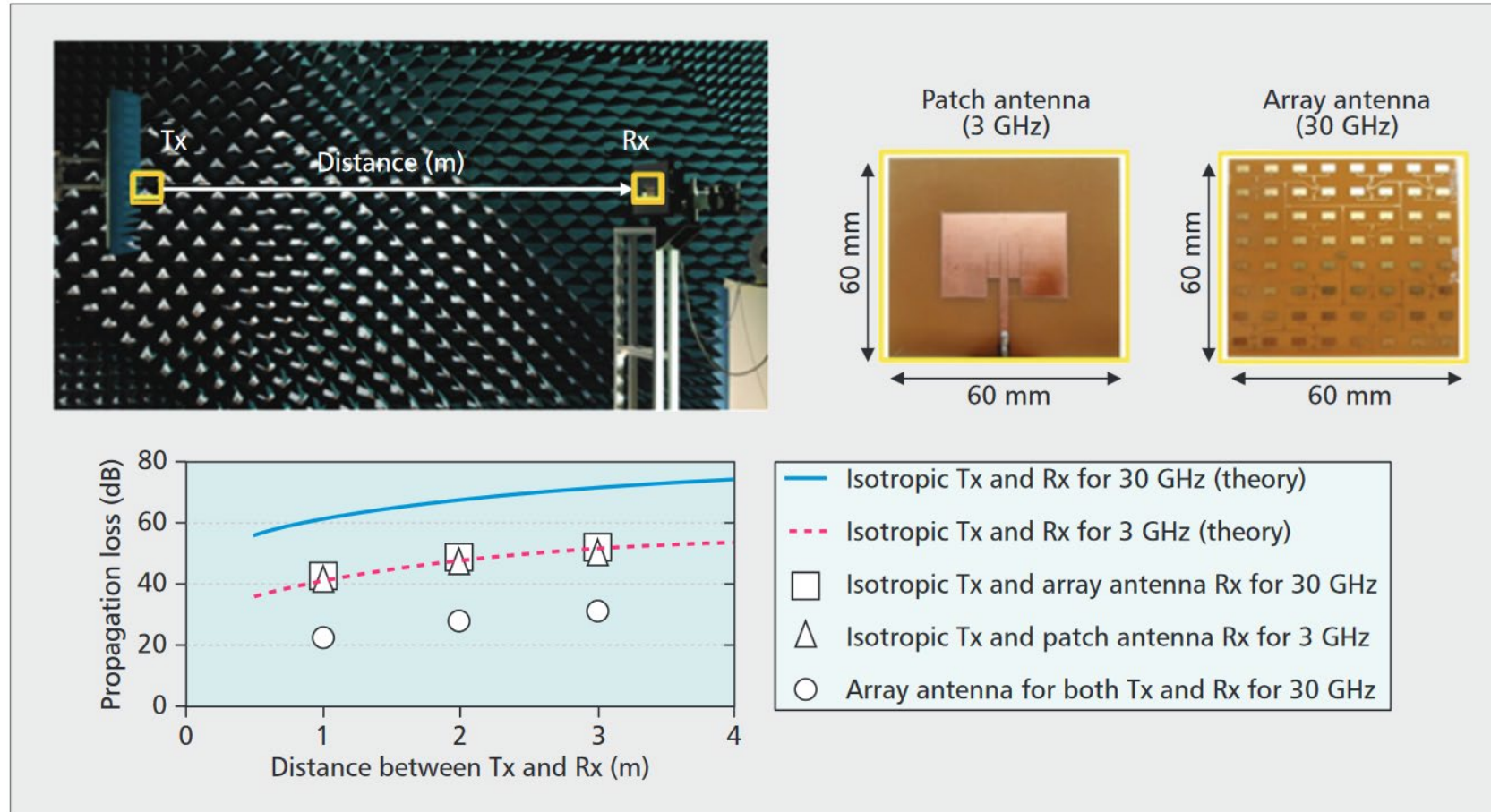
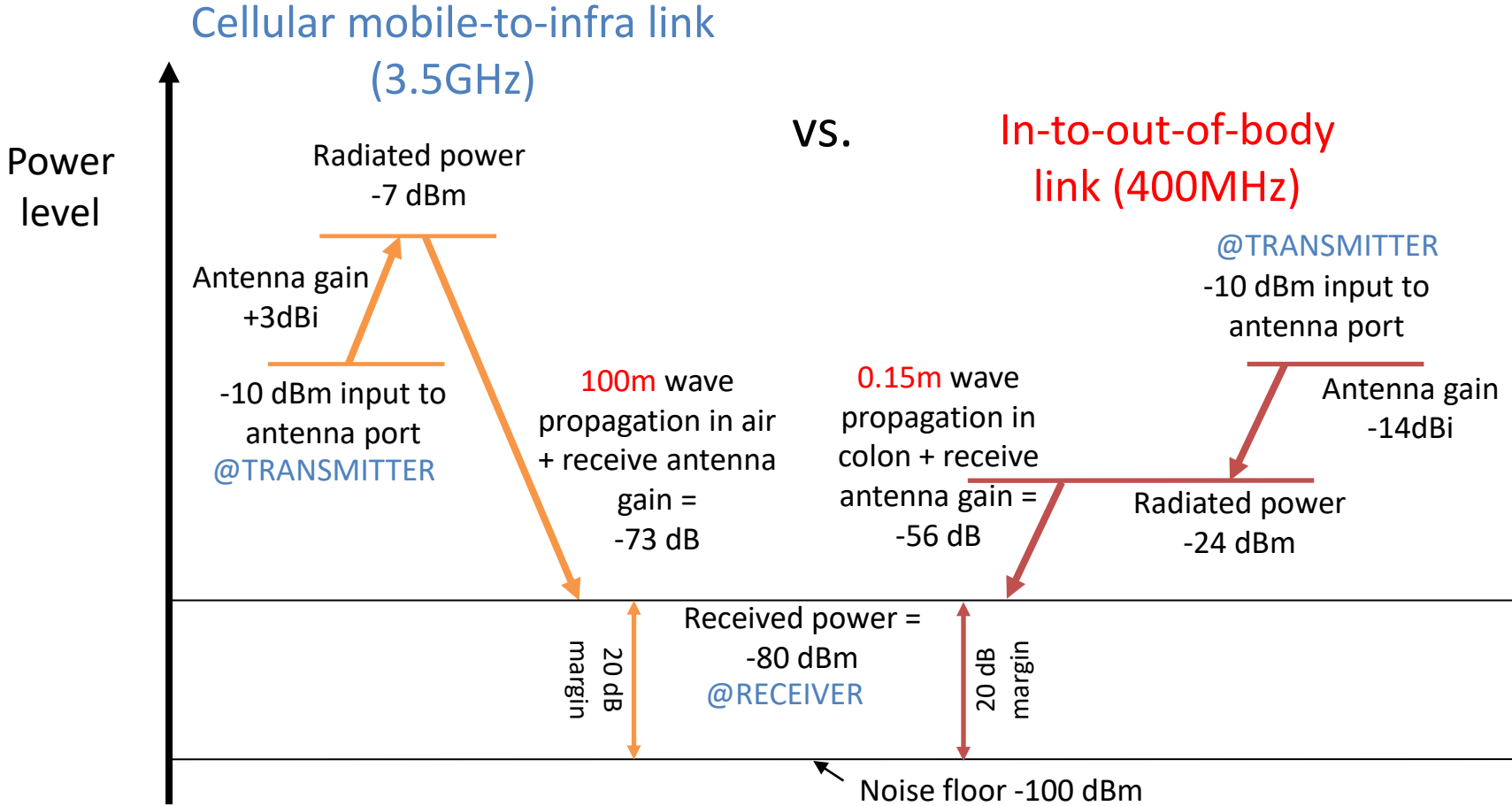


Figure 1. Results of verification measurements of propagation loss predicted by the Friis equation.



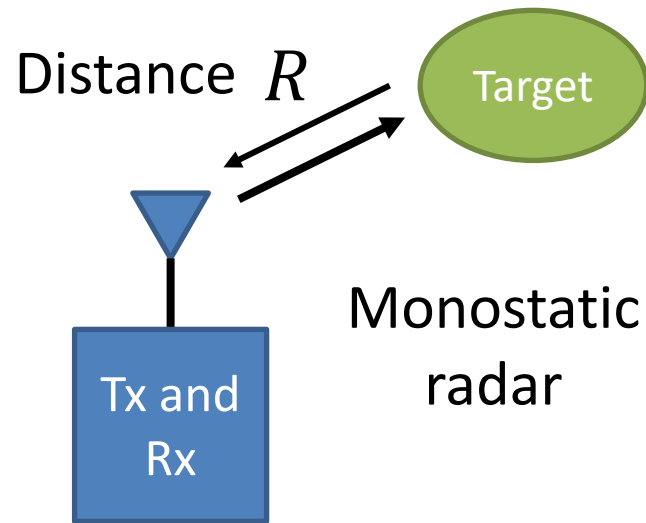
# Link budget analysis



(Here, we assume the same input power to the antenna, and the same noise floor at the receiver)

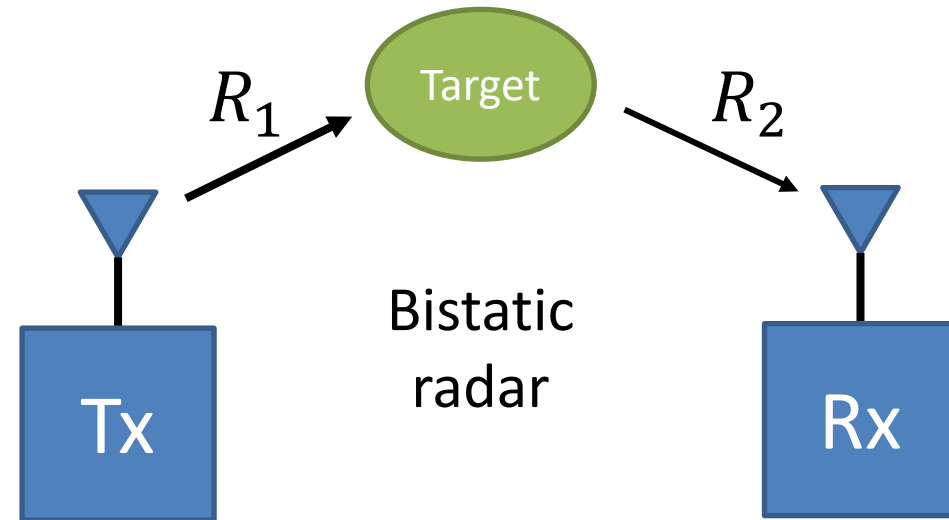
# Radar equation

- Scattering can be quantified by a radar cross section (RCS)  $\sigma$  [m<sup>2</sup>].



Power density to the target

$$S = \frac{P_t G_t}{4\pi R_1^2} \text{ [W/m}^2\text{]},$$



Scattered power from the target

$$P_s = \frac{P_t G_t}{4\pi R_1^2} \sigma \text{ [W]},$$

Received power at the Rx

$$P_r = \frac{P_t G_t G_r}{4\pi R_1^2} \left( \frac{\lambda}{4\pi R_2} \right)^2 \sigma \text{ [W]}$$