Aalto University
School of Electrical
Engineering

ELEC-E8126: Robotic Manipulation
Friction and grasping

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7.3.2022

Learning goals

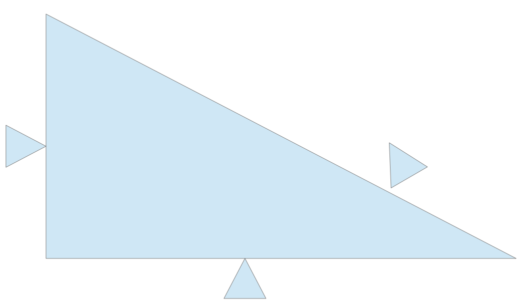
- Understand analytical models of contact with friction.
- Use those to define quality measures for grasps.

Recap: Grasp planning

- Where an object needs to be grasped in order to perform a particular task?
  - Where to place contacts on the object to immobilize it?
  - Or where to place the hand?

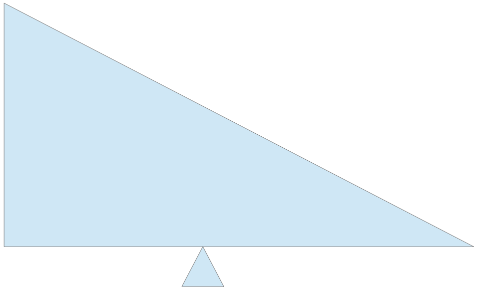
Several contacts in plane

- Can the object move? Around which point?
- What about if there's friction?



Contact with friction

- What's the range of possible forces caused by the contact?



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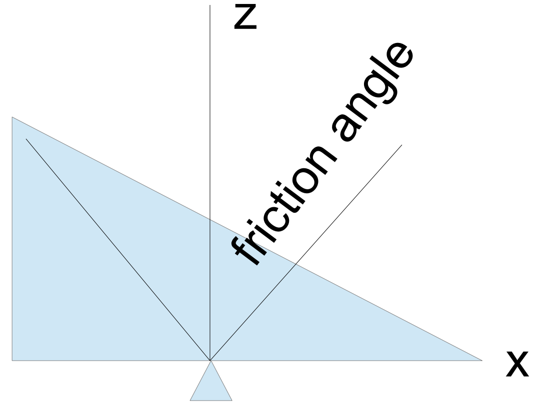
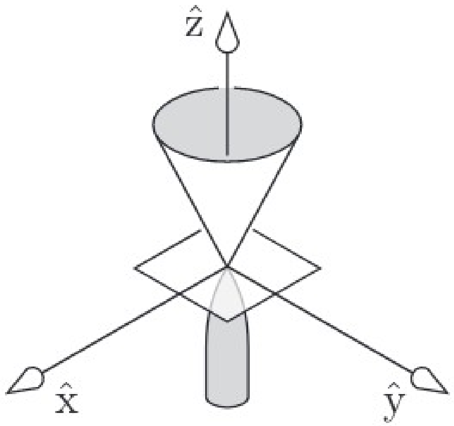
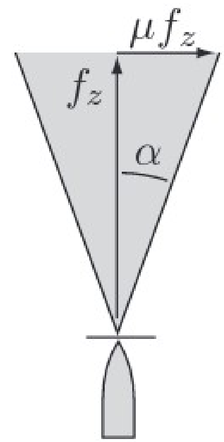


$$\sqrt{f_x^2 + f_y^2} \leq \mu f_z, f_z \geq 0$$

Friction cones





$$\sqrt{f_x^2 + f_y^2} \leq \mu f_z, f_z \geq 0$$

- In 2-D: positive span of edges

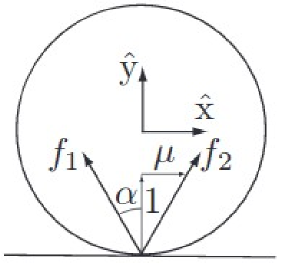
$$FC = \left\{ k_1 \begin{pmatrix} \mu \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} -\mu \\ 1 \end{pmatrix} \middle| k_1, k_2 \geq 0 \right\}$$

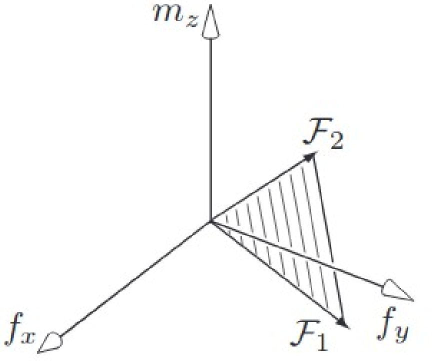
Wrench cone

contact point

contact normal

- Remember wrench for single contact $\boldsymbol{F} = (\boldsymbol{p} \times \boldsymbol{n}, \boldsymbol{n})$



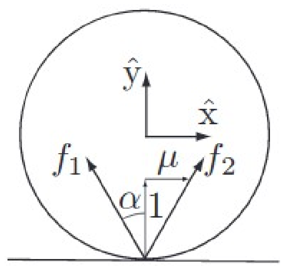


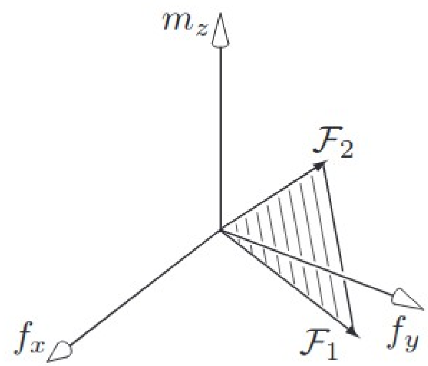
- What's now the wrench cone corresponding to the friction cone $FC = \left\{ k_1 \begin{pmatrix} \mu \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} \mu \\ 1 \end{pmatrix} \middle| k_1, k_2 \geq 0 \right\}$

Wrench cone

contact point — contact normal

- Remember wrench for single contact $\boldsymbol{F} = (\boldsymbol{p} \times \boldsymbol{n}, \boldsymbol{n})$





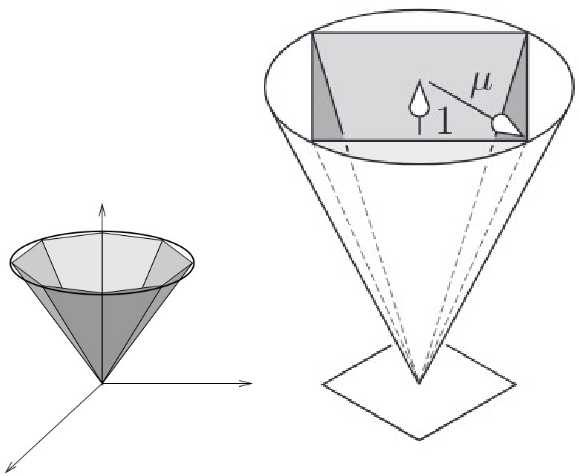
- What's now the wrench cone corresponding to the friction cone $FC = \left\{ k_1 \begin{pmatrix} \mu \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} \mu \\ 1 \end{pmatrix} \middle| k_1, k_2 \geq 0 \right\}$

- Wrench cone $WC = \{ k_1 \boldsymbol{F_1} + k_2 \boldsymbol{F_2} | k_1, k_2 \geq 0 \}$

3-D: Friction cone approximation

- In 3-D, friction cone is usually approximated, because analytical solution is difficult.
- Conservative polyhedral approximation (different number of basis forces can be used)

$$FC = \left\{ \sum_i k_i \boldsymbol{n_i} \middle| k_i \geq 0 \right\}$$



Example: With 4 basis forces,

$$\boldsymbol{n}_1 = \begin{pmatrix} \mu \\ 0 \\ 1 \end{pmatrix} \quad \boldsymbol{n}_2 = \begin{pmatrix} -\mu \\ 0 \\ 1 \end{pmatrix} \quad \boldsymbol{n}_3 = \begin{pmatrix} 0 \\ \mu \\ 1 \end{pmatrix} \quad \boldsymbol{n}_4 = \begin{pmatrix} 0 \\ -\mu \\ 1 \end{pmatrix}$$

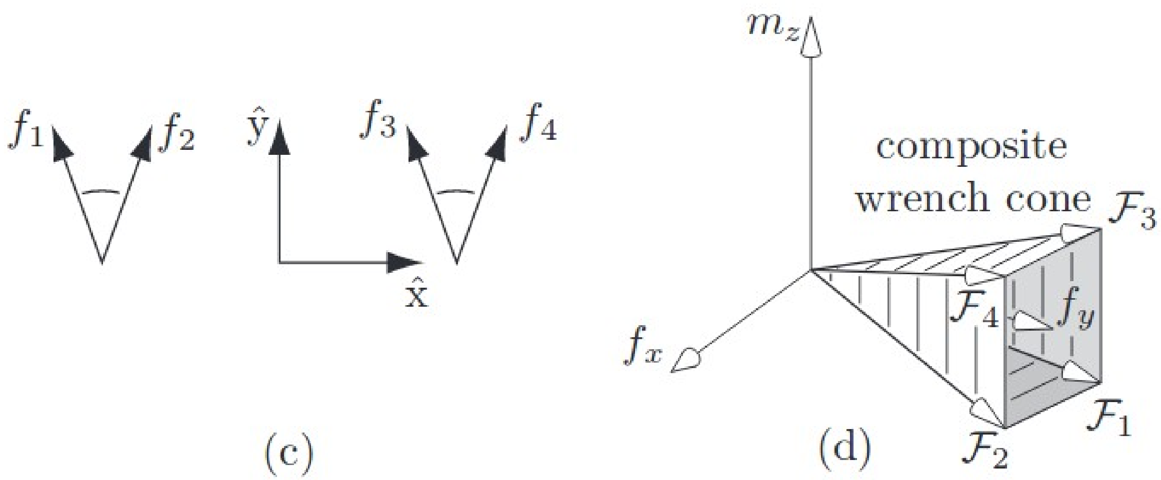
Wrench cone

$$F = (p \times n, n)$$

- WC again positive span of individual wrenches

$$WC = \left\{ \sum_i k_i \boldsymbol{F}_i \middle| k_i \geq 0 \right\}$$

- Same also across multiple contacts.



Why is F_1 different from F_3 even if f_1 and f_3 seem to be the same?

Force closure

- A grasp is *force closure* if for any external wrench there exist contact wrenches that cancel it.
- In other words, if we apply sufficient force at each contact, any <u>external wrench</u> can be compensated for.

  e.g. gravity, contact with environment

- This is equivalent to first-order form closure.
  – Contact wrenches span positively entire space

$$\left\{ \sum_i k_i \boldsymbol{F}_i \middle| k_i \geq 0 \right\} = R^6$$

- Is there a force closure grasp for any object?

Existence of force closure grasps

- Theorem: For any bounded shape that is not a surface of revolution, a force closure grasp exists (Mishra et al., 1987).

- Any non-exceptional surface requires at least p+1 contacts without friction (wrench space dimension p)
- Any non-exceptional surface can be grasped by choosing at most 2p contacts without friction.

$$p = 3 \text{ (planar)} \implies 4 \leq k \leq 6$$
$$p = 6 \text{ (spatial)} \implies 7 \leq k \leq 12.$$

With friction, 3 contacts may be sufficient in 3D.

Determining force closure

- Force-closure is equivalent to
  - Contact wrenches positively span $\mathbb{R}^p$
  - Convex hull of contact wrenches contains a neighborhood of the origin

Consider this without friction:
What are the wrenches?

How about with friction?
Draw to illustrate.

How could you check the convex hull condition?
What are the boundaries of the convex hull?

Grasp quality metrics

- Reminder: Contact forces $CF = \left\{ \sum_i k_i \boldsymbol{F}_i \middle| 0 \le k_i \le f_{max} \right\}$

  Without friction

- Approach:
  - Since contact forces at each point not known, normalize wrench basis vectors ***F*** to unit length.
    - First scale moments (torques) by characteristic length of object.
    - Origin at object CoM.
  - Construct convex hull and its supporting hyperplanes.
  - Find shortest distance from origin to any of the hyperplanes.
    - Assumes sum of forces is bounded.

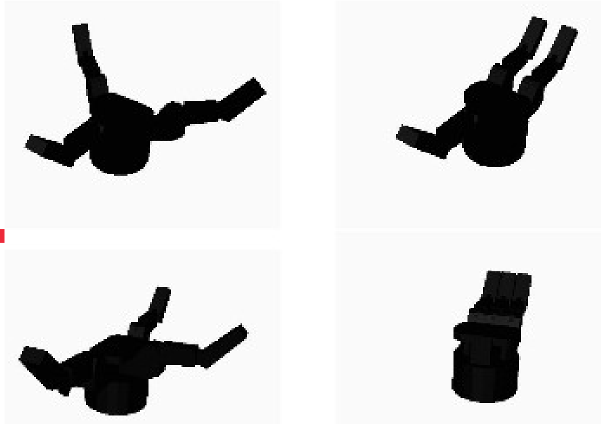
$$CF = \left\{ \sum_i k_i \boldsymbol{F}_i \middle| k_i \ge 0, \sum_i k_1 \le 1 \right\}$$

Ferrari & Canny, 1992

Sampling based grasp planning revisited

- Sampling approach
  - Choose candidate contacts.
  - Evaluate resulting grasp.

- Instead of choosing contact locations, sample location to place *preshaped* hand, and simulate where contacts happen after closing fingers.
  - Preshapes for prototypical grasps, e.g. pinch grasp, power grasp, cylindrical grasp.
  - Miller et al. 2003.



Caveats

- Quality measures are based on assumptions that are not necessarily entirely true.
- Many quality measures (with different assumptions) have been proposed.
  - Address different issues such as minimizing finger forces, contact placement accuracy, hand configuration (manipulability, joint limits), task compatibility.
  - Recent review: Roa & Suarez, “Grasp quality measures: review and performance”, 2015.

Summary

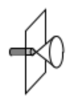
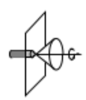
- Force closure means that a grasp is able to cancel any external wrench if contact forces are sufficiently high.
- Grasps can be planned by maximizing grasp quality metrics.

Extra topics

- Soft contacts
- Complex hands
  - Posture subspaces
- Data-driven grasping
  - Grasp databases
  - Grasping as learning problem

Modeling friction of soft fingertips

- Soft fingertips can be modeled with friction “cones” that include torsional friction.

Point contact with friction		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\sqrt{f_1^2 + f_2^2} \leq \mu f_3$ $f_3 \geq 0$
Soft-finger		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\sqrt{f_1^2 + f_2^2} \leq \mu f_3$ $f_3 \geq 0$ $\lvert f_4 \rvert \leq \gamma f_3$

Torsional friction

Grasping and complex hands

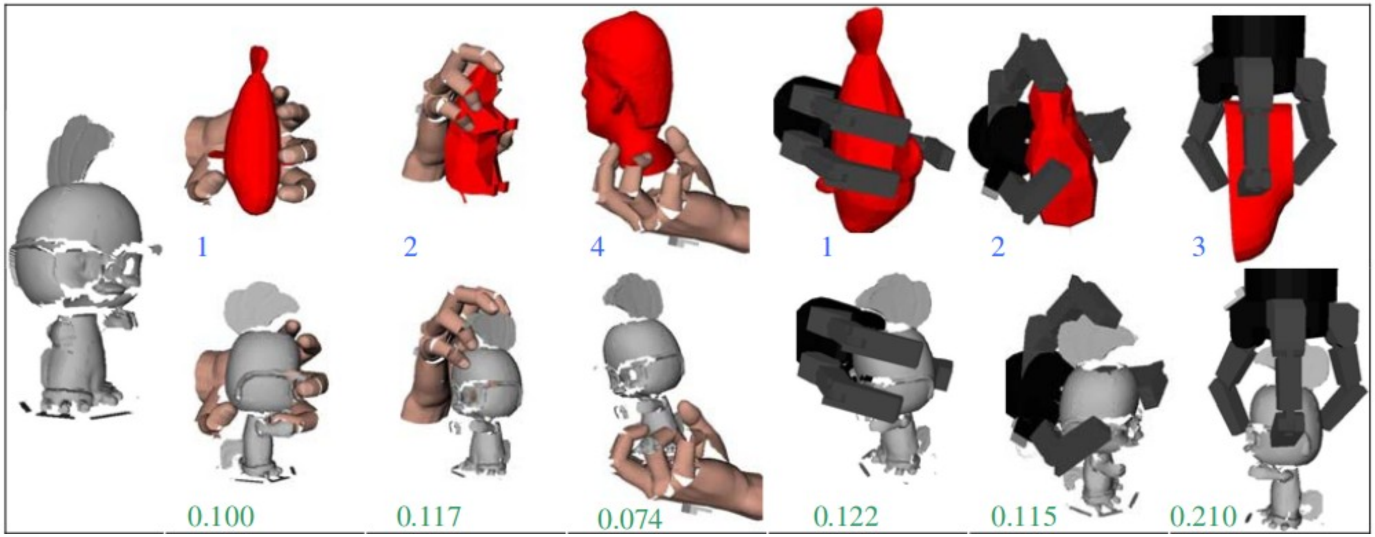
- How to plan with complex hands beyond preshapes?
- Idea: Use low-dimensional posture space.

Correlated joints

Model	DOFs	Eigengrasp 1			Eigengrasp 2		
		Description	min	max	Description	min	max
Barrett	4	Spread angle opening	 →		Finger flexion	 →	
DLR	12	Prox. joints flexion Finger abduction Thumb flexion	 →		Dist. joints flexion Prox. joints extension Thumb flexion	 →	
Robonaut	14	Thumb flexion MCP flexion Index abduction	 →		Thumb flexion MCP extension PIP flexion	 →	
Human	20	Thumb rotation Thumb flexion MCP flexion Index abduction	 →		Thumb flexion MCP extension PIP flexion	 →	

Data-driven grasping for unknown objects

- Idea: Database-driven grasping
  - Pre-compute (as above) grasps for many objects of different sizes and shapes, store in a database.
  - Measure shape of object.
  - Find closest corresponding object (or its part) in database and use its grasps.

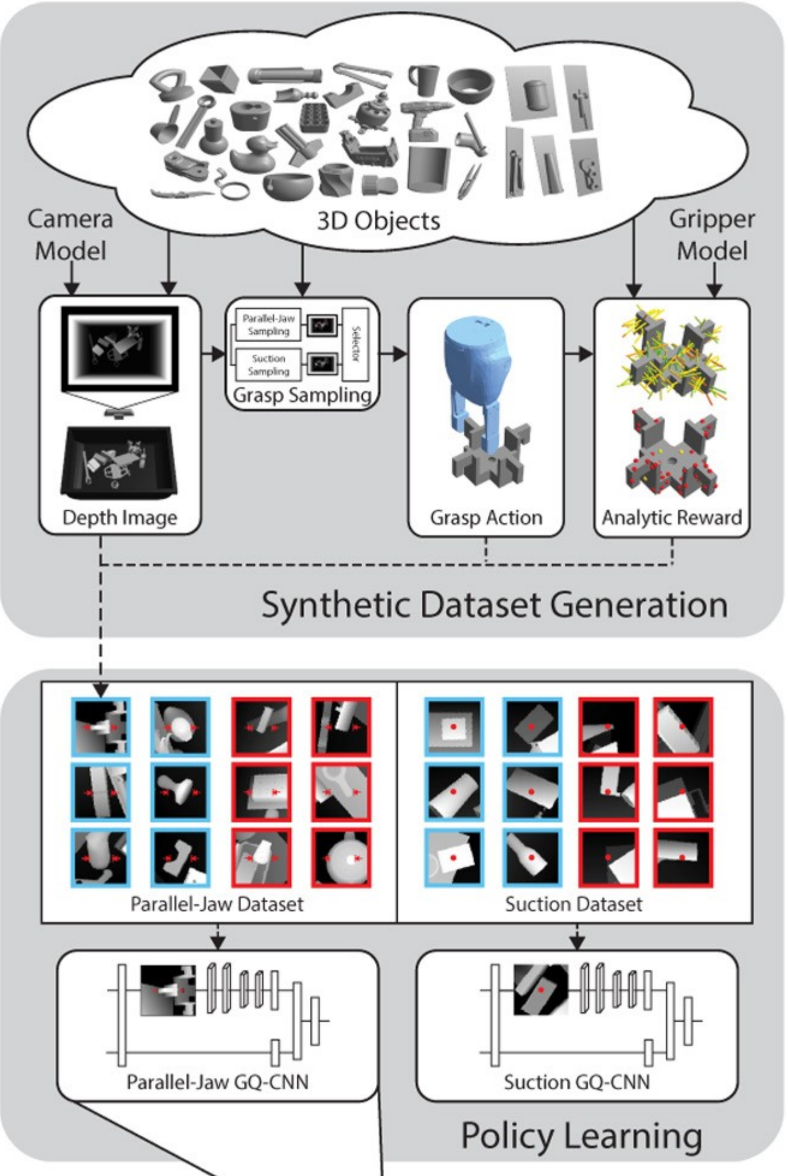


Goldfeder et al. 2009

Grasping as a learning problem

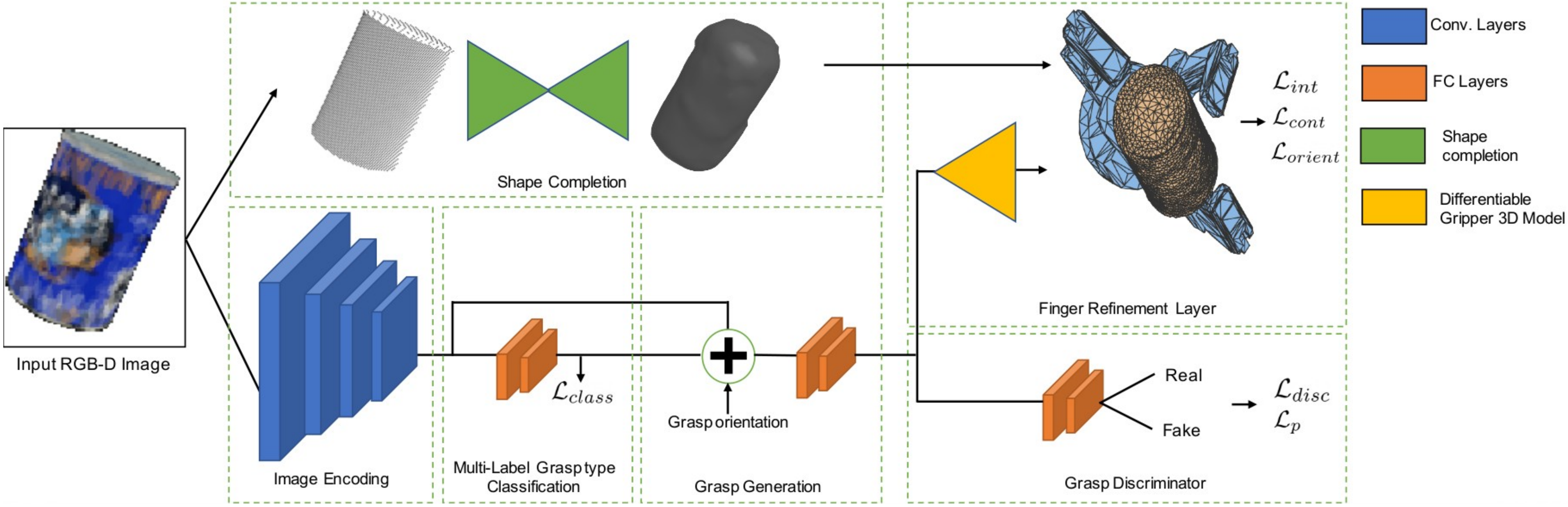
Training from physical trials an option but expensive!

- Optimization of metrics slow and difficult from visual information.
- Idea: Learn mapping from images to grasps.
  - Generate synthetic “good” grasps using existing approaches.
  - Train a neural network to predict grasp success.
- Training data metrics may still limit.



berkeleyautomation.github.io/dex-net/

Example: GanHand



Lundell et al., 2021

Next time: Closed kinematic chains

- Readings:
  - Lynch & Park, Chapter 7-7.1.3