Lecture 7

Lecturer: G. S. Paraoanu

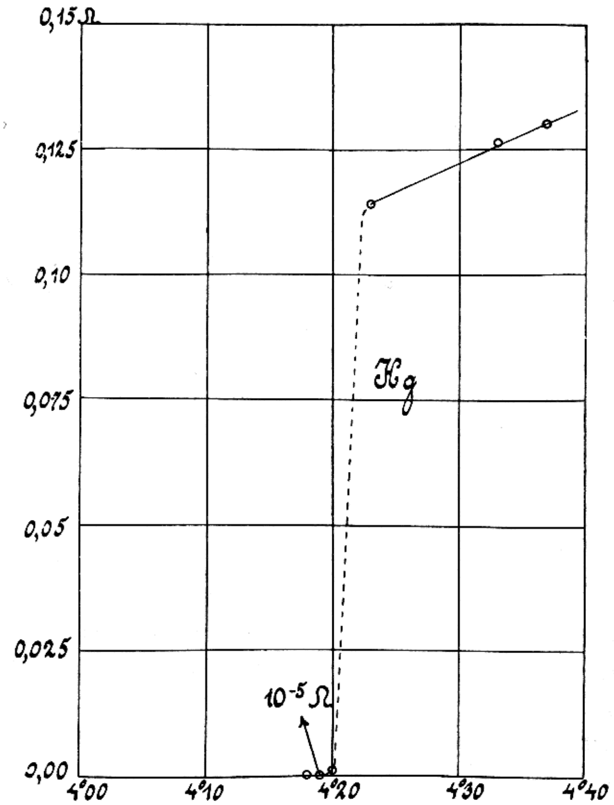
Department of Applied Physics, School of Science,

Aalto University, P.O. Box 15100, FI-00076 AALTO, Finland

I. SUPERCONDUCTIVITY

- 1911 – Heike Kamerlingh Onnes

  Electrical resistance of Hg (metal!) dropped to $< 10^{-5}\ \Omega$ at $T_c = 4.2$ K.



  Other metals become superconductors:

  $T_c = 1.2$ K for Al

  $T_c = 7.2$ K for Pb

  $T_c = 9.2$ K for Nb.

- 1986 – Discovery of high T_c compounds by J.G. Bednorz and K.A. Müller.

  $T_c = 95$ K for $YBa_2Cu_3O_{7-\delta}$

  $T_c = 125$ K for $Tl_2Ba_2Ca_2Cu_3O_{10}$

  $T_c = 9.2$ K for $HgBa_2Ca_2Cu_3O_{8+\delta}$.

  These are not metals! They are ceramic materials at room temperature!

A. Meissner Effect

– In the beginning of superconductivity research it was hoped that the electromagnetic properties could be derived from the property of infinite conductivity.

$$\sigma = \infty, \qquad \left.\begin{array}{r} \vec{\mathcal{J}} = \sigma \cdot \vec{E} \\ \vec{\mathcal{J}} = \text{finite} \end{array}\right\} \Longrightarrow \vec{E} = 0 \Longrightarrow \vec{\nabla} \times \vec{E} = 0 \tag{1}$$

$$\text{Maxwell:} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Longrightarrow \frac{\partial \vec{B}}{\partial t} = 0\ . \tag{2}$$

So $\vec{B}$ = constant inside a superconductor and also we expect it to be dependent on the way it was cooled down (e.g. either in the presence or absence of the magnetic field).

But in 1933 Meisner and Ochsenfeld discovered that $\vec{B} = 0$. The magnetic field inside the superconductor is not just constant, but it is exactly zero. Magnetic field lines are <u>expelled</u>. A superconductor is a perfect diamagnet.

<u>Theory Development</u>

- 1935 – Phenomenological theory developed by F. & H. London (two brothers!)
- 1957 – BCS (Bardeen-Cooper-Schrieffer) theory.
- high-T_C superconductivity – maybe YOU?

<u>Elements of London Theory</u>:

Consider a particle of mass m^* and charge e^*. It will turn out that $m^* = 2m_e$ and $e^* = -2e$; these particles are Cooper pairs, and a complete understanding of what they are is provided by the BCS theory.

Recall:

$\vec{B} = \vec{\nabla} \times \vec{A}$, $\qquad \vec{A}$ = magnetic vector potential, $\qquad V$ = electric potential.

Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \frac{1}{2m^*}\Big(-i\hbar\vec{\nabla} - q^*\vec{A}(\vec{r})\Big)^2\psi(\vec{r},t) + q^*V(\vec{r},t)\psi(\vec{r},t)\ , \tag{3}$$

Recall also that: $\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}V$.

Note: the Hamiltonian of a free particle in a magnetic field is $H = \frac{\Pi^2}{2m^*}$, where $\vec{\Pi}(\vec{r}) = -i\hbar\vec{\nabla} - q^*\vec{A}(\vec{r})$ is the canonical momentum.

The probability density: $\qquad P(\vec{r},t) = |\psi(\vec{r},t)|^2$

$\therefore \quad \frac{\partial P(\vec{r},t)}{\partial t} = \frac{\partial\psi^*(\vec{r},t)}{\partial t}\psi(\vec{r},t) + \psi^*(\vec{r},t)\frac{\partial\psi(\vec{r},t)}{\partial t} = \frac{i}{\hbar}\Big\{\Big[\frac{1}{2m^*}\Big(i\hbar\vec{\nabla} - q^*\vec{A}(\vec{r})\Big)^2\psi^*(\vec{r},t)\Big]\psi(\vec{r},t) - \psi^*(\vec{r},t)\Big[\frac{1}{2m^*}\Big(-i\hbar\vec{\nabla} - q^*\vec{A}(\vec{r})\Big)^2\Big]\psi(\vec{r},t)\Big\} = -\vec{\nabla}\cdot\vec{\mathcal{J}}(\vec{r},t)$.

$$\therefore \qquad \frac{\partial P(\vec{r},t)}{\partial t} = -\vec{\nabla}\cdot\vec{\mathcal{J}}(\vec{r},t)\ , \tag{4}$$

where $\vec{J}(\vec{r},t) = \frac{1}{2m^*}\Big[\Big(-i\hbar\vec{\nabla} - q^*\vec{A}(\vec{r})\Big)\psi(\vec{r},t)\Big]^*\psi(\vec{r},t) + \frac{1}{2m^*}\psi^*(\vec{r},t)\cdot\Big[\Big(-i\hbar\vec{\nabla} - q^*\vec{A}(\vec{r})\Big)\psi(\vec{r},t)\Big]$.

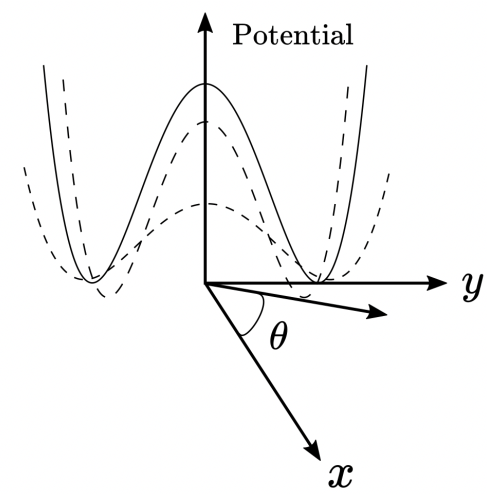
Key point: The wavefunction $\psi(\vec{r},t)$ for a superconductor can be regarded as an <u>order parameter</u> (a macroscopic wavefunction!). Let us call this "solution" ψ_s.

<u>The Ginzburg-Landau Order Parameter:</u> $\quad \psi_s(\vec{r},t) = \sqrt{n_s(\vec{r},t)}e^{i\theta(\vec{r},t)}$,

where $n_s(\vec{r},t)$ = density of superconducting particles, and $\theta(\vec{r},t)$ = superconducting phase.

Appears as a result of a <u>broken symmetry</u>.

- From now on we will assume $n_s(\vec{r},t) \equiv n_s = \text{const}$.



$\therefore \quad \vec{j}(\vec{r},t) = \frac{\hbar n_s}{m^*}\left[\vec{\nabla}\theta(\vec{r},t) - \frac{q^*}{\hbar}\vec{A}(\vec{r},t)\right]$, where the electrical current is $\vec{\mathcal{J}} = q^*\vec{j}$.

Therefore, we have the supercurrent

$$\vec{J}_s(\vec{r},t) = \frac{\hbar q^* n_s}{m^*}\left[\vec{\nabla}\theta(\vec{r},t) - \frac{q^*}{\hbar}\vec{A}(\vec{r},t)\right] = \text{superconducting current density} \; , \tag{5}$$

where $\left[\vec{\nabla}\theta(\vec{r},t) - \frac{e^*}{\hbar}\vec{A}(\vec{r},t)\right]$ is a gauge-invariant phase:

$$\begin{cases} \theta \to \theta + \frac{q^*}{\hbar}\chi \\ \vec{A} \to \vec{A} + \vec{\nabla}\chi \; . \end{cases} \tag{6}$$

Consequences:

- Let us consider $\theta = $ constant in $\vec{r}$, $\vec{\nabla}\theta = 0$.
- Perfect Conductivity

$\vec{\mathcal{J}}_s = -\frac{q^{*2}}{m^*}n_s\vec{A} \implies \frac{d\mathcal{J}_s(\vec{r},t)}{dt} = -\frac{q^{*2}}{m^*}n_s\frac{d\vec{A}(\vec{r},t)}{dt}$ (Recall Maxwell: $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ & $\vec{B} = \vec{\nabla}\times\vec{A}$), or

$$\frac{d\vec{\mathcal{J}}(\vec{r},t)}{dt} = +\frac{q^{*2}n_s}{m^*}\vec{E}(\vec{r},t) \; . \tag{7}$$

What does it mean?

Take a ballistic superelectron (no collision with atoms, impurities, etc.)

$$\left.\begin{aligned} m^* \tfrac{d\vec{v_s}}{dt} = q^* \cdot \vec{E} \\ \vec{\mathcal{J}}_s = \rho_s \cdot q^* \cdot \vec{v_s} \end{aligned}\right\} \implies \frac{d\vec{\mathcal{J}}_s}{dt} = \frac{q^{*2}\rho_s}{m^*}\vec{E} \tag{8}$$

Note the difference with respect to $\vec{\mathcal{J}} = \sigma\vec{E}$ (Ohm's law)!

- Meissner Effect

Let us look at Maxwell's equations:

$$\begin{cases} \vec{\nabla} \cdot \vec{B} = 0 \, , \\ \vec{\nabla} \times \vec{B} = \mu_0 \vec{\mathcal{J}}_s \, . \end{cases}$$

Now $\vec{B} = \vec{\nabla} \times \vec{A}$ so $\vec{\nabla} \times \vec{B} = \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \cdot \vec{A} = -\vec{\nabla}^2\vec{A}$, where we can use the Coulomb gauge $\vec{\nabla} \cdot \vec{A} = 0$.

$$\therefore \quad \begin{cases} \vec{\nabla}^2\vec{A} = -\mu_0\vec{\mathcal{J}}_s \, , \\ \vec{\mathcal{J}}_s = -\frac{q^{*2}}{m^*} n_s \vec{A} \, , \end{cases} \implies \vec{\nabla}^2\vec{A} = \frac{\mu_0 q^{*2} n_s}{m^*}\vec{A} \, . \tag{9}$$

<u>Notation:</u>

$\lambda_L = \sqrt{\frac{m^*}{\mu_0 n_s q^{*2}}}$ = London penetration length.

Since $\vec{\mathcal{J}}_s = -\frac{q^{*2} n_s}{m^*}\vec{A}$, we have

$$\begin{cases} \vec{J}_s = -\frac{1}{\mu_0 \lambda_L^2}\vec{A} \, , \\ \vec{\nabla}^2\vec{A} = \frac{1}{\lambda_L^2}\vec{A} \, . \end{cases} \tag{10}$$

$\therefore \quad \mu_0 \lambda_L^2 \vec{\mathcal{J}}_s = -\vec{A} \implies \mu_0 \lambda_L^2 \vec{\nabla} \times \vec{\mathcal{J}}_s = -\vec{\nabla} \times \vec{A} = -\vec{B},$

or $\quad \mu_0 \lambda_L^2 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{\mathcal{J}}_s) = -\vec{\nabla} \times \left(\frac{\partial \vec{A}}{\partial t} \equiv -\vec{E}\right)$, since voltage is zero and $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} V$.

But $\vec{\mathcal{J}}_s = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B}$,

$\therefore \quad \lambda_L^2 \frac{\partial}{\partial t} \cdot (\vec{\nabla} \times (\vec{\nabla} \times \vec{B})) = \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

Note $\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) - \vec{\nabla}^2 \vec{B}$, and that $\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) = 0$.

This implies $\lambda_L^2 \vec{\nabla}^2 \vec{B} = +\vec{B}$, or

$$\left[\frac{1}{\lambda_L^2} - \vec{\nabla}^2\right] \vec{B}(\vec{r}) = 0 \; . \tag{11}$$

Take $\vec{B}(\vec{r}) = (0, 0, B(z)) \implies B(z) B_0 \exp(-z/\lambda_L)$. This is the Meissner effect. The field decays exponentially in the superconductor.

To review: we found

$$\vec{\mathcal{J}}_s = -\frac{1}{\mu_0 \lambda_L^2} \vec{A} \; , \tag{12}$$

and

$$\nabla^2 \vec{A} = \frac{1}{\lambda_L^2} \vec{A} \; , \tag{13}$$

or

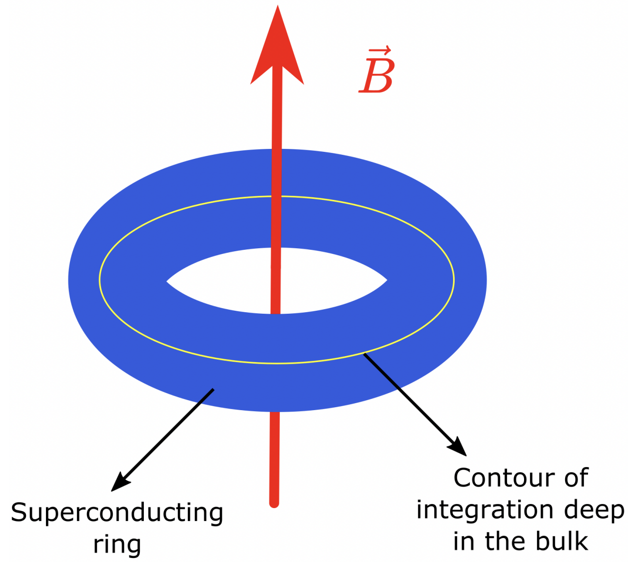
$$\frac{d\vec{\mathcal{J}}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2} \vec{E} \quad \text{— called } 1^{st} \text{ London equation,} \tag{14}$$

$$\vec{B} = -\mu_0 \lambda_L^2 \vec{\nabla} \times \vec{\mathcal{J}}_s \quad \text{— called } 2^{nd} \text{ London equation.} \tag{15}$$

So the magnetic field can penetrate at most to depths of $\simeq \lambda_L$.
Currents can flow in this region, but deep in the bulk they will be zero.

II. QUANTIZATION OF FLUX

So far we have not discussed the phase θ from the general expression of the current. Now it's time ... with a spectacular example!

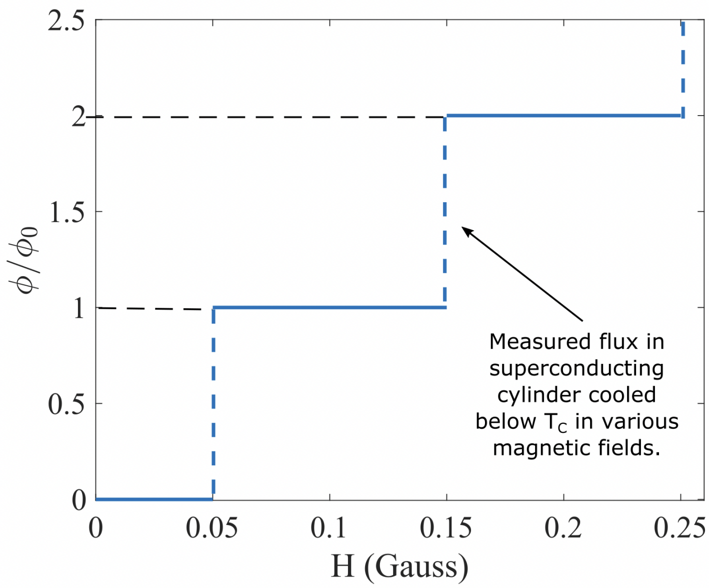


We consider a superconducting ring and choose a contour of integration deep in the bulk where $\vec{\mathcal{J}}_s(\vec{r},t) = 0$. This implies

$\hbar\vec{\nabla}\theta(\vec{r}) = e^*\vec{A}(\vec{r}) \implies \overbrace{\hbar\oint\vec{\nabla}\cdot\theta(\vec{r})d\vec{\ell}}^{2\pi n} = e^*\Phi$, where Φ = magnetic flux, and n = integer number. With $\phi = \iint \vec{B}d\vec{s}$, we have

$$\phi = \frac{2\pi n\hbar}{q^*} = \frac{h}{q^*}\cdot n\,. \tag{16}$$

The flux quantum is $\phi_0 = \frac{h}{2e} = 2.067\times 10^{-15}$ Wb, and $q^* = -2e$.



- Another useful relation: the energy-phase relationship

$$\underbrace{-\hbar\frac{\partial\theta}{\partial t}}_{\text{“change of phase”}} = \underbrace{\frac{1}{2}\frac{\mu_0\lambda_L^2}{n_s}\cdot\vec{\mathcal{J}_s}^{\,2}}_{\text{“kinetic energy”}} + \underbrace{q^*V}_{\text{“potential energy”}} \tag{17}$$

Proof:

From the Schrödinger equation $i\hbar\frac{\partial}{\partial t}\psi = \frac{1}{2m^*}\Big(-i\hbar\vec{\nabla} - q^*\vec{A}\Big)^2\psi + q^*V\psi$ we replace $\psi = \sqrt{n_s}e^{i\theta}$ where $n_s = \text{const.}$

$$\Longrightarrow \qquad -\hbar\tfrac{\partial\theta}{\partial t}\cdot\sqrt{n_s} = \tfrac{1}{2m^*}\Big(+\hbar\vec{\nabla}\theta - q^*\vec{A}\Big)^2\cdot\sqrt{n_s} + q^*V\sqrt{n_s},$$

but $\vec{\mathcal{J}_s}^{\,2} = \frac{q^{*2}n_s^2}{m^{*2}}\Big(\hbar\vec{\nabla}\theta - q^*\vec{A}\Big)^2$

$$\Longrightarrow \qquad -\hbar\tfrac{\partial\theta}{\partial t} = \tfrac{1}{2}\underbrace{\frac{m^*}{n_s^2q^{*2}}}_{\equiv\frac{\mu_0\lambda_L^2}{n_s}}\mathcal{J}_s^2 + q^*V.$$

Let us recap a bit:

Electrodynamics of superconductors is described by

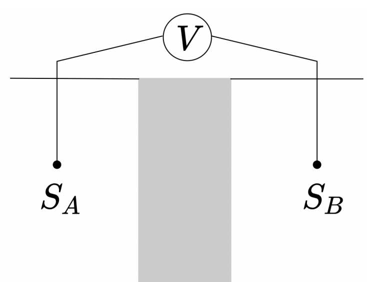
$$\begin{cases} 1^{st} \text{ London equation} & \frac{d\vec{\mathcal{J}}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2} \vec{E} \ , \\ 2^{nd} \text{ London equation} & \vec{B} = -\mu_0 \lambda_L^2 \vec{\nabla} \times \vec{\mathcal{J}}_s \ . \end{cases} \tag{18}$$

Here $\vec{\mathcal{J}}_s = \frac{\hbar q^* n_s}{m^*} \left[\vec{\nabla}\theta - \frac{q^*}{\hbar}\vec{A} \right]$ or
$\vec{\mathcal{J}}_s = -\frac{\phi_0}{2\pi\mu_0\lambda_L^2} \left[\vec{\nabla}\theta + \frac{2\pi}{\phi_0}\vec{A} \right]$, where the London penetration length is $\lambda_L^2 = \frac{m^*}{\mu_0 n_s q^{*2}}$ and $\phi_0 = \frac{h}{2e}$ = flux quantum, $q^* = -2e$.

The quantity: $\vec{\nabla}\theta + \frac{2\pi}{\phi_0}\vec{A}$ = gauge-invariant phase gradient.

III. JOSEPHSON EFFECT

- What happens when we put a voltage across a weak link between two superconductors?



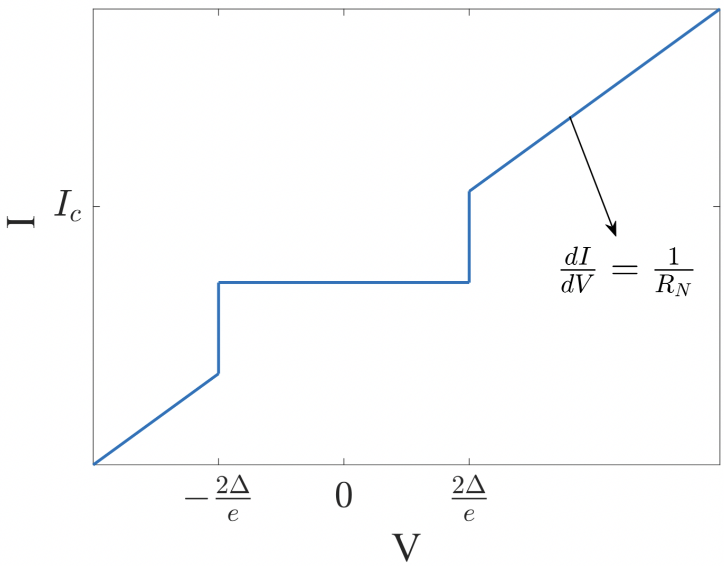
Weak link = can be a:

S-I-S (insulator between two superconductors)

S-N-S (a metal in-between)

S-s-S (a constriction)

- What we measure:

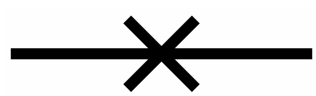


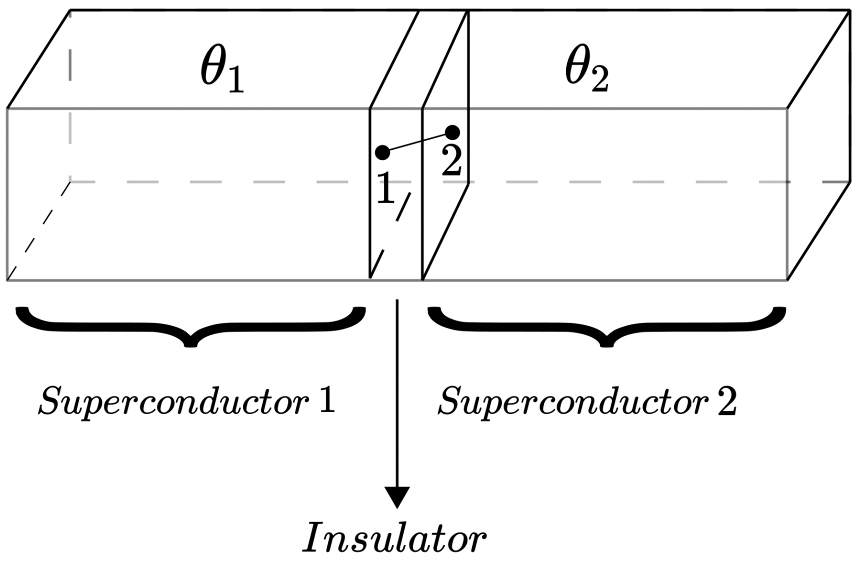
Δ = superconducting gap, $\Delta = 1.764 k_B T_c$, T_c = critical temperature (from BCS theory), R_N = normal-state resistance.

– Currents flowing for $|V| \geq \frac{2\Delta}{e}$ are no surprise – they are associated with breaking the Cooper pairs by the voltage.

But, at $V = 0$ there is a current flowing, with max. value $= I_c$ (critical current of the junction). This is the Josephson effect.

– Circuit symbol:





1. Current-phase:

   Consider the gauge-invariant phase difference, obtained by integrating the gauge-invariant phase gradient,
   $\varphi = \int_1^2 d\vec{r}\left(\vec{\nabla}\theta + \frac{2\pi}{\phi_0}\vec{A}\right) = \theta_2 - \theta_1 + \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\vec{A}(\vec{r},t).$

   This is the only quantity which is gauge-invariant and includes the difference in phases $\theta_2 - \theta_1$ as we cross the insulator.

   So perhaps $\mathcal{J}_s = \mathcal{J}_s(\varphi)$, that is, a function of φ. Which one? Well, we should have also:

   1. periodicity $\mathcal{J}_s(\varphi) = \mathcal{J}_s(\varphi + 2\pi n)$
   2. $\mathcal{J}_s(0) = 0$ (no current when there is no phase difference).

$$\Longrightarrow \quad \mathcal{J}_s(\varphi) = \mathcal{J}_c \sin\varphi + \underbrace{\sum_{m=2}^{\infty} \mathcal{J}_m \sin m\varphi}_{\text{This can be neglected.}} \quad , \tag{19}$$

where $\mathcal{J}_c$ is constant and is the critical Josephson current density.

Therefore for a given device we will have

$$I = I_c \sin\varphi \ , \tag{20}$$

$I_c =$ critical Josephson current.

2. Voltage-phase

Consider again the gauge-invariant phase difference
$\varphi = \theta_2 - \theta_1 + \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\vec{A}(\vec{r},t)$

Recall now the energy-phase relationship
$-\hbar\frac{\partial\theta}{\partial t} = \frac{1}{2}\frac{\mu_0\lambda_L^2}{n_s}\mathcal{J}_s^2 + q^*V.$
This relation is valid for the phases θ_1 and θ_2 inside the superconductors 1 and 2.
$\implies \quad \frac{\partial\varphi}{\partial t} = -\frac{1}{\hbar}\frac{\mu_0\lambda_L^2}{2n_s}\Big(\mathcal{J}_s^2(2) - \mathcal{J}_s^2(1)\Big) - \frac{q^*}{\hbar}\Big(V(2) - V(1)\Big) + \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\frac{\partial\vec{A}}{\partial t}$, but $\mathcal{J}_s(1) \equiv \mathcal{J}_s(2)$ (conservation of charge or Kirchoff's current law).

$\implies \frac{\partial\varphi}{\partial t} = \frac{2\pi}{\phi_0}\int_1^2 d\vec{r}\Big(\vec{\nabla} + \frac{\partial\vec{A}}{\partial t}\Big)$, but $\vec{E} = -\vec{\nabla}V - \frac{\partial\vec{A}}{\partial t}$

$\implies \frac{\partial\varphi}{\partial t} = -\frac{2\pi}{\phi_0}\underbrace{\int_1^2 d\vec{r}\cdot\vec{E}}_{\equiv -(V_2 - V_1) = -V}$, where $V \equiv V_2 - V_1$.

$$\implies \qquad \frac{\partial\varphi}{\partial t} = \frac{2\pi}{\phi_0}V\ . \tag{21}$$

$$\implies \qquad I = I_c\sin\varphi \qquad \text{current-phase relation}\ , \tag{22}$$

$$\frac{\partial\varphi}{\partial t} = \frac{2e}{\hbar}V \qquad \text{phase-voltage relation}\ , \tag{23}$$

or: $\quad V = \frac{\partial}{\partial t}\Big(\frac{\phi_0}{2\pi}\cdot\varphi\Big)$ very similar to Faraday's law.

- Consequences:

DC Josephson Effect: $\quad V = 0 \implies \frac{\partial\varphi}{\partial t} = 0 \implies \varphi = \text{const.}$
$I = I_c\sin\varphi$ — the current can reach a max. value of I_c.

AC Josephson Effect: $\quad V = \text{const} \neq 0 \implies \varphi = \frac{2e}{\hbar}V\cdot t.$
$\implies \quad I = I_c\sin\Big(\frac{2e}{\hbar}V\cdot t\Big) = I_c\sin\Big(2\pi\frac{V}{\phi_0}t\Big).$

$f_J = \frac{V}{\phi_0}$ = Josephson frequency $= 483 \times 10^{12}\ V_0$ (Hz).

Josephson Inductance: $\quad \frac{\partial I}{\partial t} = \frac{\partial I}{\partial \varphi} \cdot \frac{\partial \varphi}{\partial t} = I_c \cos\varphi \cdot \frac{2\pi}{\phi_0} V$

or: $\quad V = L_J(\varphi)\frac{\partial I}{\partial t}$, where $L_J(\varphi)$ = Josephson inductance.

$L_J(\varphi) = \frac{\phi_0}{2\pi I_c \cos\varphi}$ — depends on phase! Can be ∞ if $\varphi = \frac{\pi}{2} + n\pi$.

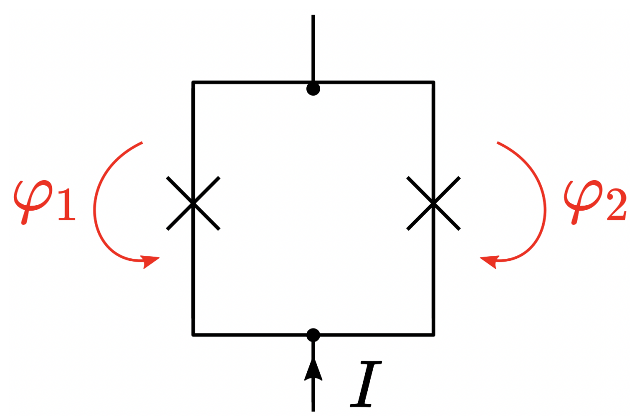
Josephson Energy: $\quad E_J = \int dt I \cdot V = \int d\varphi \cdot I_c \sin\varphi \cdot \frac{\phi_0}{2\pi}$

$E_J = -\frac{I_c \phi_0}{2\pi} \cos\varphi = -E_J \cos\varphi$.

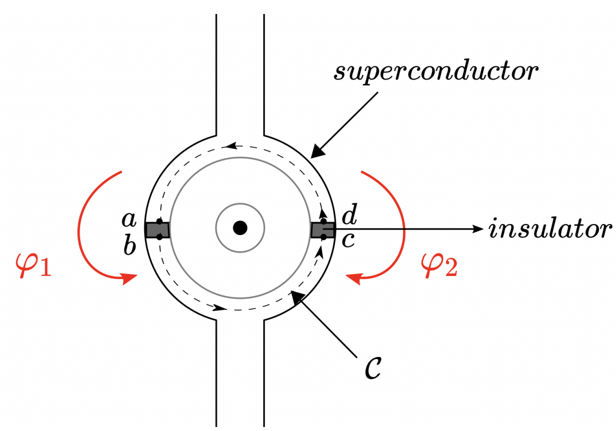
$E_J = \frac{\phi_0 I_c}{2\pi}$ = Josephson energy.

IV. APPLICATION: THE DC–SQUID

– Superconducting quantum interference device:



How it is fabricated:



$\oint_{\mathcal{C}} \vec{\nabla}\theta \cdot d\vec{r} = 2\pi n = (\theta_b - \theta_a) + (\theta_c - \theta_b) + (\theta_d - \theta_c) + (\theta_a - \theta_d)$, where

$$\begin{cases} \theta_b - \theta_a = \varphi_1 - \frac{2\pi}{\phi_0} \int_a^b \vec{A} d\vec{r} \\ \theta_c - \theta_b = \int_b^c d\vec{r} \cdot \vec{\nabla}\theta = \qquad \underbrace{-\frac{2\pi}{\phi_0}\mu_0\lambda_L^2 \int_b^c d\vec{r}\vec{\mathcal{J}}_s}_{0, \quad \mathcal{J}_s=0 \text{ inside the superconductor}} \qquad -\frac{2\pi}{\phi_0}\int_b^c d\vec{r}\cdot\vec{A} \\ \theta_d - \theta_c = -\varphi_2 - \frac{2\pi}{\phi_0} \int_c^d \vec{A} d\vec{\ell} \\ \theta_a - \theta_d = \int_d^a d\vec{r} \cdot \vec{\nabla}\theta = \underbrace{-\frac{2\pi}{\phi_0}\mu_0\lambda_L^2 \int_d^a d\vec{r}\vec{\mathcal{J}}_s}_{0} - \frac{2\pi}{\phi_0}\int_d^a d\vec{r}\cdot\vec{A} \end{cases}$$

$\therefore \quad \oint_{\mathcal{C}} \vec{\nabla}\theta \cdot d\vec{r} = \varphi_1 - \varphi_2 - \frac{2\pi}{\phi_0} \underbrace{\oint_{\mathcal{C}} d\vec{r}\cdot\vec{A}}_{=\phi}$, where ϕ = the magnetic flux piercing the SQUID.

$$\phi_1 - \phi_2 = 2\pi n + \frac{2\pi\phi}{\phi_0} \ . \tag{24}$$

So $\quad I = I_1 + I_2 = I_c \sin\varphi_1 + I_c \sin\varphi_2 = 2I_c \sin\frac{\varphi_1+\varphi_2}{2}\cos\frac{\varphi_1-\varphi_2}{2}$.

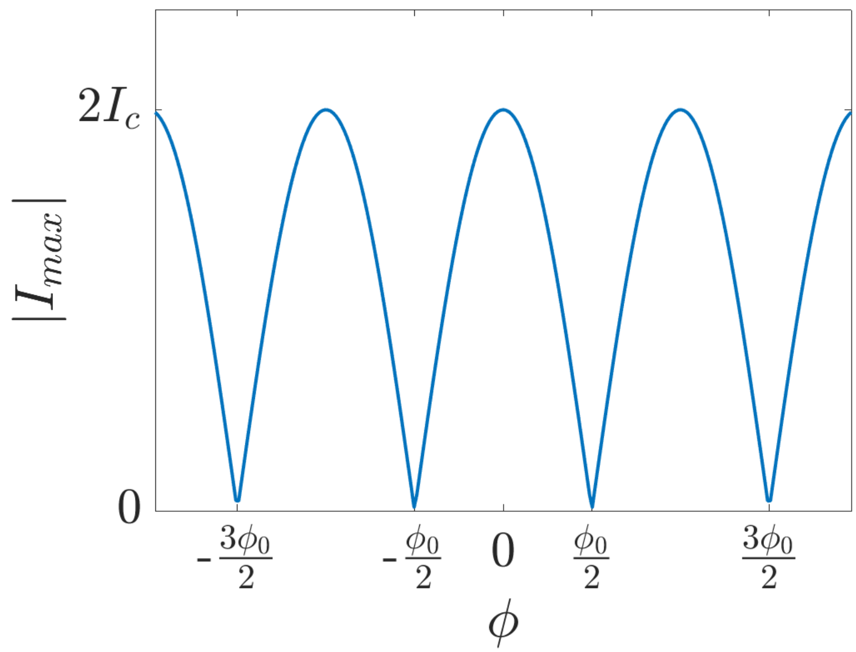
Let $\varphi \equiv \frac{\varphi_1-\varphi_2}{2} \implies I = 2I_c \sin\varphi\cos\left(\frac{\pi\phi}{\phi_0} + \pi n\right)$.

$$I \equiv I_{\max}(\phi)\cdot\sin\varphi \ , \tag{25}$$

where $I_{\max}(\phi) = 2I_c\cos\left(\frac{\pi\phi}{\phi_0} + \pi n\right)$.

The SQUID behaves as a single Josephson junction with critical current controlled by the magnetic flux.

The maximum current will be



- $|I_{\max}|$ never exceeds $2I_c$.
- $I_{\max}$ can be zero! This is understood as destructive interference of the currents in the two branches of the SQUID.

References

- Terry P. Orlando and Kevin A. Delin — Foundations of Applied Superconductivity.
- D.R. Tilley and J. Tilley — Superfluidity and Superconductivity.
- Antonio Barone and Giafranco Paternò — Physics and Applications of the Josephson Effect.