IQM

WE BUILD QUANTUM COMPUTERS

Jan Goetz

www.meetiqm.com

Lecture notes on PHYS-C0254 Quantum Circuits

About me: From science to startups

- Originally from Neuss (near Cologne, GER)
- 2006 – 2011: Physics student in Munich (TUM)
- 2012 – 2017: PhD student in Munich (WMI)
- 2017 – 2019: Marie-Curie Fellow in Helsinki (Aalto)
- 2019 – today: Co-founding CEO of IQM
- 2021 – today: Board Member EIC &QuIC

- I like dogs
- I like sports
- I have strong eye rings (but I sleep well)
- I like standing at the bar

- I am a team person (collaboration)
- I can give away tasks (trust)
- I like making big plans (ambition)

IQM in brief

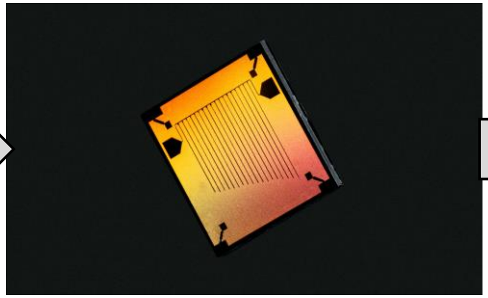
Quantum-computer scale-up

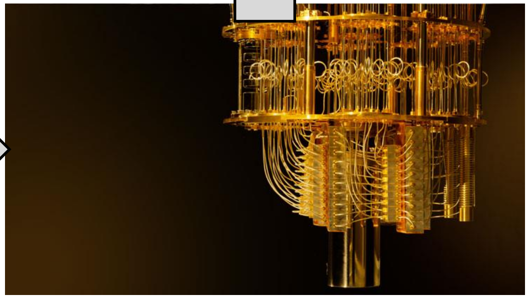
- Providing quantum computers based on superconducting technology
- DeepTech Scale Up, > 140 people strong
- Secured > M70 EUR funding
- Offering:
  - On-premises systems for research and supercomputing centers
    - 2 systems sold, 1 delivered
  - HPC integration of quantum computers
  - Co-design approach for application-specific quantum computers

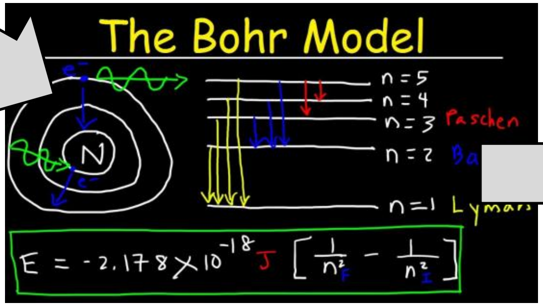
How does everything fit into the big picture?

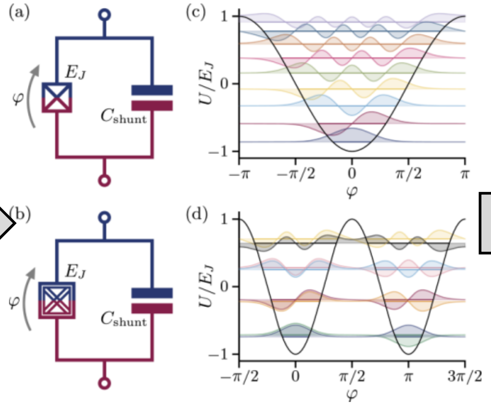
- Scientists, atoms, circuits, qubits, quantum computing

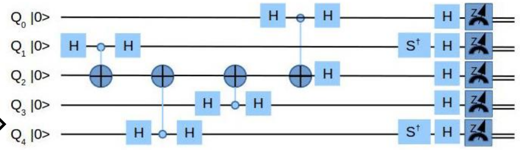
What is required?











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Di Vincenzo Criteria and where you can find them in this course

Statement of the criteria

1. A scalable physical system with well characterized qubit
2. The ability to initialize the state of the qubits to a simple fiducial state
3. Long relevant decoherence times
4. A "universal" set of quantum gates
5. A qubit-specific measurement capability

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Agenda for lectures 7-11

7. Quantization of electrical networks
    a. Harmonic oscillator: Lagrangian, eigenfrequency
    b. Transfer step: LC oscillator, Legendre transform to Hamiltonian
    d. Quantization of oscillators
8. Superconducting quantum circuits
    a. Qubits: Transmon qubit, Charge qubit, Flux qubit 1st DiVincenzo criteria
    b. Circuit-QED: Rabi model
    c. Rotating Wave approximation: Jaynes-Cummings model
9. Single-qubit operations:
    a. Initialization 2nd DiVincenzo criteria
    b. Readout 5th DiVincenzo criteria
    c. Control:T1, T2 measurements, Randomized benchmarking 3rd DiVincenzo criteria
10. Two-qubit operations: Architectures for 2-qubit gates 4th DiVincenzo criteria
    a. iSWAP
    b. cPhase
    c. cNot
11. Challenges in quantum computing
    a. Scaling
    b. SW-HW gap
    c. Error-correction

Agenda for today

7. Quantization of electrical networks
   a. Harmonic oscillator: Lagrangian, eigenfrequency
   b. Transfer step: LC oscillator, Legendre transform to Hamiltonian
   d. Quantization of oscillators

a.

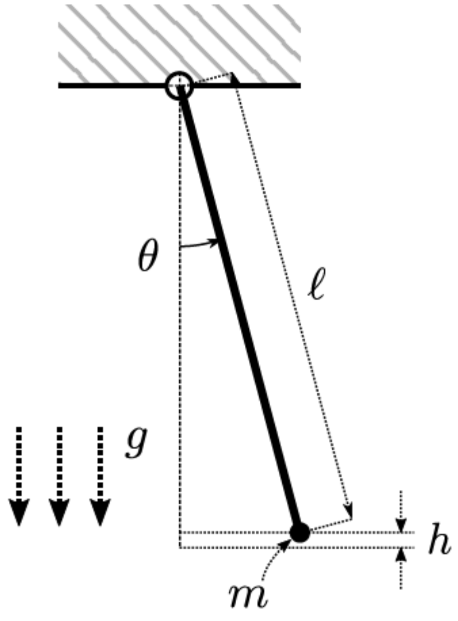


Figure 1: Classical pendulum.

b.

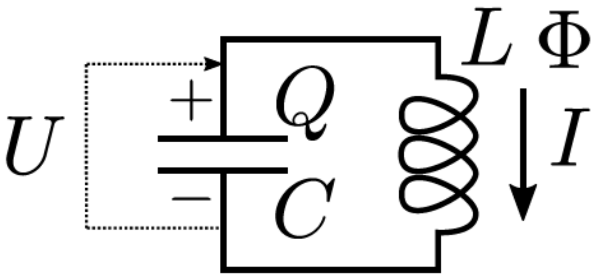
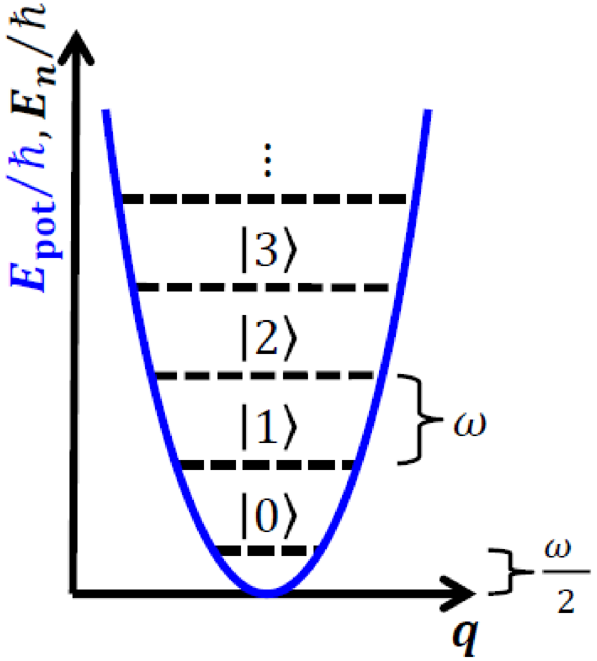


Figure 2: Superconducting LC oscillator.

c.



General note: Harmonic oscillators

- General note: In physics, many phenomena can be explained by harmonic oscillators. They are the standard tool in our physics toolbox.
- Usually, there are two important variables involved like position and momentum, x and p.
- One can often find analogies where two system variables are equivalent to x and p. For example, in an LC oscillator these are charge and flux.

Short review: Lagrangian & Hamiltonian

- During this course, Lagrangian and Hamiltonian mechanics are used for analyzing quantum computing circuits.

- Recall that the Lagrangian is defined as the kinetic energy *T* minus the potential energy *V*:

$$L \equiv T - V \, .$$

- Quite often the Hamiltonian is representing the total energy of the system:

$$H = T + V \, .$$

Legendre transformation

Short review: Classical oscillator*

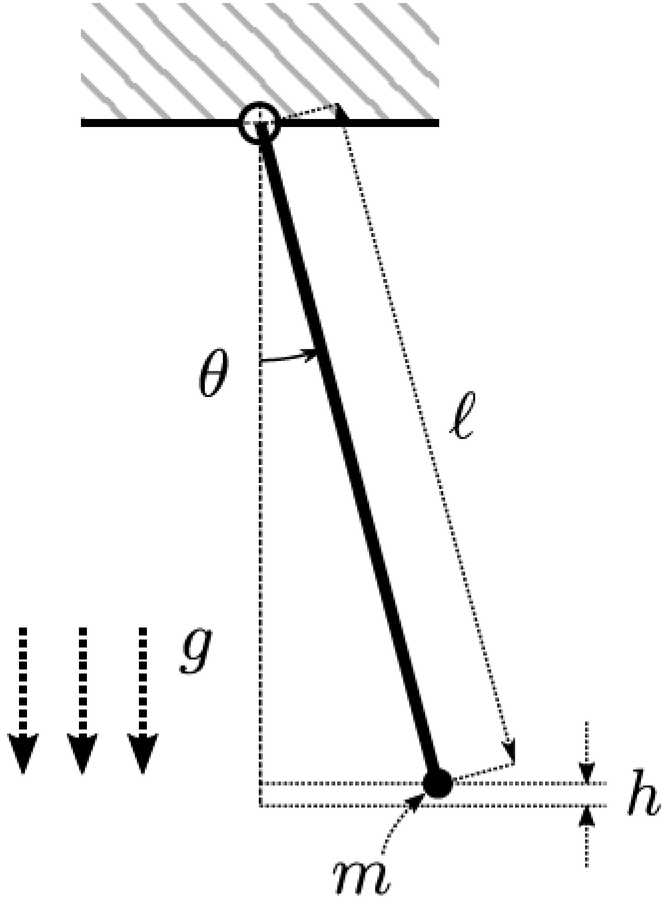


Figure 1: Classical pendulum.

The Euler-Lagrange equation states

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

Since T does not depend on q

$$\frac{\partial L}{\partial q} = -\frac{\partial V}{\partial q}$$

Since $p = \partial L / \partial \dot{q}$ we obtain Newton´s equation of motion

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{\partial V}{\partial q}$$

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Short review: Classical oscillator*

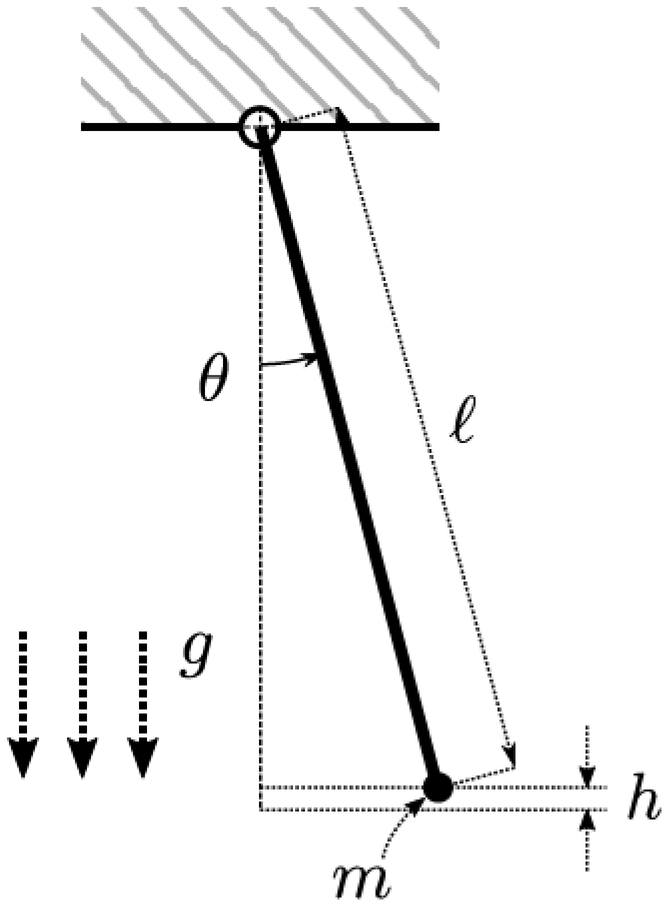

Figure 1: Classical pendulum.

The kinetic energy is

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m \ell^2 \dot{\theta}^2$$

The potential energy is

$$V = mgh = mg\ell\,(1 - \cos\theta) \approx \frac{1}{2} mg\ell\theta^2$$

$$L \equiv T - V\,.$$

We introduce generalized coordinates p and q

$$q = \ell\,\theta\,,$$

$$p \equiv \frac{\partial L}{\partial \dot{q}} \approx \frac{\partial}{\partial \dot{q}} \left(\frac{1}{2} m \dot{q}^2 - \frac{mg}{2\ell} q^2 \right) = m\dot{q} = m\ell\,\dot{\theta}$$

* Details in lecturenotes-EQS_Helsinki_2019.pdf

Short review: Classical oscillator*

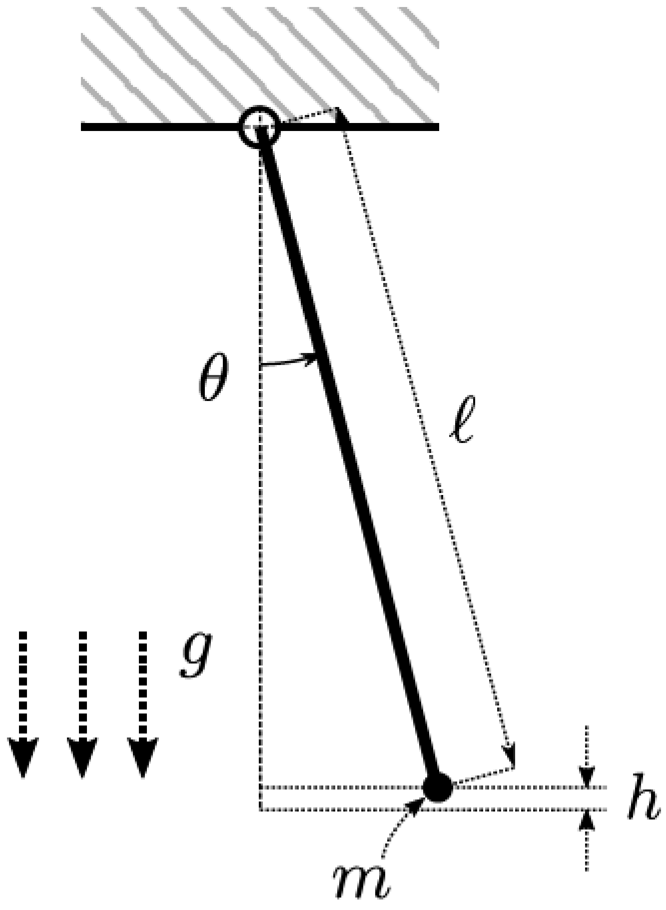

Figure 1: Classical pendulum.

Applying our example to the Euler-Lagrange equation gives

$$\dot{p} = -mg\,\theta$$

In addition, we can independently differentiate *p* wrt time:

$$\dot{p} = m\ell\,\ddot{\theta}$$

Together this yields:

$$m\ell\,\ddot{\theta} + mg\theta = 0\,,$$

$$\ddot{\theta} + \frac{g}{\ell}\theta = 0\,.$$

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Short review: Classical oscillator*

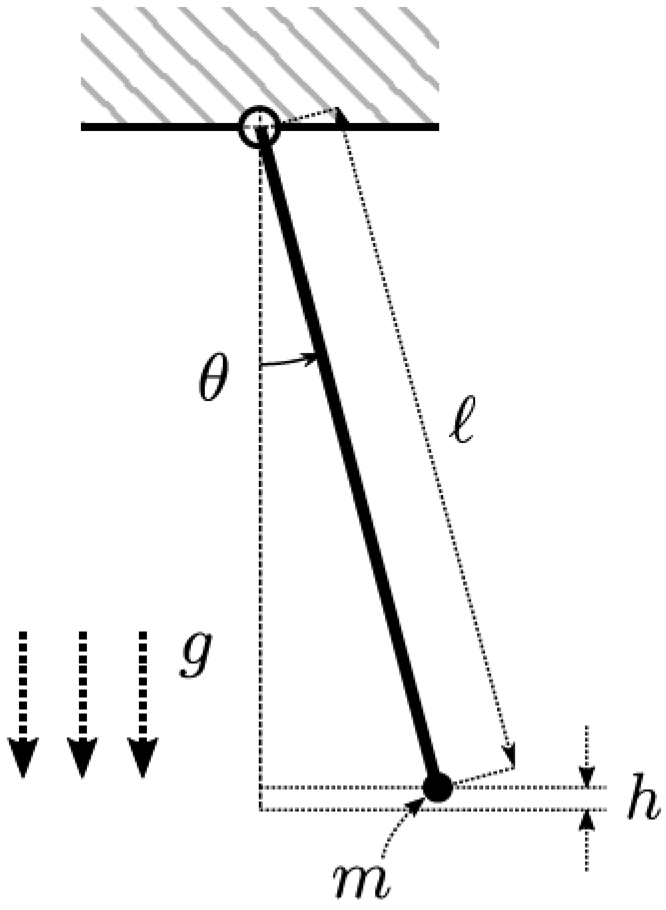

Figure 1: Classical pendulum.

Because we are smart, we chose the trial function

$$\theta = C\exp(i\omega t)$$

Inserting this function into differential equation yields:

$$i^2\omega^2 C\exp(i\omega t) + \frac{g}{\ell}C\exp(i\omega t) = 0$$

Solving this equation provides the well-known result:

$$\omega = \sqrt{g/\ell}$$

Key takeaway: Starting from energy considerations, we can derive the eigenfrequency of the system

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Agenda for today

7. Quantization of electrical networks
   a. Harmonic oscillator: Lagrangian, eigenfrequency
   b. Transfer step: LC oscillator, Legendre transform to Hamiltonian
   d. Quantization of oscillators

a.

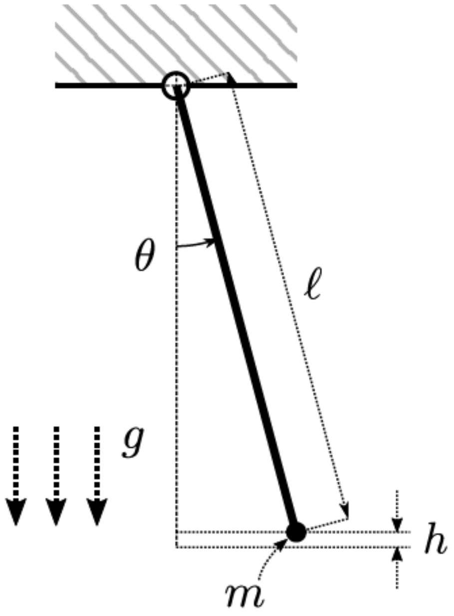

Figure 1: Classical pendulum.

b.

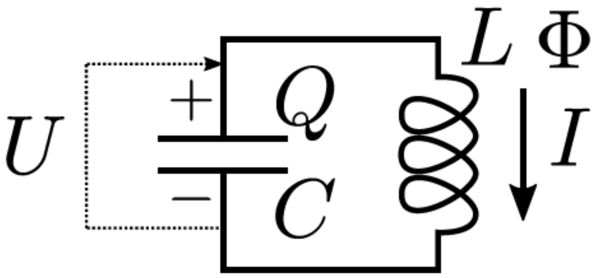

Figure 2: Superconducting LC oscillator.

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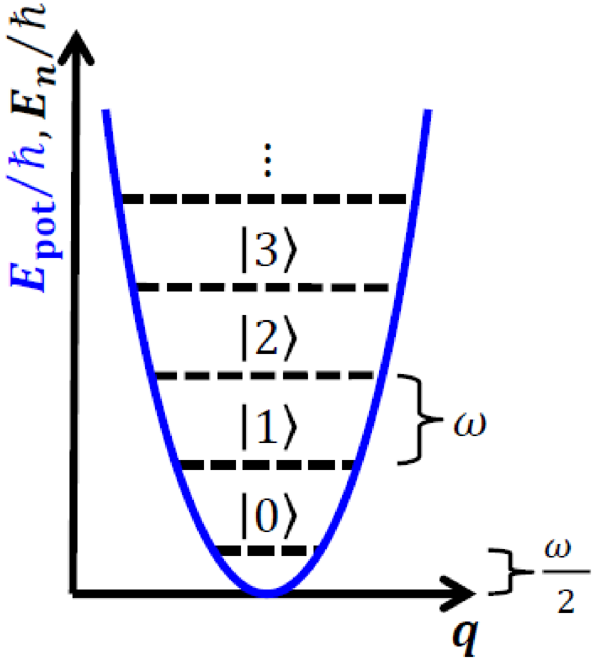
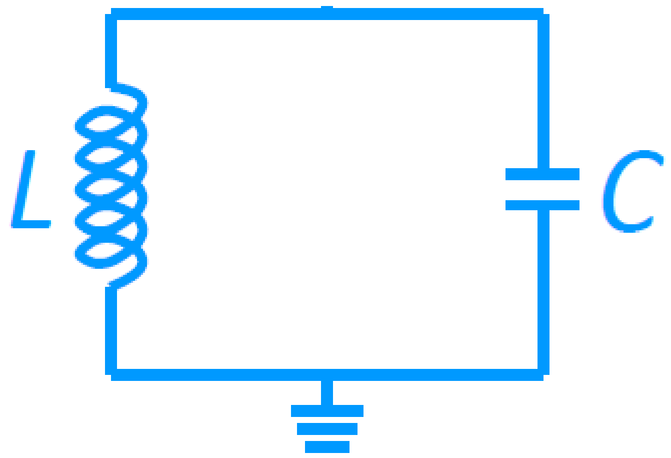


General note: LC oscillators

- General note: Once you understand the harmonic oscillator, you can easily apply the concept to any other oscillator.

Momentum $\hat{p}$	$\leftrightarrow$ Charge $\hat{q}$
Position $\hat{x}$	$\leftrightarrow$ Flux $\hat{\Phi}$
Mass m	$\leftrightarrow$ Capacitance C
Resonance frequency ω_r	$\leftrightarrow$ $\omega_r = 1/\sqrt{LC}$



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Transfer step: LC oscillator*

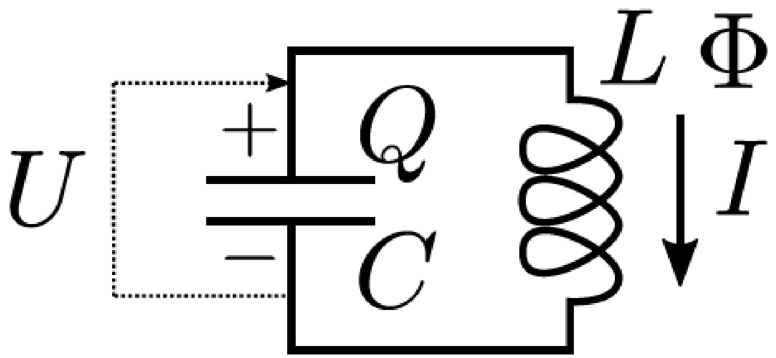

Figure 2: Superconducting LC oscillator.

We assume an electrical circuit consisting of inductance L and capacitance C. The charge stored in the capacitor is

$$Q = CU$$

The power fed into the circuit is $P = UI$ and consequently

$$P = U\dot{Q}$$

Hence, the potential energy stored in the system is

$$V = \int_{t_0}^{t_1} P \, \mathrm{d}t = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QU = \frac{1}{2}CU^2$$

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Transfer step: LC oscillator*

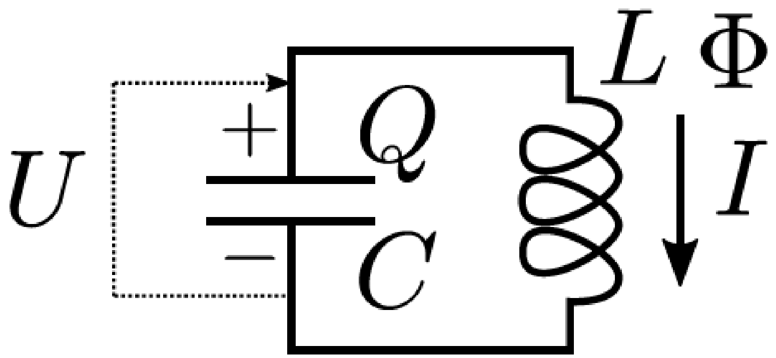

Figure 2: Superconducting LC oscillator.

For the magnetic flux F in a coil, it holds that

$$\Phi = LI$$

Lenz law tells us that

$$\dot{\Phi} = U$$

Hence, the kinetic energy stored in the system is

$$T = \int_{t_0}^{t_1} P \, \mathrm{d}t = \int_{t_0}^{t_1} UI \, \mathrm{d}t = \frac{1}{2} LI^2 = \frac{\Phi^2}{2L} .$$

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Transfer step: LC oscillator*

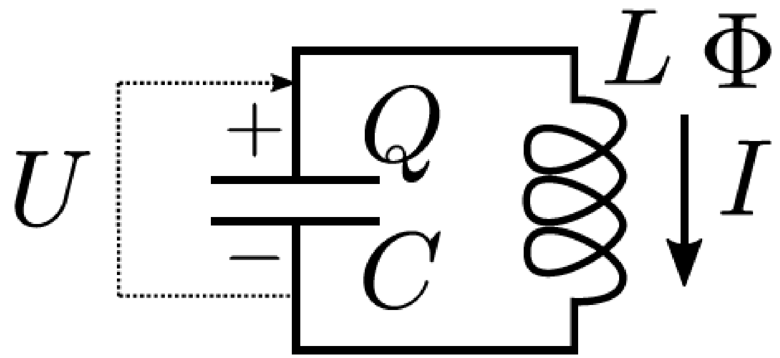

Figure 2: Superconducting LC oscillator.

To apply Lagrangian mechanics, we use the previous results

$$V = \frac{1}{2}CU^2 = \frac{Q^2}{2C}\,, \quad T = \frac{1}{2}LI^2 = \frac{1}{2}L\dot{Q}^2\,,$$

allowing us to write the Lagrangian as

$$L = T - V = \frac{1}{2}L\dot{Q}^2 - \frac{Q^2}{2C}\,.$$

To derive the equation of motion, we again introduce generalized coordinates

$$q = Q\,,$$

$$p \equiv \frac{\partial L}{\partial \dot{q}} = L\dot{Q} = -LI = -\Phi\,.$$

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Transfer step: LC oscillator*

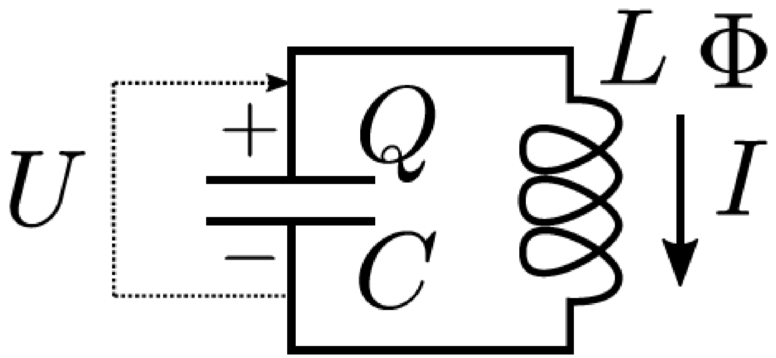

Figure 2: Superconducting LC oscillator.

Remind yourself again of Euler-Lagrange:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

Using the above results gives the equation of motion for charge:

$$\ddot{Q} + \frac{1}{LC} Q = 0$$

Using a similar ansatz for the trial function yields the resonance frequency

$$\omega = \frac{1}{\sqrt{LC}} \quad \Longleftrightarrow \quad \text{Pendulum: } \omega = \sqrt{g/\ell}$$

Key takeaway: Starting from energy considerations, we can derive the eigenfrequency of the system

* Details in lecturenotes-EQS_Helsinki_2019.pdf

Legendre transformation to Hamiltonian*

Hamiltonian gives two 1st order differential equations, while Euler Lagrange gives one 2nd order

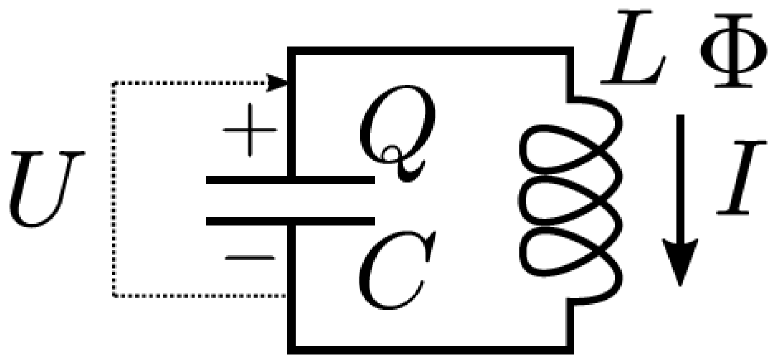

Figure 2: Superconducting LC oscillator.

The general definition for a Hamiltonian is

$$H \equiv \dot{q}p - L\ .$$

We take the total time derivative to analyze the system dynamics:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \ddot{q}p + \dot{q}\dot{p} - \frac{\partial L}{\partial q}\dot{q} - \frac{\partial L}{\partial \dot{q}}\ddot{q} - \dot{L}$$

To solve this equation, we use

$$p \equiv \partial L / \partial \dot{q}$$

$$p = p(t) \text{ only, so } \mathrm{d}p/\mathrm{d}t = \dot{p}$$

Legendre transformation to Hamiltonian*

Hamiltonian gives two 1st order differential equations, while Euler Lagrange gives one 2nd order

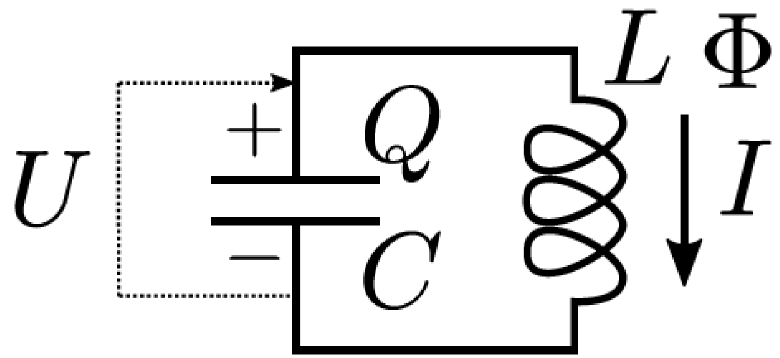

Figure 2: Superconducting LC oscillator.

Using the above terms in the total time derivative yields

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \ddot{q}p + \dot{q}\dot{p} - \frac{\partial L}{\partial q}\dot{q} - p\ddot{q} - \dot{L}$$

Simplifying this formula further results in

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \dot{q}\left[\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}\right] - \dot{L}$$

The Lagrangian is time independent and due to Euler Lagrange, the parentheses are also zero, hence

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0$$

We find that the Hamiltonian is a constant of motion, i.e. energy is conserved in the system

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Legendre transformation to Hamiltonian*

Hamiltonian gives two 1st order differential equations, while Euler Lagrange gives one 2nd order

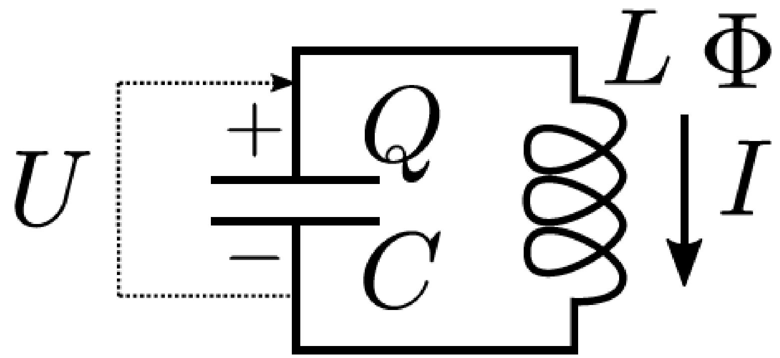

Figure 2: Superconducting LC oscillator.

We can use the general definition for the Hamiltonian to find

$$H = \dot{Q}(L\dot{Q}) - \left(\frac{1}{2}L\dot{Q}^2 - \frac{Q^2}{2C}\right) = \frac{1}{2}L\dot{Q}^2 + \frac{Q^2}{2C}$$

Inserting the standard definitions, we find

$$H = \frac{1}{2}LI^2 + \frac{Q^2}{2C} = \frac{\Phi^2}{2L} + \frac{Q^2}{2C}$$

Hence, the Hamiltonian represents the total energy of the system

$$H = T + V\ .$$

Key takeaway: Starting from Lagrangian, we can derive the total energy of the system. This is necessary to derive energy quantization.

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   a. Harmonic oscillator: Lagrangian, eigenfrequency
   b. Transfer step: LC oscillator, Legendre transform to Hamiltonian
   d. Quantization of oscillators

a.

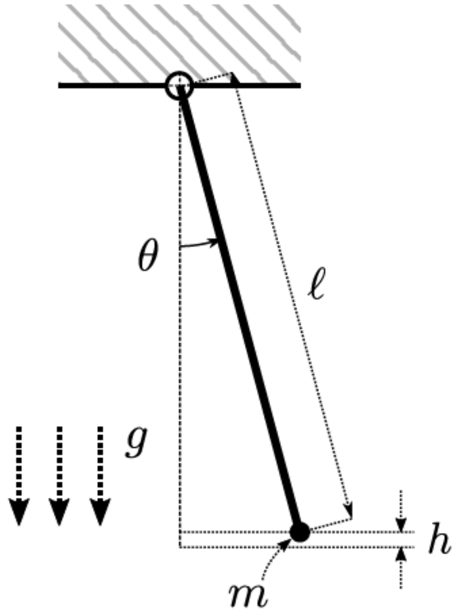

Figure 1: Classical pendulum.

b.

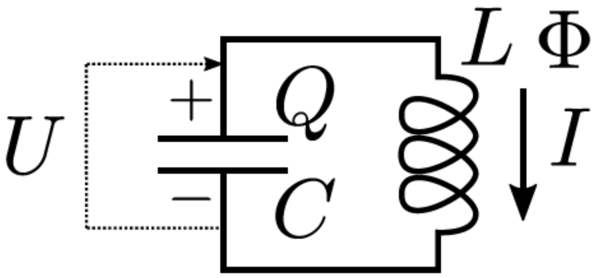

Figure 2: Superconducting LC oscillator.

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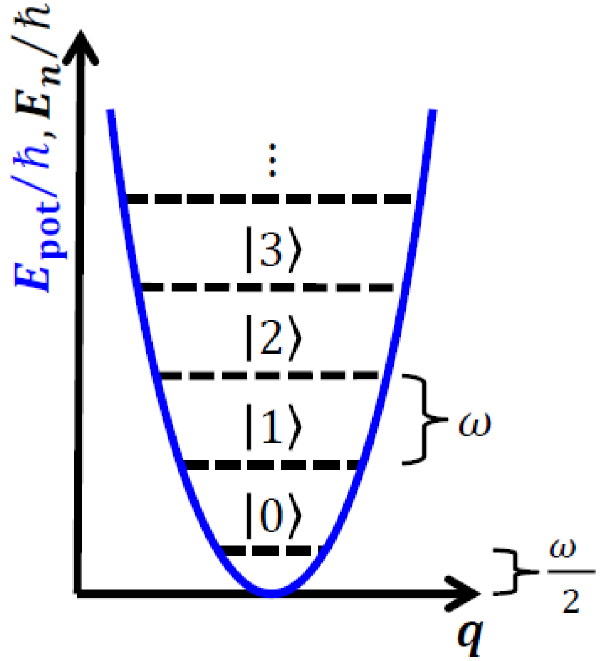
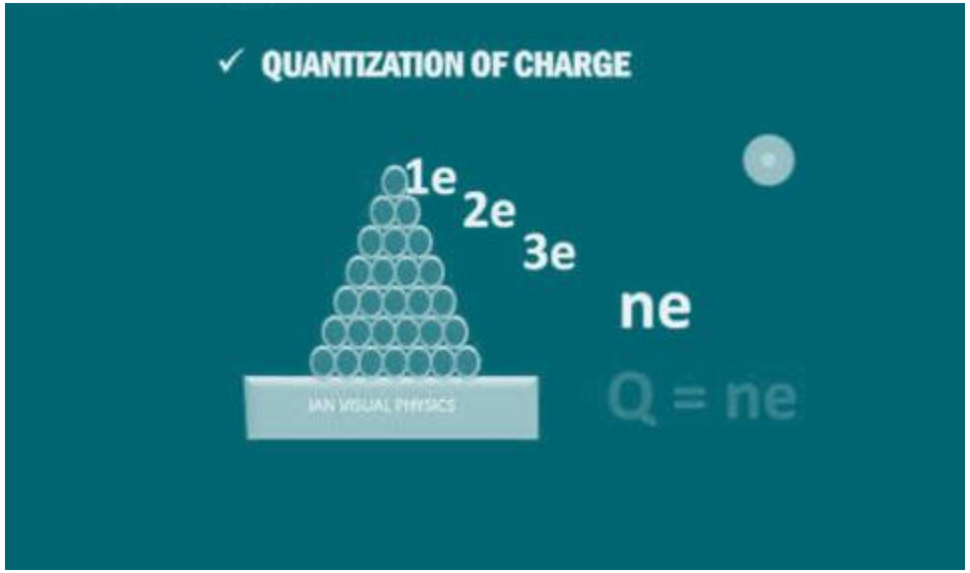
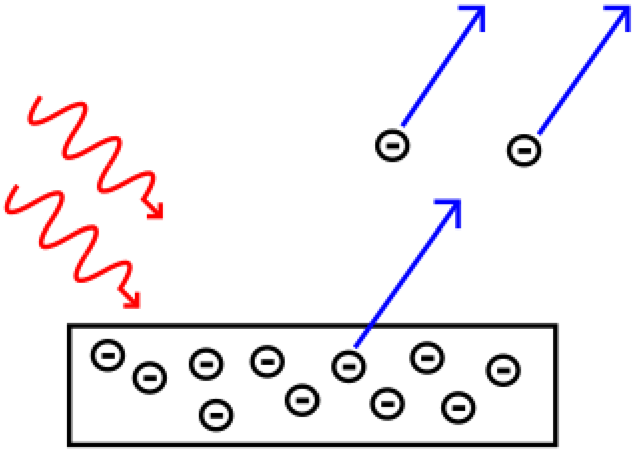


Classical → Quantum

Photoelectric effect →
Electromagnetic field is quantized →
Energy is quantized: $E = \hbar\omega.$





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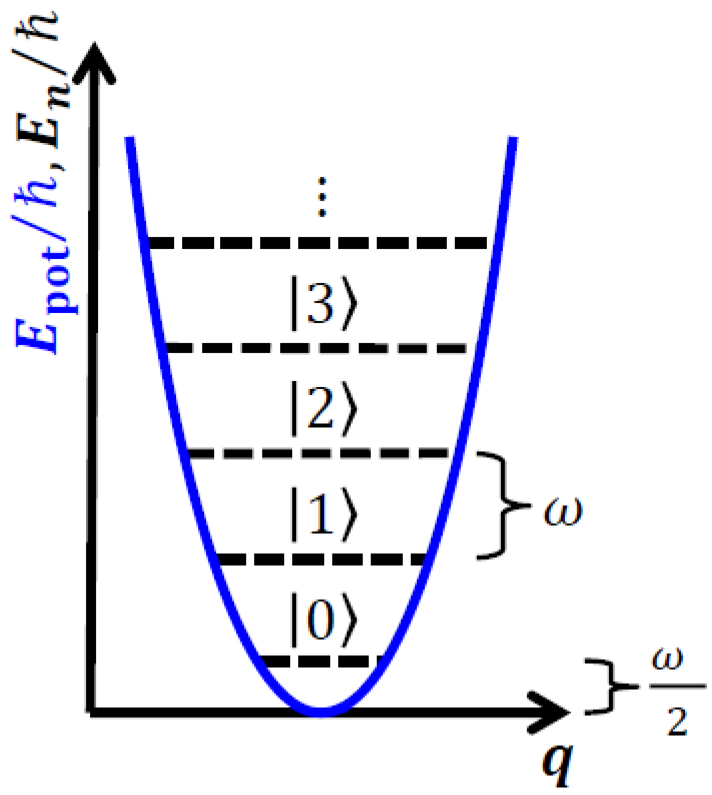
General note: Quantization of oscillators

- General note: Quantization means we see the effect of single particles or excitations.
- In a harmonic oscillator, the energy is quantized equidistantly.
- Energy quantization can be seen as counting the number of photons stored in the oscillator.

Quantization of an oscillator



In quantum mechanics, variables are replaced by operators:

$$q \to \hat{q} : \mathscr{H} \to \mathscr{H} \ ,$$
$$p \to \hat{p} : \mathscr{H} \to \mathscr{H} \ ,$$

For practical reasons, we often use matrix representations

$$q_{k\ell} = \langle e_k | \, \hat{q} \, | e_\ell \rangle$$

Two conjugate variables follow the commutation relation

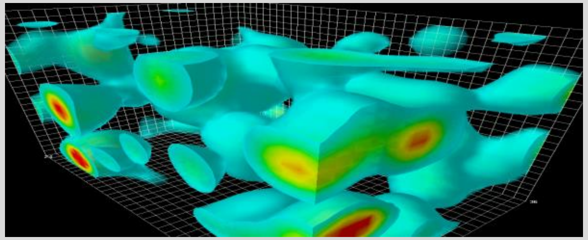
$$[\hat{p}, \hat{q}] \equiv \hat{p}\hat{q} - \hat{q}\hat{p} = -i\hbar$$

Examples

Charge $\hat{q}$
Flux $\hat{\Phi}$

$$\begin{pmatrix} (O\psi)_1 \\ (O\psi)_2 \\ \dots \\ (O\psi)_i \\ \dots \end{pmatrix} = \begin{pmatrix} O_{11} & O_{12} & \dots & O_{1j} & \dots \\ O_{21} & O_{22} & \dots & O_{2j} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ O_{i1} & O_{i2} & \dots & O_{ij} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \\ \psi_j \\ \dots \end{pmatrix}$$

Vacuum fluctuations:



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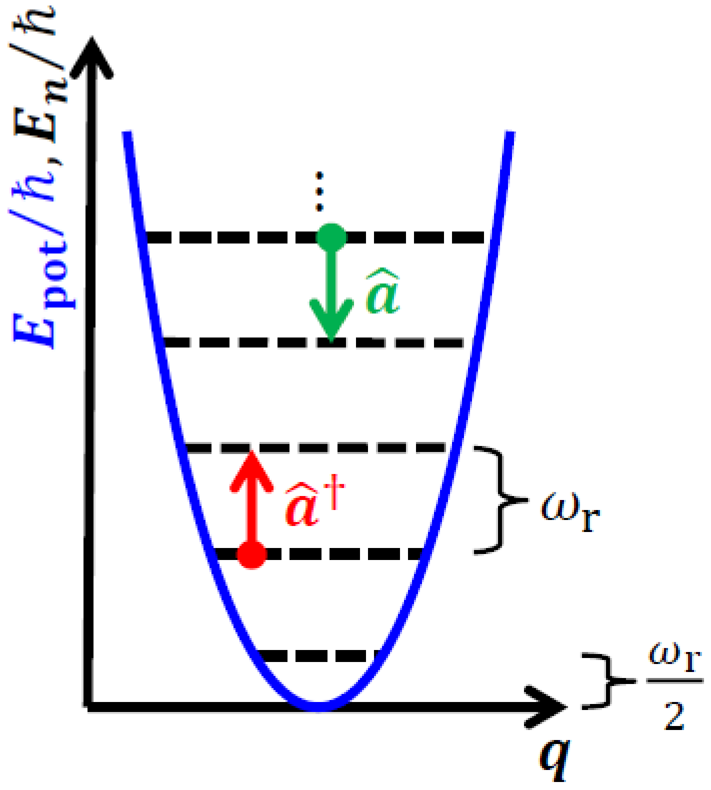
Quantization of an oscillator



For pedagogical reasons, it is convenient to transform systems into the basis of number states (give matrix representation of a)

$$a^\dagger = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & \dots \\ \sqrt{1} & 0 & 0 & \dots & 0 & \dots \\ 0 & \sqrt{2} & 0 & \dots & 0 & \dots \\ 0 & 0 & \sqrt{3} & \dots & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \dots \\ 0 & 0 & 0 & \dots & \sqrt{n} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad a = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \dots & 0 & \dots \\ 0 & 0 & \sqrt{2} & 0 & \dots & 0 & \dots \\ 0 & 0 & 0 & \sqrt{3} & \dots & 0 & \dots \\ 0 & 0 & 0 & 0 & \ddots & \vdots & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \sqrt{n} & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & \ddots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

These can be interpreted as ladder operators raising and lowering the excitation number

$$\hat{a} \equiv \frac{\omega_r C\hat{\Phi} + i\hat{q}}{\sqrt{2\omega_r C\hbar}}$$ is the annihilation operator

$$\hat{a}^\dagger \equiv \frac{\omega_r C\hat{\Phi} - i\hat{q}}{\sqrt{2\omega_r C\hbar}}$$ is the creation operator

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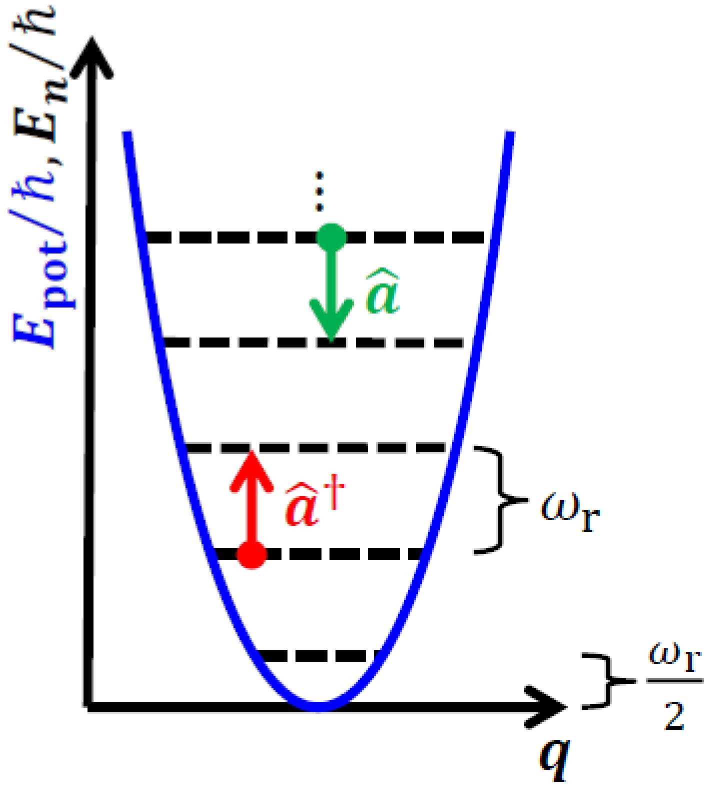
Quantization of an oscillator



→ When applied to a Fock state, $\hat{a}$ annihilates a photon inside the oscillator

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

→ When applied to a Fock state, $\hat{a}^\dagger$ creates a photon inside the oscillator

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

Their product gives the excitation number of a system

$$\hat{n} \equiv \hat{a}^\dagger\hat{a}$$

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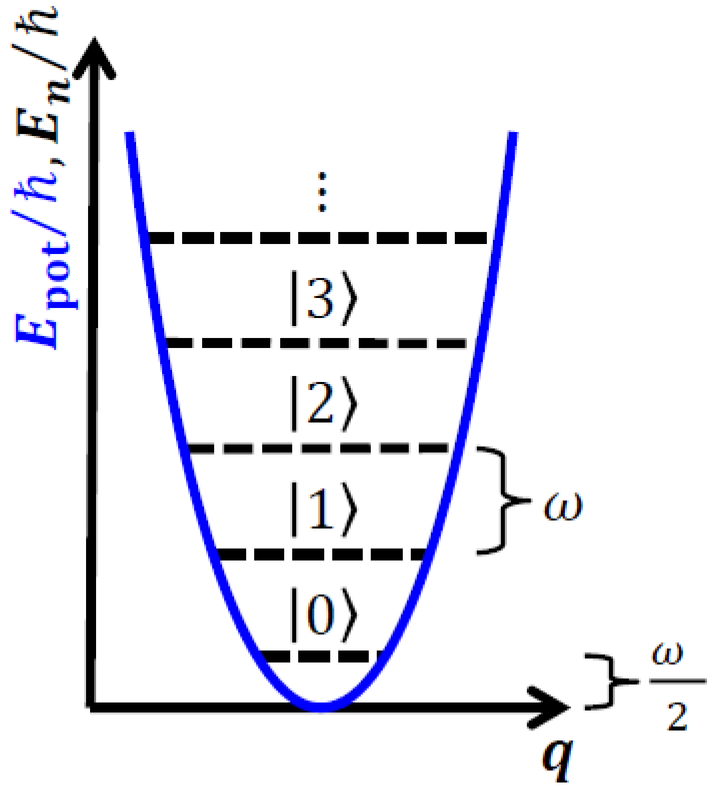
Quantization of the LC oscillator



For the superconducting resonator, we have

$$\hat{H} = \frac{\hat{p}^2}{2L} + \frac{\hat{q}^2}{2C} \; .$$

We aim to diagonalize H into a form involving only one operator. This can be done by a change of variables.

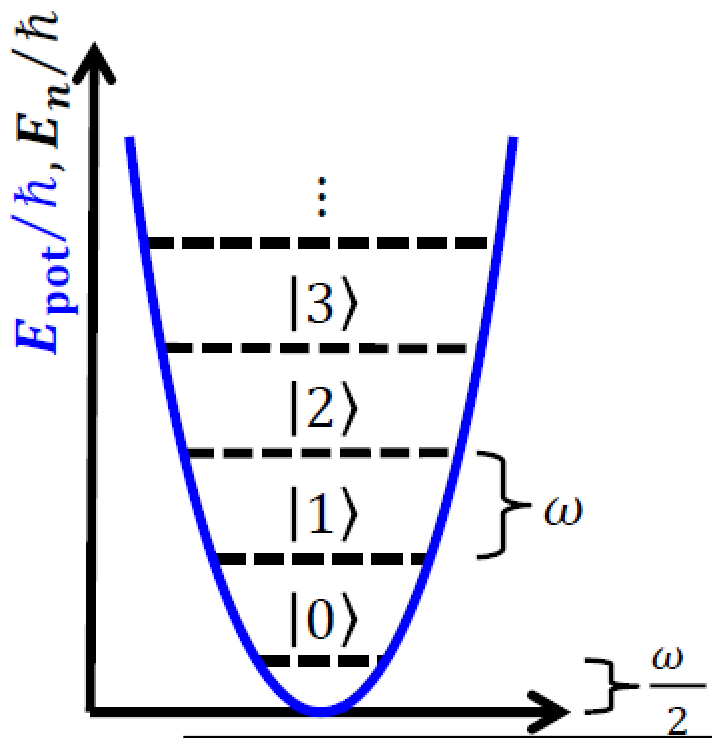
$$\hat{p} = \sqrt{\frac{\hbar\omega L}{2}}\left(\hat{a} + \hat{a}^\dagger\right) , \quad \hat{q} = \sqrt{\frac{\hbar\omega C}{2}} i \left(\hat{a} - \hat{a}^\dagger\right) \; .$$

Here ω is a free scalar parameter, which we will choose later. The square root factors have been inserted for convenience.

$(\hat{a} + \hat{a}^\dagger)$ and $i(\hat{a} - \hat{a}^\dagger)$ are Hermitian and independent.

 * Details in lecturenotes-EQS_Helsinki_2019.pdf

Quantization of the LC oscillator



The previous Hamiltonian becomes

$$\hat{H} = \frac{\left[\sqrt{\frac{\hbar\omega L}{2}}\left(\hat{a}+\hat{a}^\dagger\right)\right]^2}{2L} + \frac{\left[\sqrt{\frac{\hbar\omega C}{2}}i\left(\hat{a}-\hat{a}^\dagger\right)\right]^2}{2C}$$

$$= \frac{\hbar\omega}{4}\left(\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}\right)$$

$$= \frac{\hbar\omega}{2}\left(\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}\right) .$$

Using $[\hat{a}, \hat{a}^\dagger] = 1$ it follows that $\hat{a}\hat{a}^\dagger = \hat{a}^\dagger\hat{a} + 1$

$$\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

Key takeaway: The total energy of the system is given by vacuum fluctuations (+1/2) and the number of photons stored at frequency ω

 * Details in lecturenotes-EQS_Helsinki_2019.pdf

Agenda for today (done)

7. Quantization of electrical networks
   a. Harmonic oscillator: Lagrangian, eigenfrequency
   b. Transfer step: LC oscillator, Legendre transform to Hamiltonian
   d. Quantization of oscillators

a.

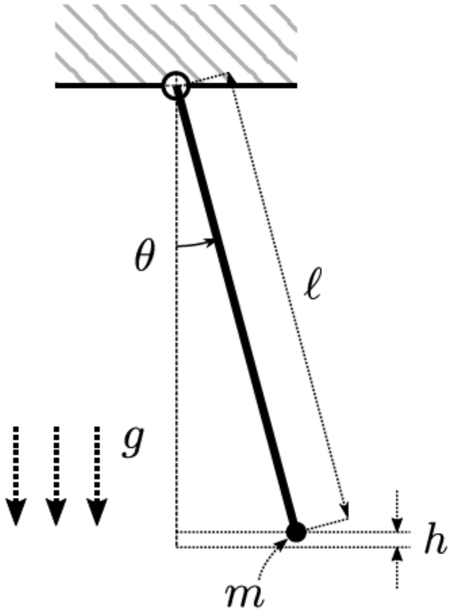


Figure 1: Classical pendulum.

b.

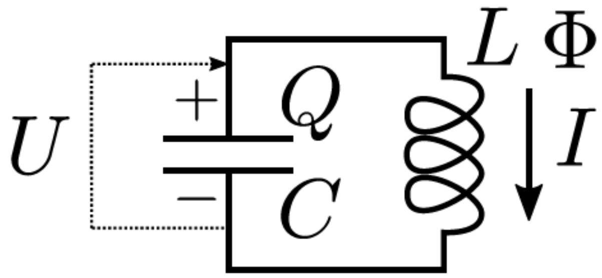
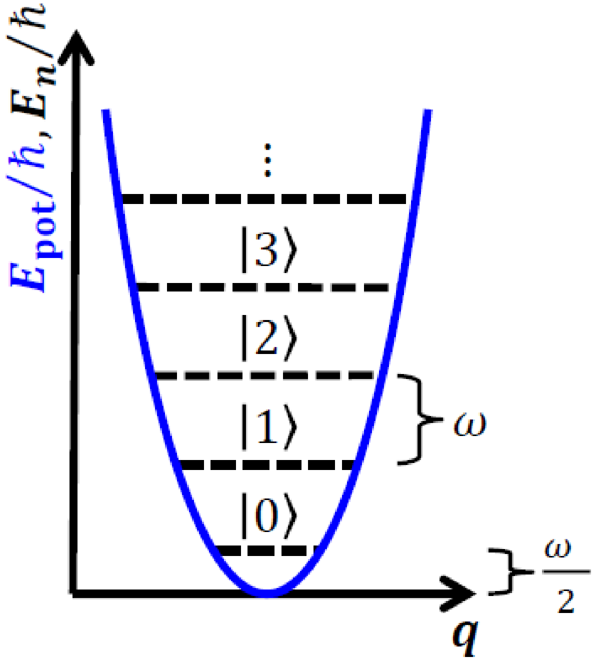


Figure 2: Superconducting LC oscillator.

c.



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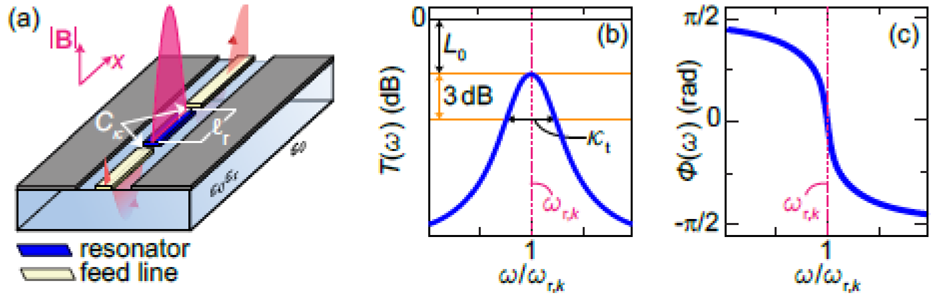
Add-on: Vacuum fluctuations & thermal photons (if time allows)



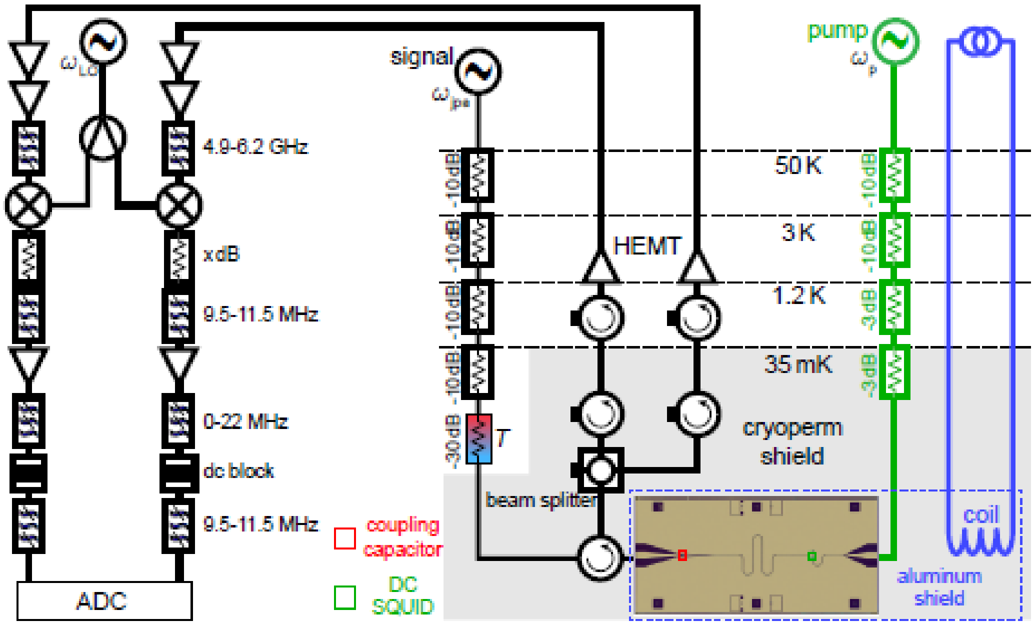
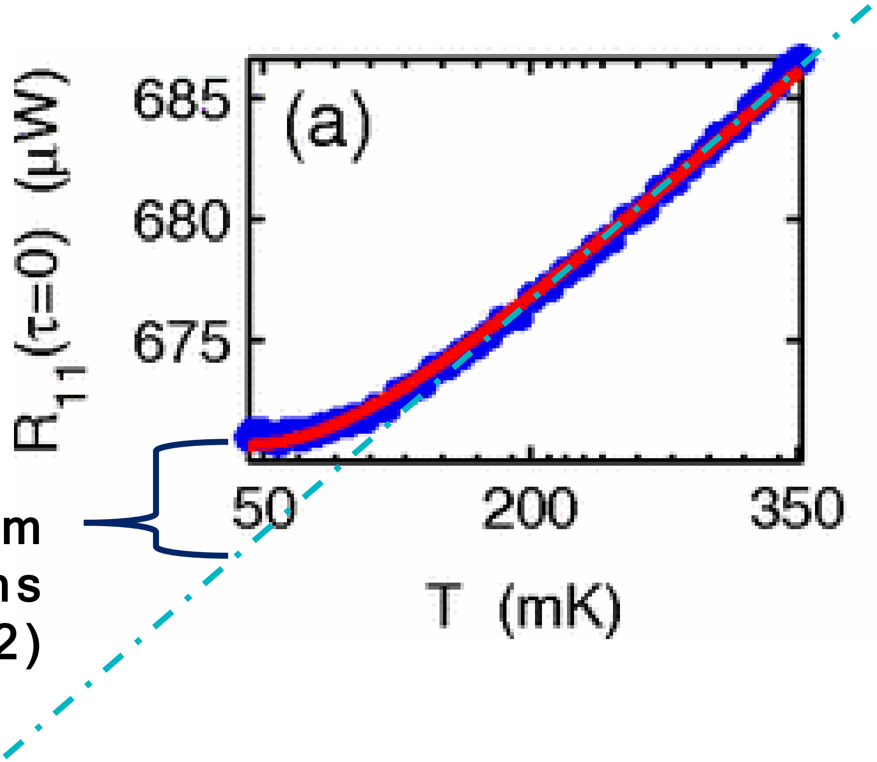


Figure 4.38: Schematics of the dual-path setup.



Vacuum fluctuations (+1/2)

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