Logistics

- Referee report assignments to be posted on MyCourses
  - Complete the poll if you haven't yet.
- Presentation topics yet to be assigned
  - Complete the poll if you haven't yet.
- Student presentation date moved from May 26 to ...

Lecture 2
Spatial equilibrium: canonical urban models

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ECON-L6000 - Urban and Regional Economics
Aalto University School of Business

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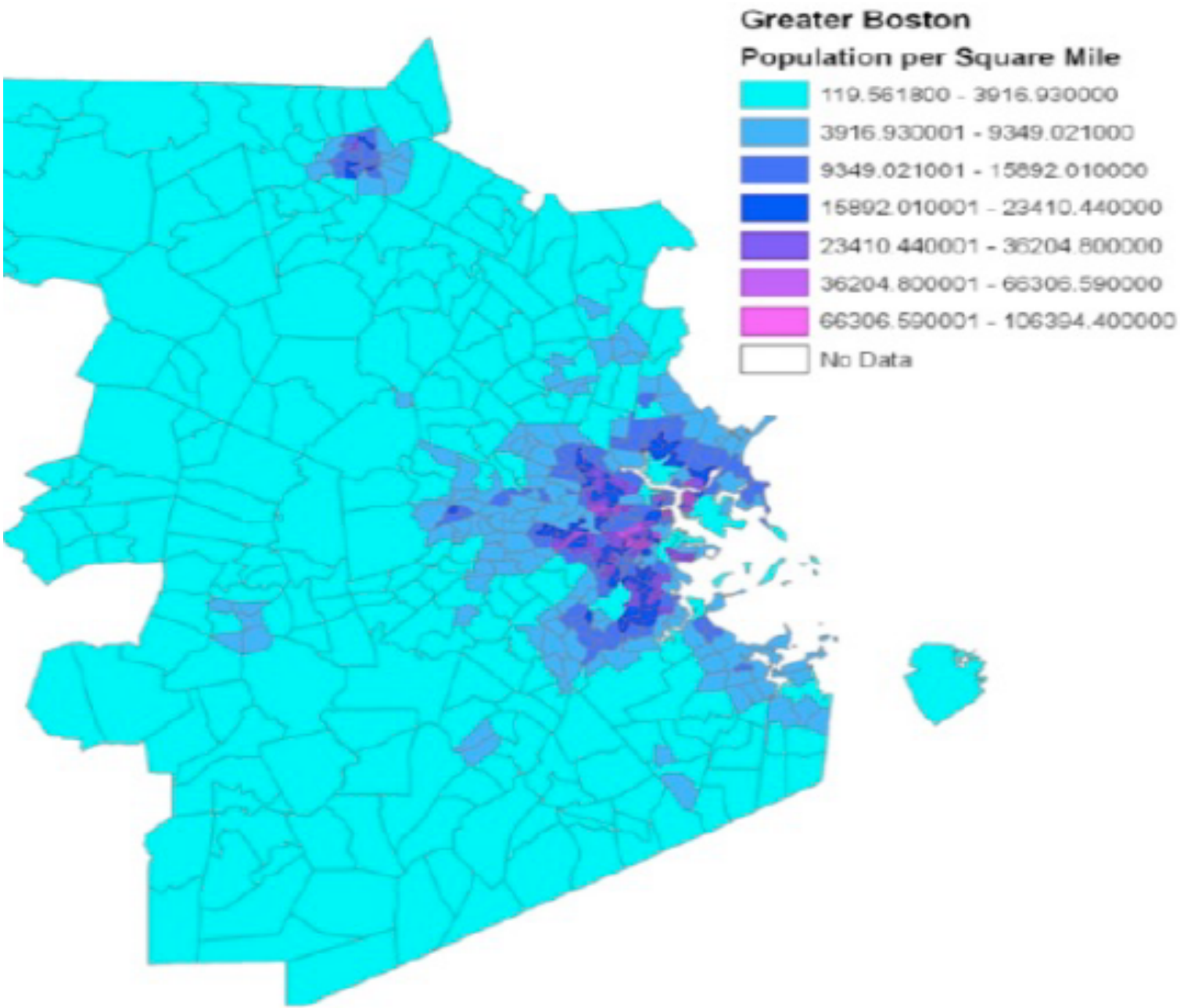
Today's agenda

1. Spatial equilibrium **within** cities
   - The Alonso-Muth-Mills model
   - a.k.a the monocentric city model
   - following *Alonso (1964)*, *Mills (1967)* and *Muth (1969)*.

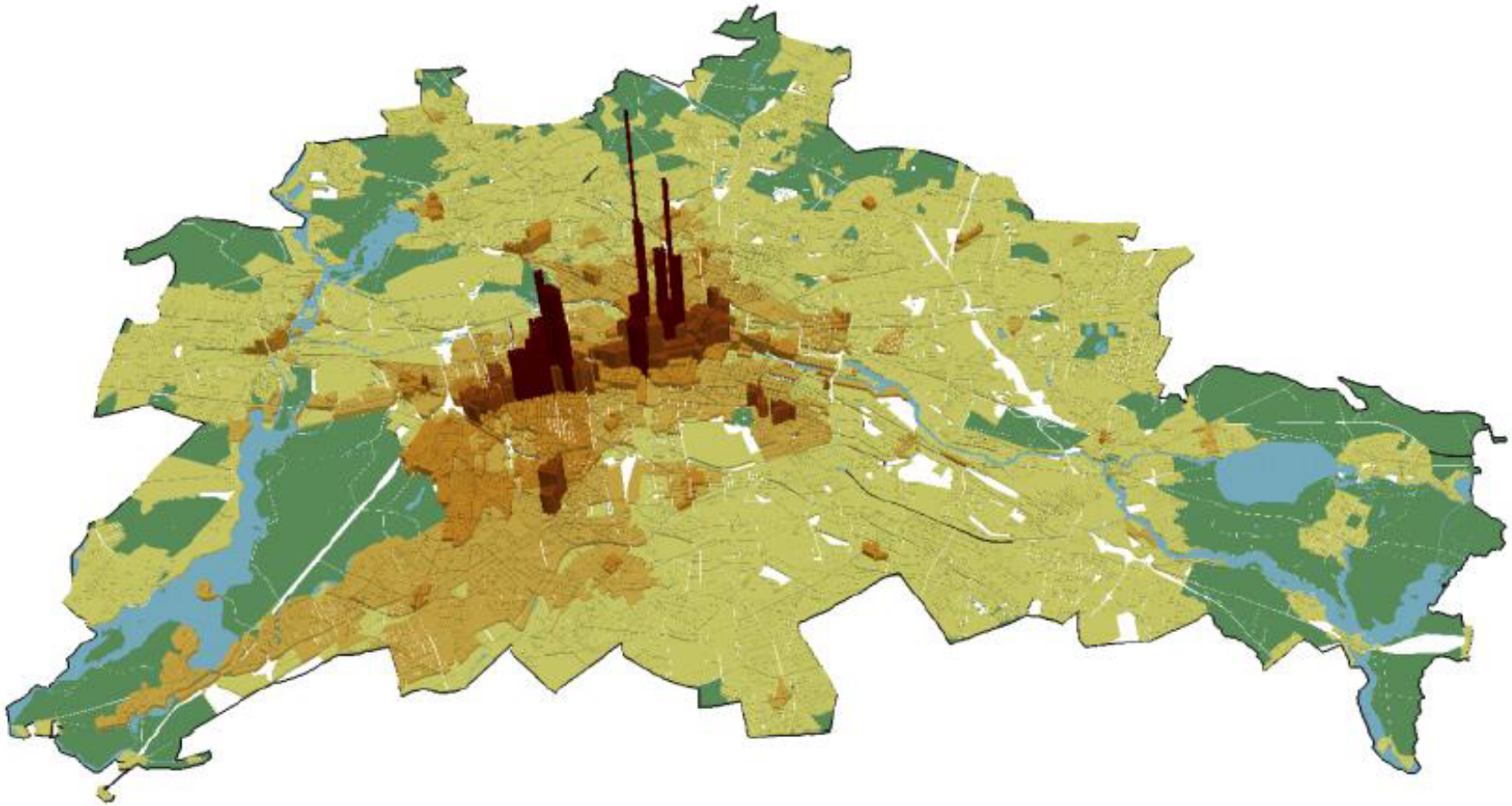
2. Spatial equilibrium **across** cities
   - The Rosen-Roback model
   - following *Rosen (1974)* and *Roback (1982)*.

Higher population density near city centers



Taller structures near city centers

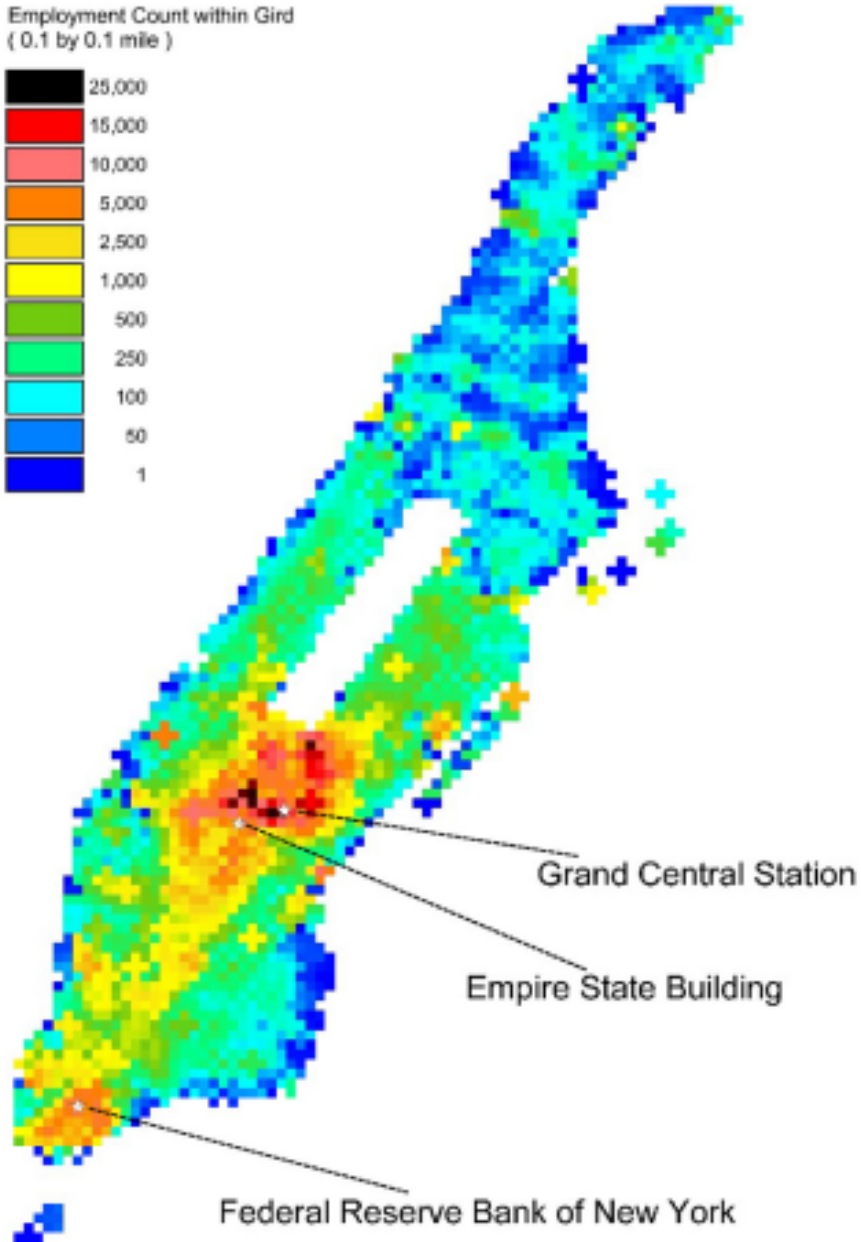
More expensive land near city centers



Land prices in Berlin

from *Ahlfeldt et. al (2015)*

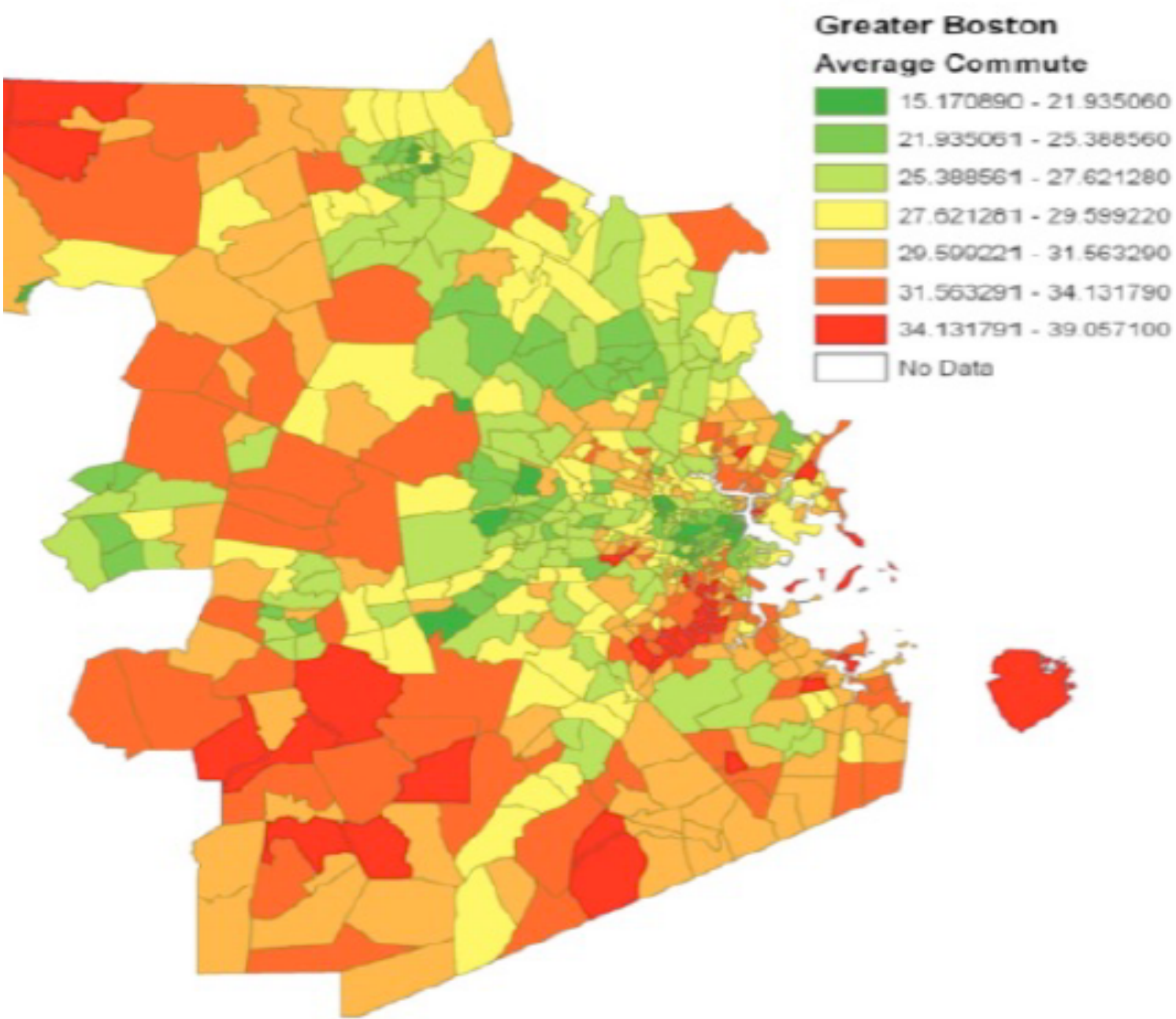
Higher employment density near city centers



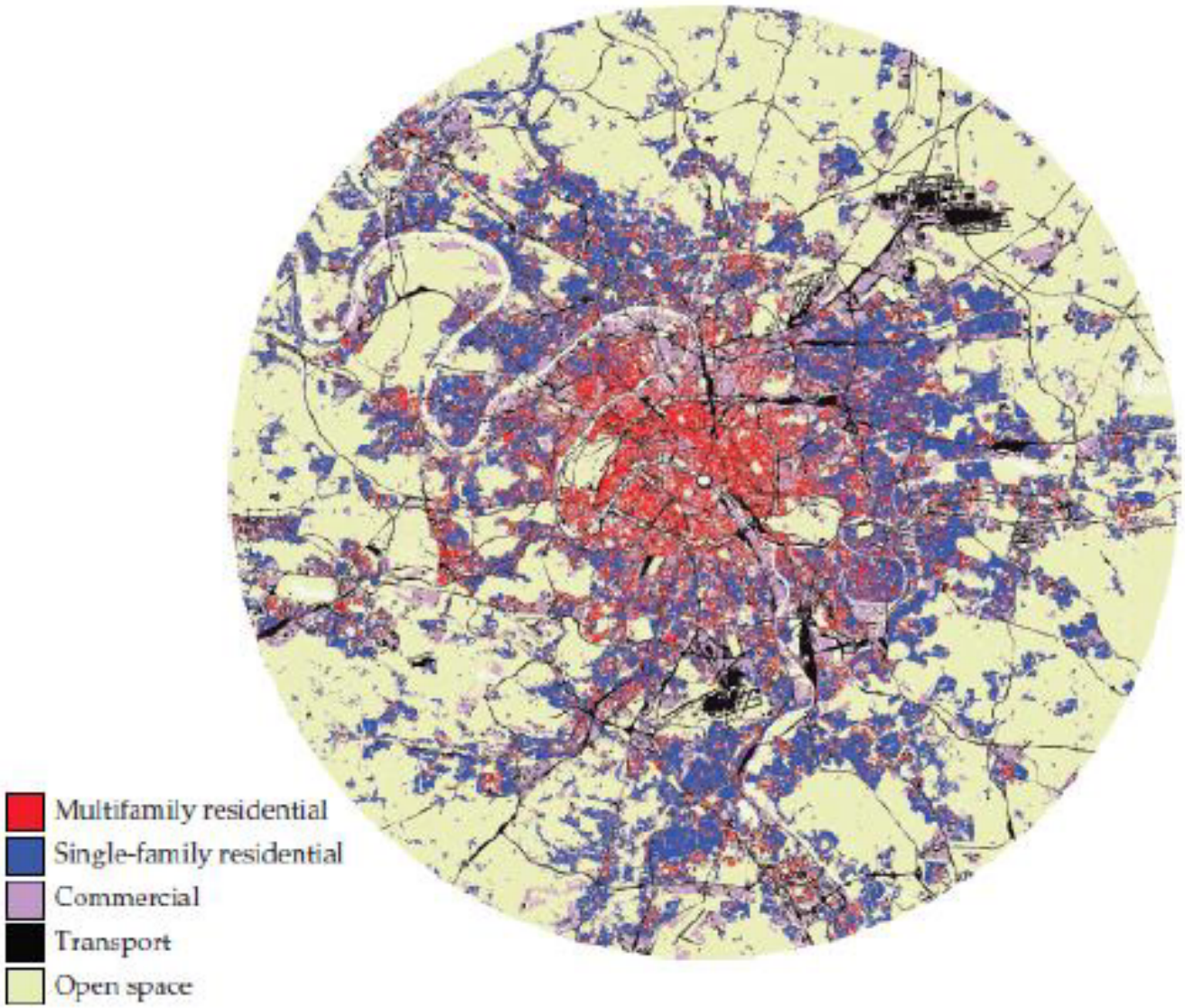
Employment density in Manhattan

from *Liu et. al (2020)*

Shorter commutes near city centers



Land Use in Paris



Duranton, G. & D. Puga. 2015. Urban Land Use. In G. Duranton, J.V. Henderson, W.C. Strange (ed.), *Handbook of Regional and Urban Economics*, Vol 5, 467-560

Monocentric city model: Residents

Homogeneous urban residents:

- with income y
- commute to a job in the city center
- **choose** distance x from city center to reside in
- face commuting costs $T(x)$ that are increasing with distance: $T'(x) > 0$
  - locations are identical in all directions
- **choose** consumption of:
  - housing space: q
  - a composite good: c
- face prices $p(x)$ of housing space that varies with location
- face a constant price=1 of composite good

Monocentric city model: Utility

Urban residents:

- **maximize** a quasi-concave utility function $v(c, q)$
- s.t. income constraint $y = T(x) + p(x)q + c$ (i.e., no saving)

Monocentric city model: Utility

Urban residents:

- **maximize** a quasi-concave utility function $v(c, q)$
- s.t. income constraint $y = T(x) + p(x)q + c$ (i.e., no saving)

In a spatial equilibrium, utility

- must be same for everyone
  - regardless of consumption and location choices
  - given homogeneity
- equals some constant u
- (recall from micro theory: Hicksian approach)

Monocentric city model: Residential choices

Optimal housing consumption satisfies the f.o.c.:

$$p(x) = \frac{\partial v}{\partial q} \Big/ \frac{\partial v}{\partial c}$$

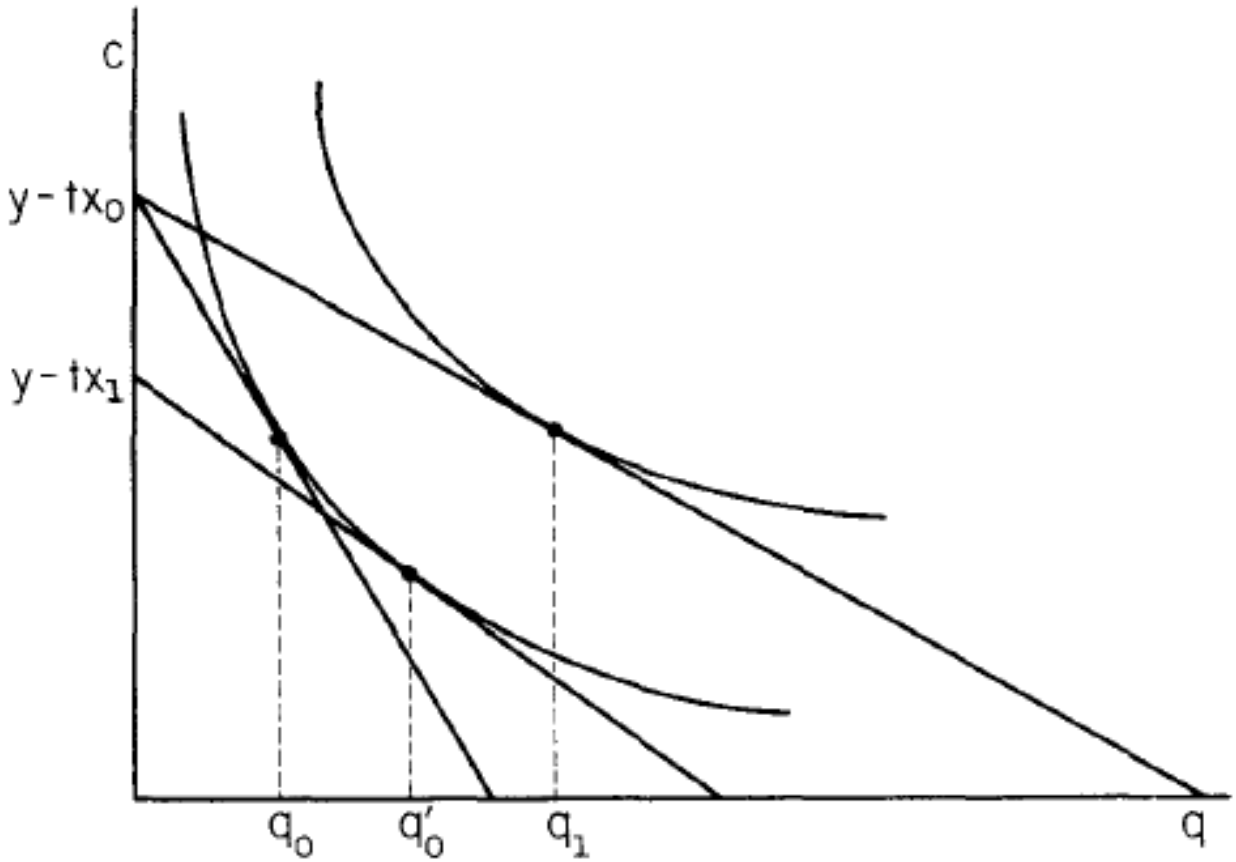


Fig. 1 from
Brueckner (1987)

Monocentric city model: Residential choices

1. Housing price $p(x)$ decreases with distance
   - $\frac{\partial p}{\partial x} < 0$
   - and commuting cost
2. Housing consumption q increases with distance
   - $\frac{\partial q}{\partial x} > 0$
   - and commuting cost
3. Higher utility $u \iff$ higher housing consumption q and lower prices p
   - assuming housing is normal good
   - $\frac{\partial p}{\partial u} < 0$ and $\frac{\partial q}{\partial u} > 0$
   - note: holding income fixed

Monocentric city model: Housing supply

Housing production

- uses land l and capital N
- according to concave constant returns function $H(N, l)$
- faces rental prices r of land and i of capital
- **maximizes** profit: $pH(N, l) - iN - rl$

$$= l\big(ph(S) - iS - r\big)$$

where $S \equiv N/l$ is capital-land ratio and $h(S) \equiv H(S, 1)$ is floor space per unit of land

  - first-order condition: $i = ph'(S)$

- is perfectly competitive
  - zero profit condition: $r = ph(S) - iS$

Monocentric city model: Housing supply

1. Land rent r is decreasing with distance x
   - $\frac{\partial r}{\partial x} < 0$
   - land is cheaper farther from the center

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   - buildings are shorter farther from the center

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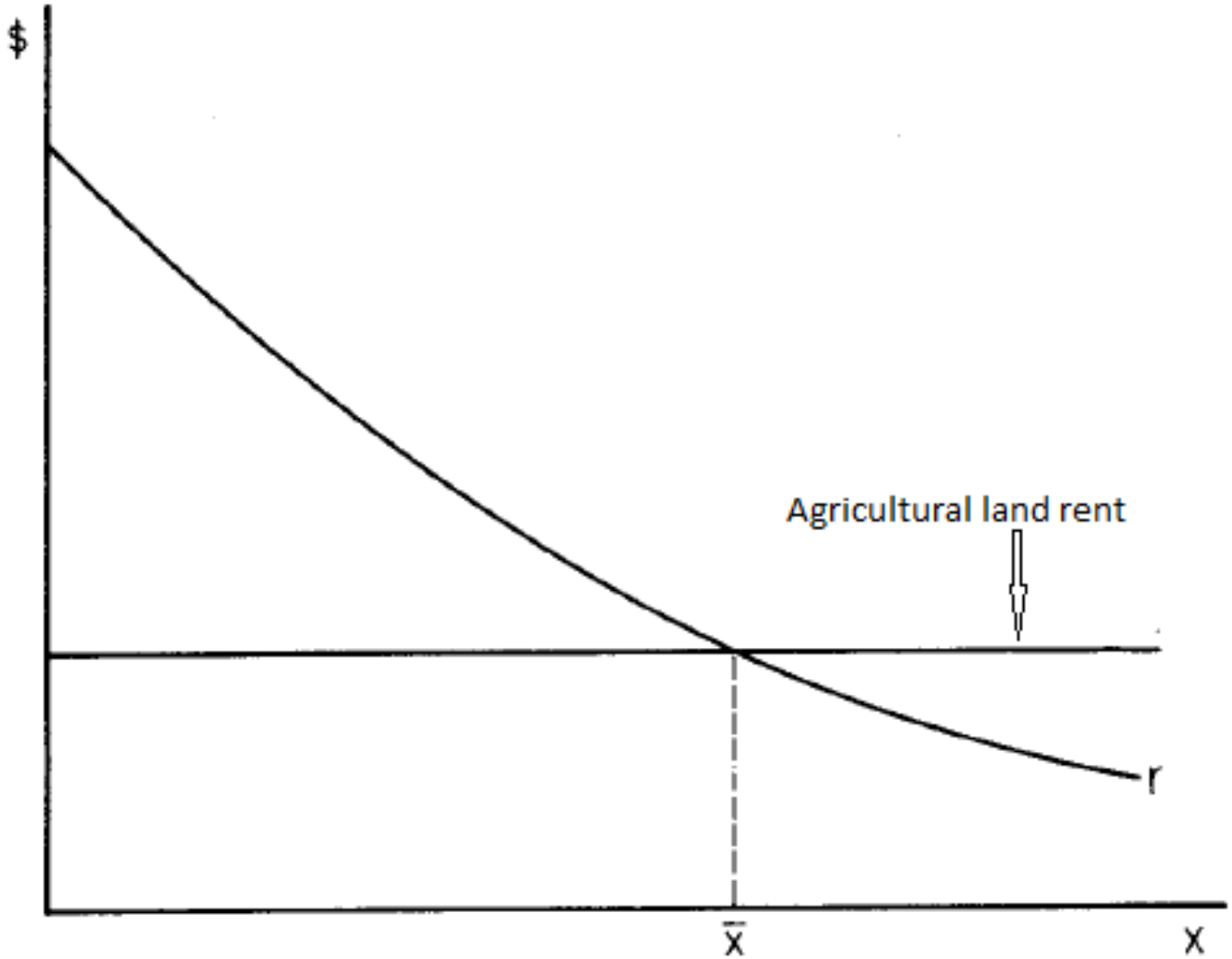
   - $\frac{\partial r}{\partial x} < 0$
   - land is cheaper farther from the center
2. Capital-land ratio S is decreasing with distance x
   - $\frac{\partial S}{\partial x} < 0$
   - buildings are shorter farther from the center
3. Population density $h(S)/q$ is decreasing with distance x
   - since $\frac{\partial q}{\partial x} > 0$ and $\frac{\partial S}{\partial x} < 0$.

Monocentric city model: city size and utility

We require two further equilibrium conditions to determine city population and area:

1. Housing producers outbid agricultural users for all the land used for urban housing.
   - land rents $r(x)$ in the city should exceed land rent $r(\bar{x})$ at the distance $\bar{x}$ to the city boundary
2. Total city population L should fit inside $\bar{x}$.

Monocentric city model: Land rent at boundary



Open vs. closed city

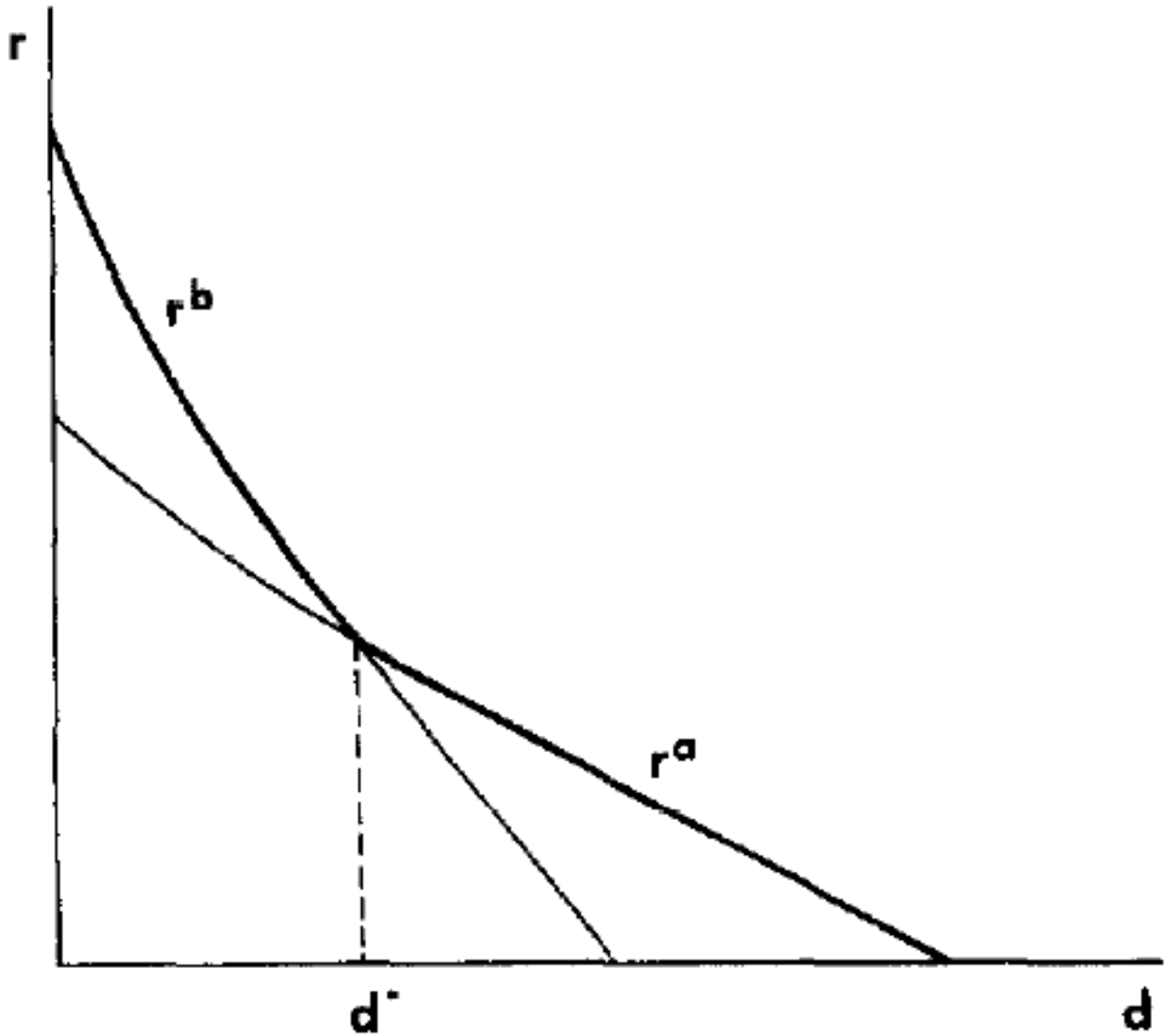
- Open-city model: people move from elsewhere and there is a spatial equilibrium across cities.
  - fixed (reservation) utility, endogenous population
- Closed-city model: no mobility across cities
  - fixed population, endogenous utility

Monocentric city model: Extensions

- Heterogeneous incomes (Wheaton, 1976; Glaeser, Kahn and Rapapport 2008)
- Travel mode choice (LeRoy and Sonstelie, 1983)
- Decentralized employment (Fujita and Ogawa, 1982)
- Many other variants!

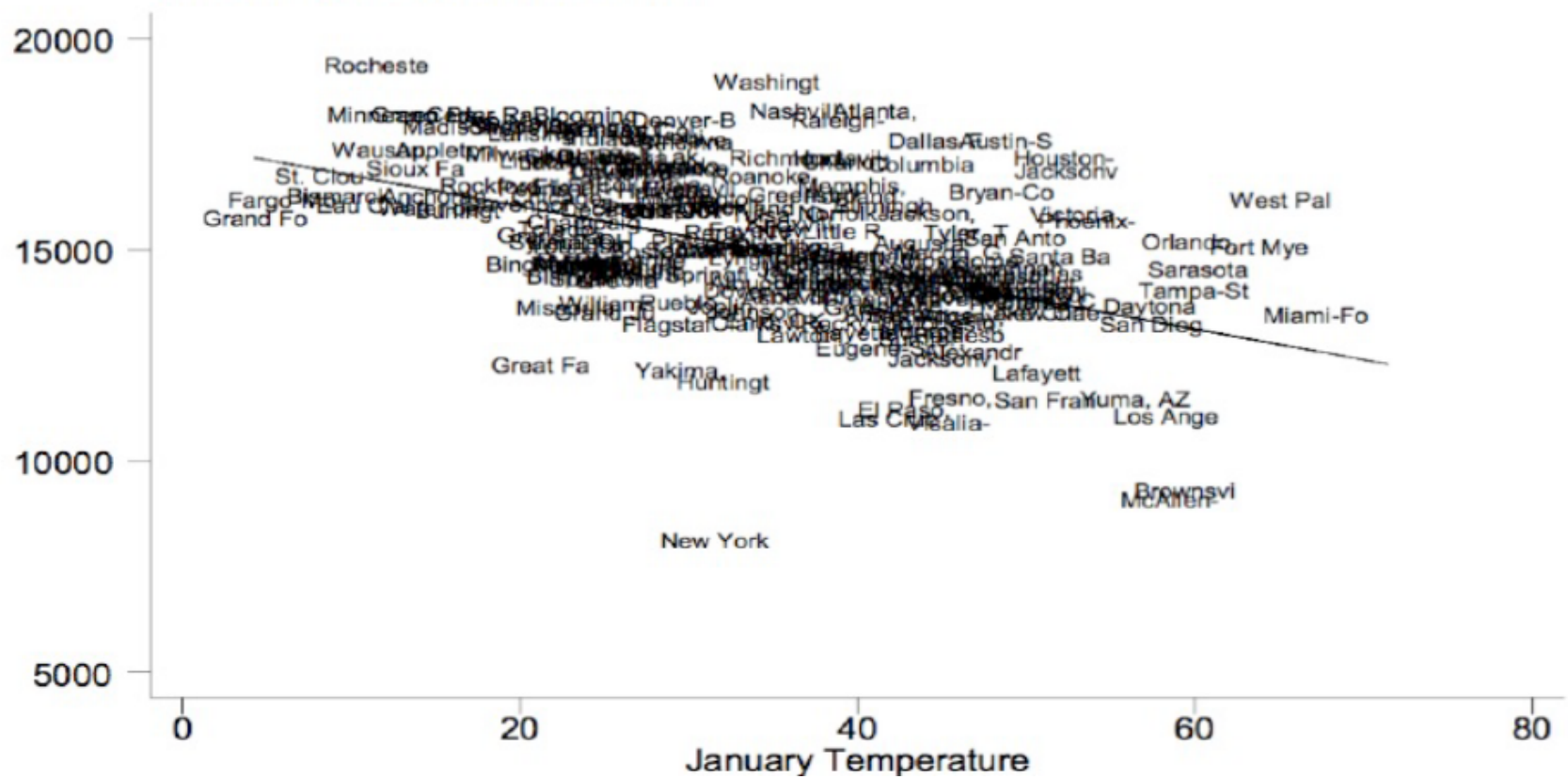
Monocentric city model: Bid-rent functions

Residents may face different bid-rent gradients (over distance):



Spatial equilibrium across cities

Income and climate



Rosen-Roback model

1. Discrete location choices
2. Spatial equilibrium for mobile workers/consumers
3. Zero profit condition and spatial equilibrium for mobile firms
4. Zero profit condition for suppliers of housing and non-tradable goods

Rosen-Roback model: Intuition

Individual utility over wages (Y_c), prices (P_c) and amenities (A_c) in place c:

$$V\left(Y_c, P_c, A_c\right)$$

must be constant across locations and equal reservation utility $\bar{U}$.

Rosen-Roback model: Intuition

Individual utility over wages (Y_c), prices (P_c) and amenities (A_c) in place c:

$$V\big(Y_c, P_c, A_c\big)$$

must be constant across locations and equal reservation utility $\bar{U}$.

If $V\big(Y_c, P_c, A_c\big) = V\big(Y_c - P_c, 0, A_c\big)$,

$$\frac{d(Y_c - P_c)}{dA_c} = -\frac{V_A\big(Y_c - P_c, 0, A_c\big)}{V_Y\big(Y_c - P_c, 0, A_c\big)}$$

Higher amenities correspond to lower real incomes.

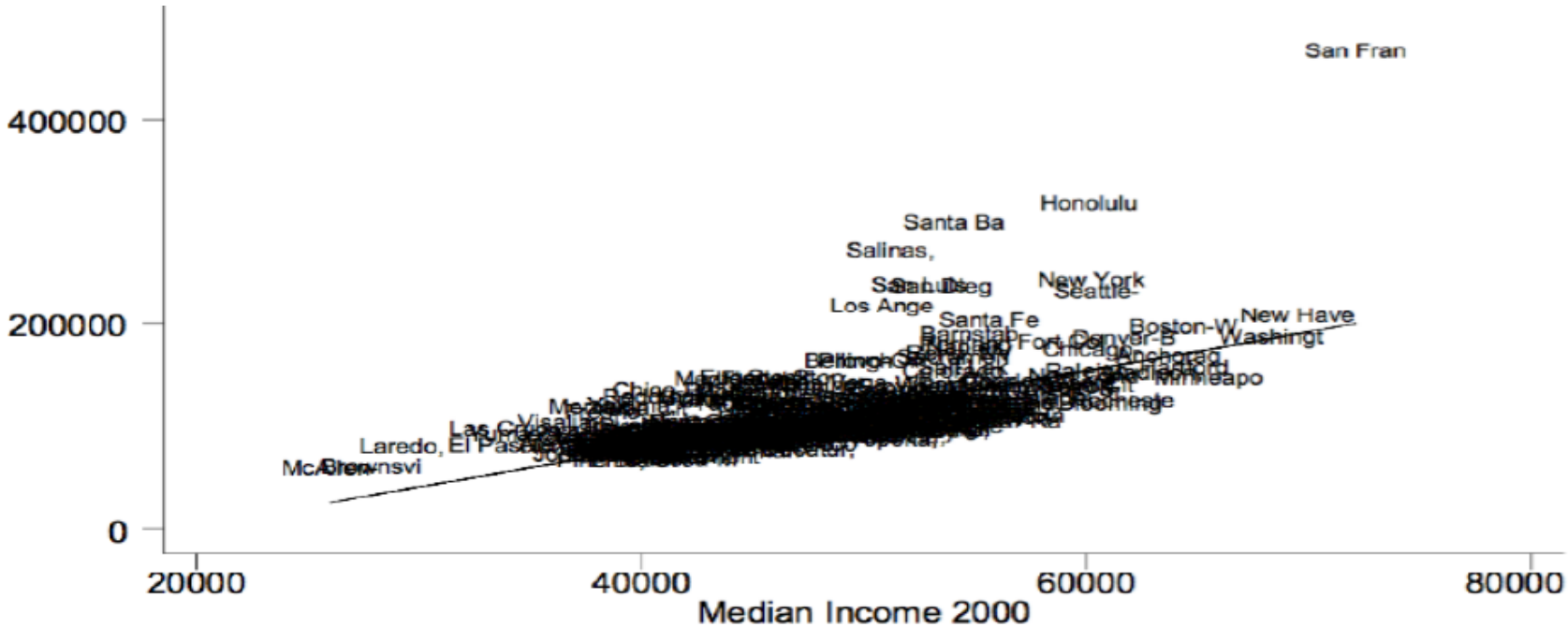
Rosen-Roback model: Intuition

$$V\left(Y_c, P_c, A_c\right) = \bar{U}$$

Holding amenities constant:

- any increase in prices must be offset by an equivalent increase in incomes
- any increase in incomes must be offset by an equivalent increase in prices
- Compensating differential!

House prices and income



Rosen-Roback model: Key take-aways

- Population rises with productivity, amenities and land supply
- Incomes rise with productivity, and decrease with amenities and land supply
- Land prices rise with productivity and amenities, and decrease with land supply
- Changes in population, incomes and land prices can be used to study changes in amenities, productivity and land supply.

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