# Lecture 5 Equilibrium sorting and estimating preferences for amenities

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### Recap: Tiebout (1956)

- ► How can we estimate the demand for public goods that are not explicitly traded in formal markets?
- ➤ Tiebout: households "vote with their feet" for their preferred combination of local public goods

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- "Tiebout bias": say, we regress public good expenditures (A) on neighborhood demographic characteristics (median incomes y, tax rates τ, etc.)?
  - ▶ simultaneity problem: A is determined by neighborhood composition, but composition is determined by households (with heterogeneous preferences for A) "sorting" across neighborhoods.

#### Recap: WTP for amenities through housing

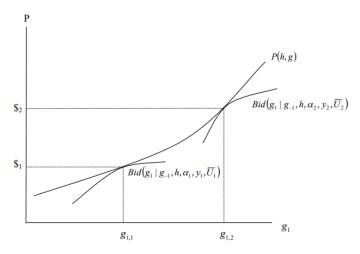


Figure Bid Functions for Housing as a Function of  $g_1$  in Hedonic Equilibrium

from Kumminoff, Smith, and Timmins (2013)

#### Recap: WTP for amenities through housing

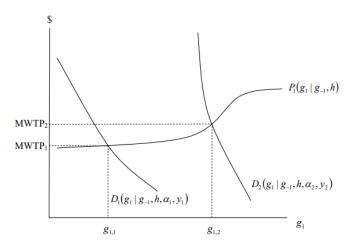


Figure Implicit Price Function for  $g_1$  and Demand Curves for Two Households

#### Recap: Hedonic Property Value Models

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#### Empirical challenges include:

- 1. Identifying WTP requires households to be able to choose continuous quantities of the amenity
- 2. Unobserved endogenous amenities
  - Bayer, Ferreira, and McMillan (2007): school boundary discontinuity doubles WTP estimates of school quality in San Francisco when they do not control for local demographic composition.
- 3. Not informative beyond marginal effects (e.g., large shocks, in different settings, etc.)

#### **Equilibrium Sorting Models**

- combine information from the hedonic price function
  - ► Rosen (1974), ...
- with a formalization of the choice process that underlies market sorting of heterogeneous agents
  - McFadden (1974), Berry, Levinsohn, and Pakes (1995), ...
- while recognizing that characteristics of the objects of choice may be determined endogenously
  - ▶ Epple and Sieg (1999), Bayer and Timmins (2007), ...
- ▶ to understand "general equilibrium" feedback effects between economic agents and their environments.

#### Today's agenda

- 1. Equilibrium sorting theory
  - investigate existence and uniqueness of equilibria
  - generalizable implications for equity and efficiency
- 2. Equilibrium sorting estimation
  - infer preferences for amenities from observable characteristics of households and their location choices
  - predict market responses to large policy shocks to amenities

# Equilibrium Sorting Theory

#### Setup

Urban landscape includes n = 1, ..., N houses across j = 1, ..., J neighborhoods.

Houses are a bundle of physical housing characteristics  $h_n$  and neighborhood amenities  $g_i$ .

Household consume a numeraire b.

Households are heterogeneous in:

- observable characteristics d
- ightharpoonup unobservable features of their preferences lpha

#### Setup

Household's decision problem is subject to a budget constraint:

$$\max_{n \in j,b} U(b, h_n, g_j; \alpha_i, d_i)$$
s.t.  $y_{i,j} = b + P_{n \in j}$ 

#### where

- $\triangleright$   $y_{i,j}$  is income
- $ightharpoonup P_{n \in j}$  is expenditure on house n in neighborhood j

In equilibrium, households occupy utility-maximizing location and nobody wants to move, given prices, housing characteristics, amenities, and incomes.

#### Setup

#### Three key assumptions:

- Full information
  - All households share the same objective evaluation of housing characteristics and amenities
- ► Free mobility
  - ► Households can move freely across all neighborhoods in the choice set.
- No discrimination
  - every household faces the same schedule of housing prices

#### Compared to other differentiated product models

Features that distinguish the location choice problem:

- 1. Mix of public and private goods
- Endogeneous characteristics determined by the sorting process
- 3. Heterogeneous preferences + heterogeneous landscape
- 4. Multiple equilibria
  - uniqueness (and analytical solvability) requires either additional restrictions on structure of preferences or less endogeneity

#### Households' choice problem

$$\max_{n \in j, b} U(b, h_n, g_j; \alpha_i, d_i) \text{ s.t. } y_{i,j} = b + P_{n \in j}$$

#### Can be depicted in two stages:

- 1. choose the optimal quantities of housing and numeraire in each neighborhood
- 2. choose the neighborhood that maximizes utility

#### Household sorting

#### Suppose:

- 1. 1-dimensional public good  $g_i$
- 2. homogeneous preferences  $\alpha$  (HHs differ only in income)

#### Household sorting

If indifference curves in the (g, p) plane are strictly increasing in income:

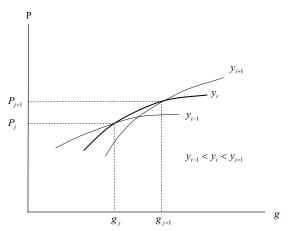


Figure The Single Crossing Condition: Indifference Curves for Three Households

### Single crossing condition

Households typically do not perfectly stratify by income, maybe because they have heterogeneous tastes  $\alpha$  for amenities.

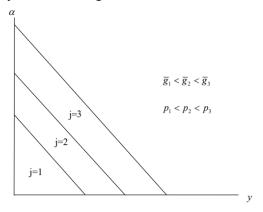


Figure Partition of Households into Communities by Preferences and Income

from Kumminoff, Smith, and Timmins (2013)

#### Single crossing condition

Implies 3 necessary properties of any sorting equilibrium:

- 1. Boundary indifference
  - ▶ HHs on the border between two nbhds in  $(\alpha, y)$  space is indifferent between the nbhds.
- 2. Increasing bundles
  - Ranking of nbhds by public goods provision must match the ranking by price.
- 3. Stratification
  - ▶ HHs of each type are stratified across the J ordered locations by  $(\alpha|y)$  and by  $(y|\alpha)$

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#### Fernandez and Rogerson (1996):

- ➤ Setup: single public good (school quality) is increasing in average income of neighborhood residents
- Show: Sorting equilibrium is inefficient! Why?
- Migrating HHs do not internalize the effect of their location choices on the current residents of their destination nbhds.
- School finance reforms which are most effective at inducing migration to poorer nbhds tend to be Pareto improving.

#### Benabou (1993):

- Adds production sector with complementarity between high and low skill labor.
- Higher skill HHs have incentive to segregate from lower skill HHs
- ► Segregation raises cost of education in low skill nbhds
- ...increases unemployment, decreases production, and worsens inefficiency from stratification.

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- ...increases unemployment, decreases production, and worsens inefficiency from stratification.

**Benabou (1996)**: Minor differences in preferences can create a "tipping" effect, leading to large stratifications by income.

# Equilibrium Sorting Estimation

#### **Equilibrium Sorting Estimation**

#### Three dominant approaches:

- 1. Pure Characteristics Sorting Model (PCM)
  - ► Epple and Sieg (1999)
- 2. Random Utility Sorting Model (RUM)
  - Bayer, McMillan, and Rueben (2004)
- 3. General Equilibrium Sorting Model (GEM)
  - Ferreyra (2007) and others...

- ightharpoonup condense housing characteristics into a single housing "quantity" number  $\bar{h}(h_n)$
- housing characteristics are separable from the effect of amenities and numeraire on utility
- HHs choose from a discrete number of nbhds and then, conditional on nbhd choice, from a continuous quanity of housing in the nbhd.

Indirect utility function:

$$\begin{split} V_{i,j} &= \left\{ \alpha_i \left( \overline{g}_j \right)^{\rho} + \left[ \exp \left( \frac{y_i^{1-\nu} - 1}{1 - \nu} \right) \exp \left( -\frac{\beta \left( p_j^{\eta + 1} \right) - 1}{1 + \eta} \right) \right]^{\rho} \right\}^{\frac{1}{\rho}}, \\ where &\quad \overline{g}_j = \gamma_1 g_{1,j} + \gamma_2 g_{2,j} ... + \gamma_{K-1} g_{K-1,j} + \gamma_K \overline{\xi}_j, \qquad F(\alpha, y) \sim lognormal. \end{split}$$

- ▶ HHs agree on ranking of nbhds by  $\bar{g}$ .
- no idiosyncratic shocks
- Roy's identity yields housing demand:

$$\bar{h}_{i,j} = \beta p_j y_i^{\nu}$$

#### 2-stage estimation:

1. Use *stratification* property to express quantiles of income distributions in each nbhd as function of model parameters.

Then, minimize difference between observed and predicted income quantiles.

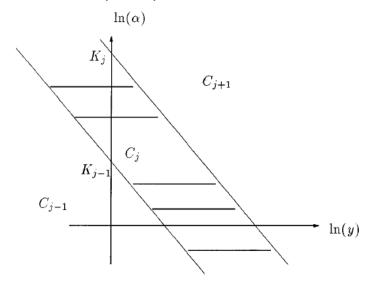


FIG. —Distribution of households across communities, given the parameterization of the model.

- Use increasing bundles and boundary indifference properties to develop an instrumental variable estimator to identify remaining parameters
  - $\blacktriangleright$  since amenities  $\bar{g}$  may be correlated with unobserved amenities  $\bar{\xi}$
  - ▶  $p_j$ ,  $\bar{g}_j$  and  $y_j(\alpha)$  all follow the same ranking.
  - Assume:  $\bar{\xi}$  may affect level of nbhd income but not the ranking of nbhds

# Bayer, McMillan, and Rueben (2004)

Much more flexible probabilistic approach:

- ► HHs may differ in relative preferences for housing characteristics and amenities
- Random utility specification with idiosyncratic taste shock for every choice alternative

$$\begin{split} V_{i,n\in j} &= \alpha_h^i h_n + \alpha_g^i g_j + \alpha_c^i c_{i,j} + \alpha_p^i p_n + \overline{\xi}_n + \varepsilon_n^i, \\ \text{where} \quad \alpha_a^i &= \alpha_{0,a} + \sum_{i=1}^R \alpha_{r,a} d_r^i, \quad \text{and} \quad \varepsilon_j^i \sim \text{iid type I extreme value.} \end{split}$$

▶ Adaptation of Berry, Levinsohn, and Pakes (1995) to minimize difference between predicted and observed location choices

# Bayer, McMillan, and Rueben (2004)

To deal with endogenous amenities:

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To deal with endogenous amenities:

rely on amenity discontinuity at boundaries

To deal with endogenous prices:

- ▶ instrument for price in nbhd j is a function of housing characteristics and exogenous amenities in all other nbhds
- because price depends on characteristics of closest substitutes
- but utility from residing in j is unaffected by exogenous attributes of other nbhds

# Ferreyra (2007) and others ...

Alternatively: rather than develop instruments for endogeneous components of the equilibrium, model the mechanisms that underlie the endogeneity.

- much simpler utility framework (typically Cobb-Douglas)
- ▶ idiosyncratic shocks (from iid type 1 EV distribution)
- shape of preference heterogeneity is pre-specified

# Comparing the Empirical Sorting Models

- 1. Choice Set
- 2. Preference heterogeneity
- 3. Instruments

#### **Policy Evaluations**

$$V(g_{1j}, g_{1,j}, p_j; \alpha_i, y_i) = V(g_{1j}^*, g_{-1,j}, p_j; \alpha_i, y_i - WTP_{PE}).$$

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- 1. using hedonic value models
  - unlikely to hold for large shocks to amenities
  - but restrictions on preferences are less arbitrary (relative to structural models)
- 2. using equilibrium sorting models
  - Closing the model
  - Frictions and dynamics
  - ► Multiple equilibria

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