

# Comparison of most used activation functions in deep neural networks and their circuit realizations in analog and digital neural networks

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**ELEC-L352001: Postgraduate  
Course in Electronic Circuit Design**

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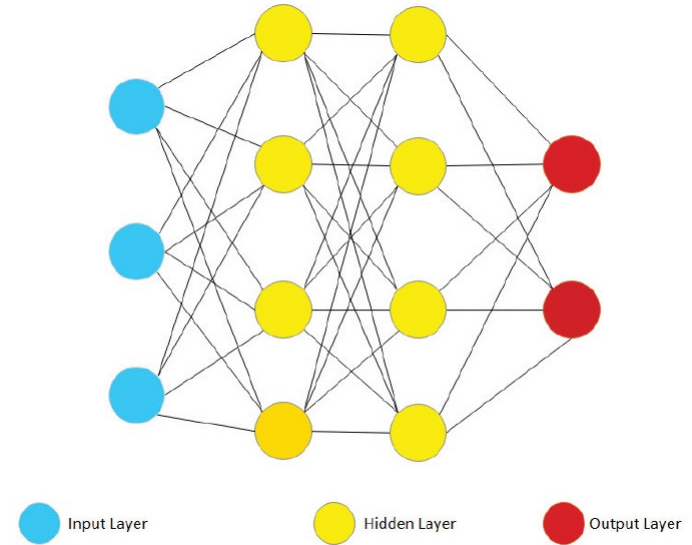


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- **Introduction**
- **Theory**
  - Sigmoid function
  - Hyperbolic tangent
  - Rectified linear units
- **Comparison**
- **Hardware**
- **Assignment**

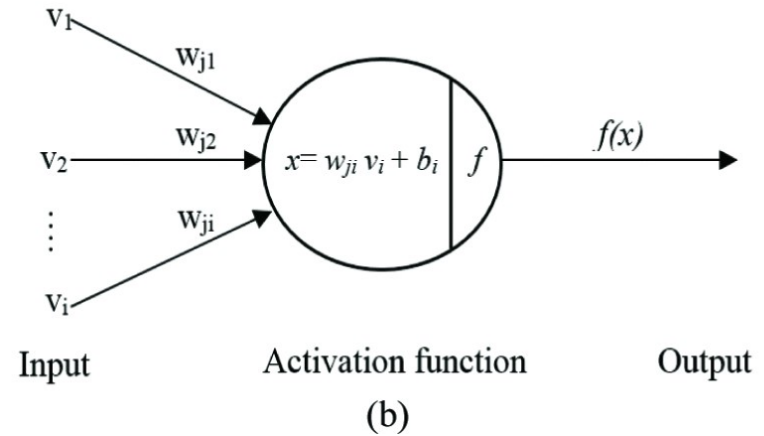
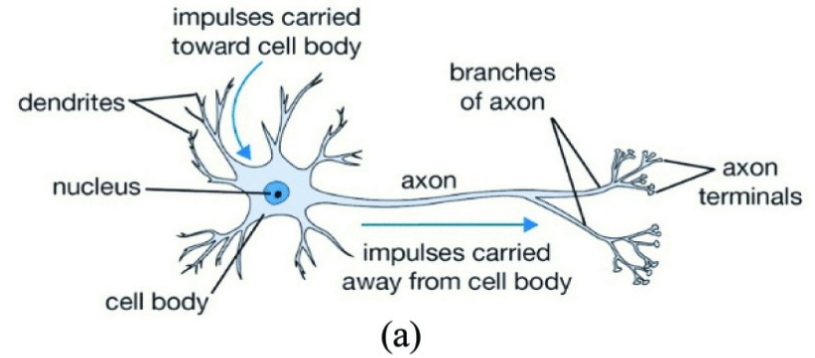
# Introduction

- Main functional elements of neural networks are neurons
- Used for solving complex problems such as pattern classification, clustering, prediction, control and function approximation
- Deep networks have layers between input and output
- More calculations, more layer, more complex



# Neurons

- Inputs are weighted
- Adder sums input data together
- Activation function decides if neuron activated or not
- Activation functions can be linear or non-linear
- Non-linear are used for more complex calculations



# Most used activation functions

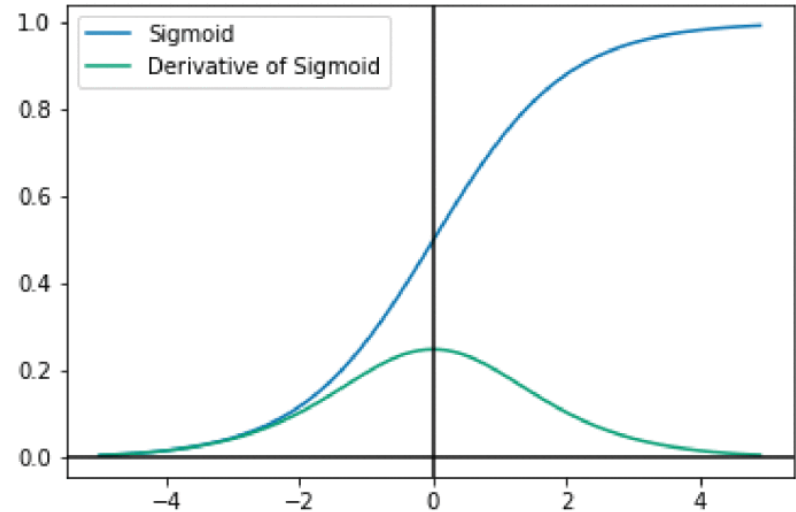
- Sigmoid function
- Hyperbolic tangent
- Rectified linear units

# Theory

# Sigmoid function

- Saturated
- Centered at 0.5
- Gradient vanished especially in deep networks
- Soft saturation results in the difficulties of training a deep neural network
- Not used in deep networks
- Hard to optimize

$$f(x) = \frac{1}{1 + e^{-x}}$$



# Improvements

- **Orthogonal weight initialization can increase performance of sigmoid network**
- **Pre-training**
- **Adding noise to activation function**
- **Hyperbolic tangent**

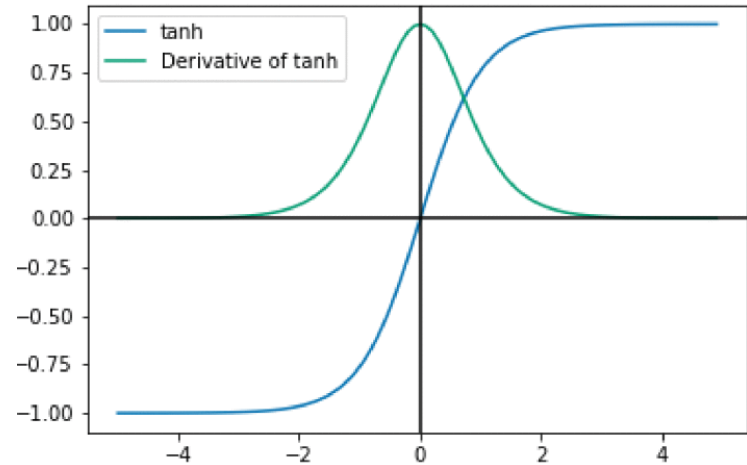


# Hyperbolic tangent

- Saturated
- Centered at zero
- Derivative is steeper
- Faster than sigmoid
- Lower error
- Still vanishes in deep networks

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

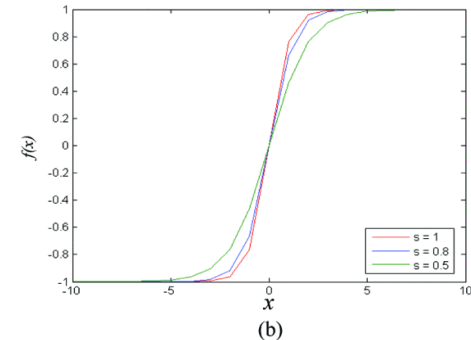
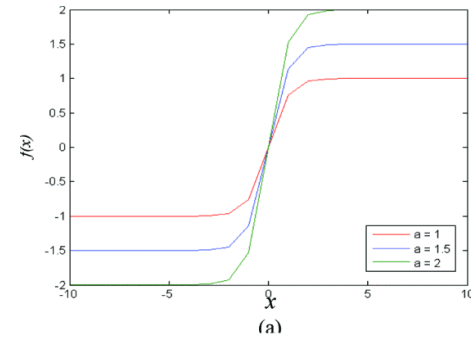
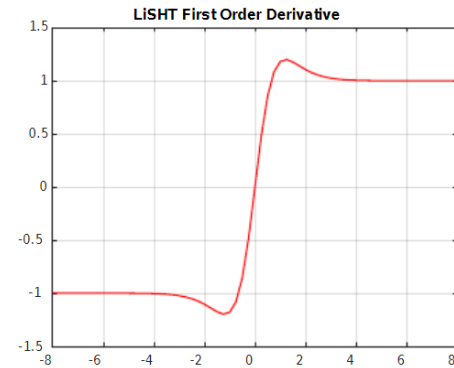
$$\tanh(x) = 2\text{sigmoid}(2x) - 1$$



# Improvements

- Linear scaling to tackle its gradient diminishing problems
- Linearly Scaled Hyperbolic Tangent (LiSHT)
- Adaptive hyperbolic tangent

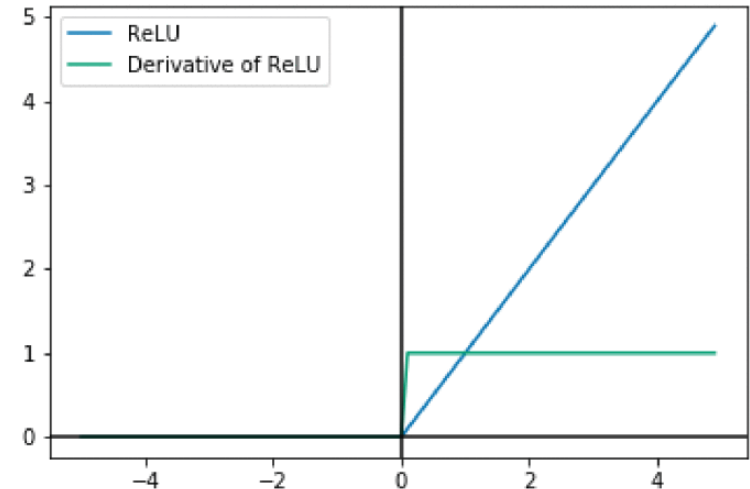
$$f_s(x) = a \frac{\exp(2sx) - 1}{\exp(2sx) + 1}$$



# Rectified linear units

- ReLU
- Most popular
- Simple and fast in training
- Not saturated
- Linear for positive values
- Zero for negative values
- “Dead neuron” or “dying ReLU”
- Bias shift

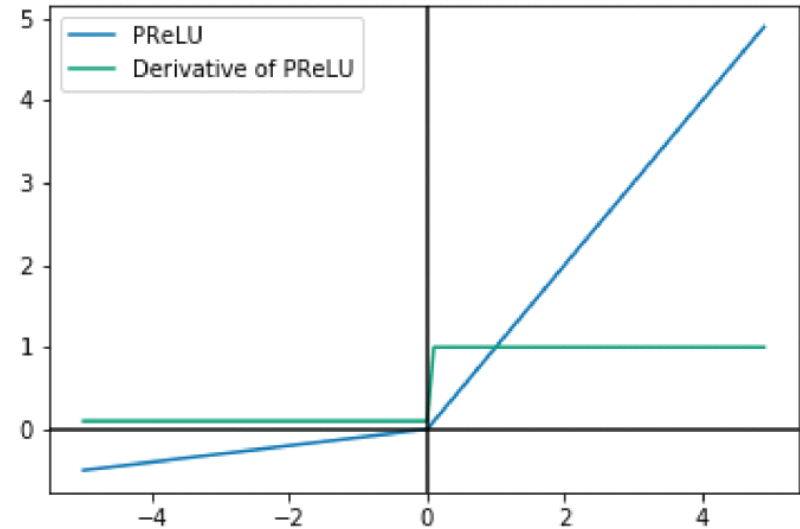
$$\text{ReLU}(x) = \max(0, x)$$



# Leaky ReLU

- Leaky ReLU, LReLU
- Parametric leaky ReLU, PReLU, if  $\alpha$  is learnable
- Attempt to fix “dying ReLU”
- Possible to perform back propagation

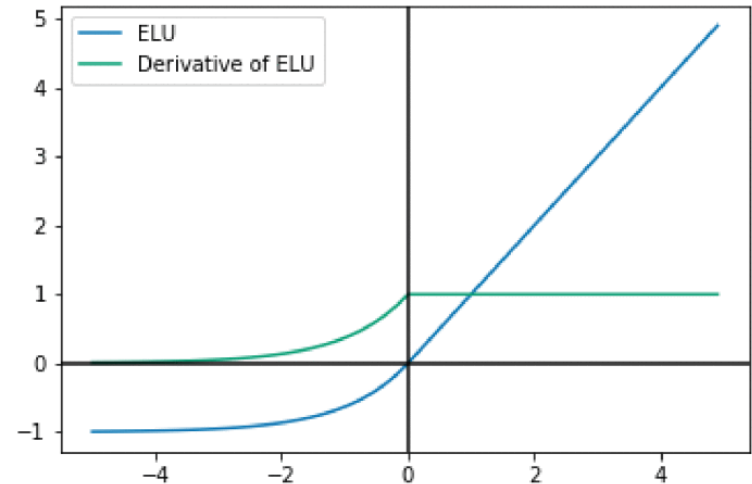
$$\text{PReLU}(x) = \max(0, x) + \alpha * \min(0, x)$$



# Exponential Linear Unit

$$\text{ELU}(x) = \max(0, x) + \min(0, \alpha(e^x - 1))$$

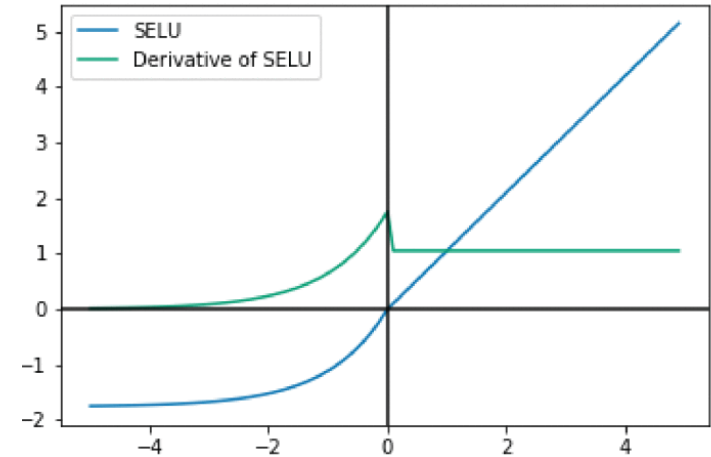
- ELU
- For negative values increases exponentially
- Same benefits as from Leaky ReLU
- Reduces bias shift problem, which is defined as the change of a neuron's mean value due to weights update



# Scaled Exponential Linear Unit

$$\text{SELU}(x) = \gamma * (\max(0, x) + \min(0, \alpha (e^x - 1)))$$

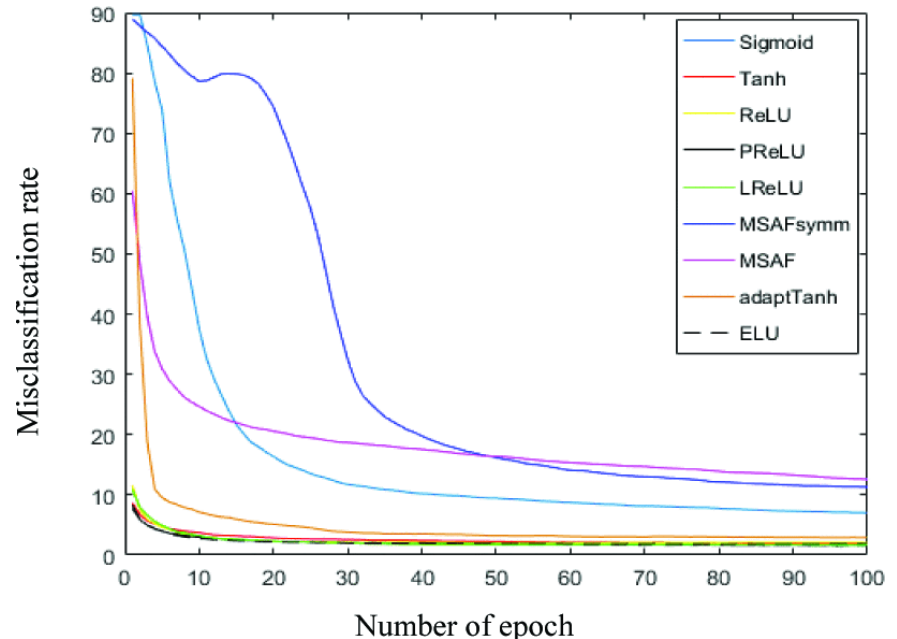
- SELU
- Self-normalizing
- Converges towards zero mean and unit variance even under the presence of noise



# Comparison 1

- M. M. Lau and K. Hann Lim
- Four layers, feedforward
- Initialization for saturated activation functions with small random Gaussian weight initialization
- Unsaturated activation function, the weight initialization were using Xavier weight initialization
- Training: 60000 images  
Testing: 10000 images

Activation Functions	Misclassification rate	Pre-train
Sigmoid	7.01	Yes
Hyperbolic Tangent	1.86	Yes
MSAF	12.59	Yes
MSAF_symmetrical	11.28	Yes
ReLU	2.08	No
LReLU	1.68	No
PReLU	1.6	No
ELU	1.88	No
Adaptive tanh	2.93	No



# Comparison 2

- **B. Ding, H. Qian and J. Zhou**
- **Deep convolutional neural network**
- **Training: 60000 samples**  
**Testing: 10000 samples**

Activation function	Parameter	Error (%)
Sigmoid	-	1.15
Tanh	-	1.12
ReLU	-	0.8
RReLU	$a = 0.5$	0.99
ELU	$\alpha = 1$	1.1

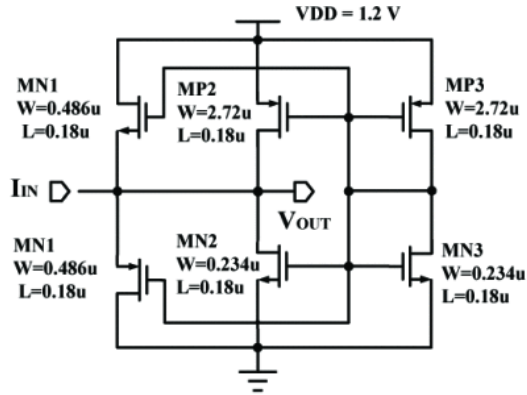


# Hardware

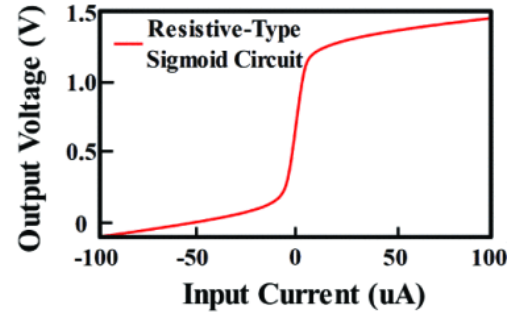
# Hardware implementations

- **May be categorized into three approaches:**
  - Approximation
    - Taylor
    - Piecewise linear
    - Approximation of first derivative
  - Lookup Table (LUT) based
  - Hybrid Approaches

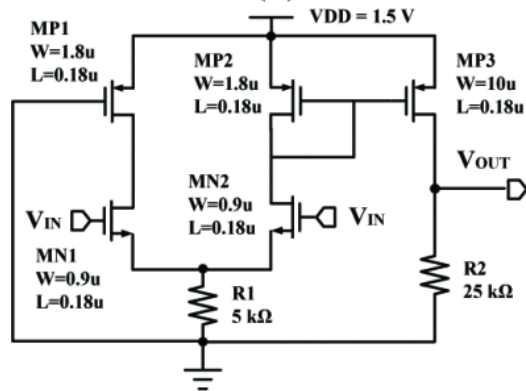
# Sigmoid



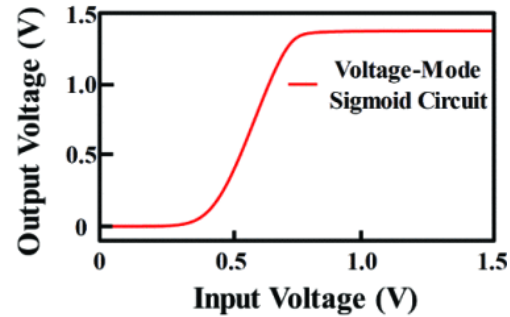
(a)



(b)



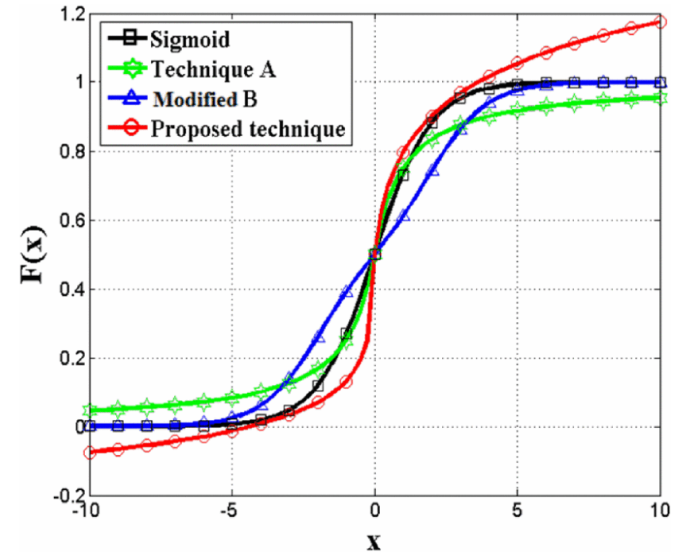
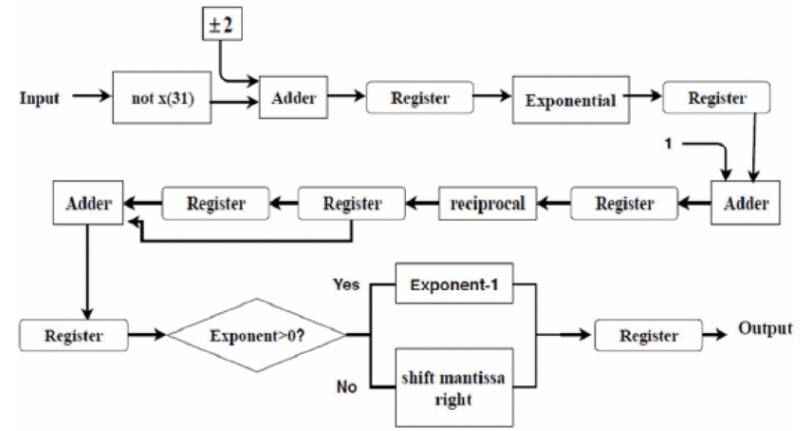
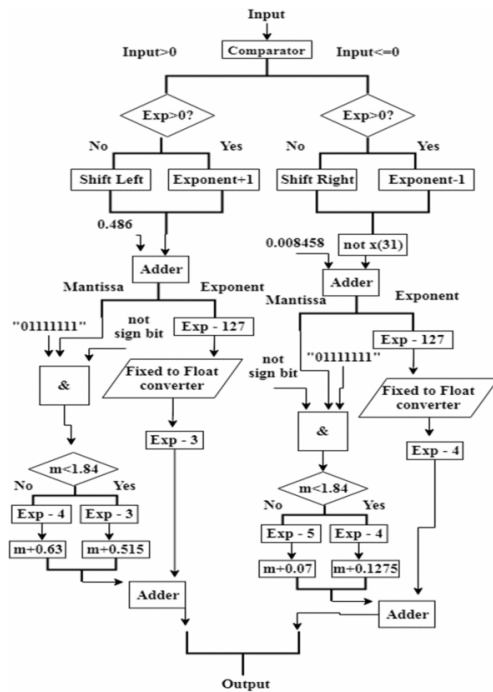
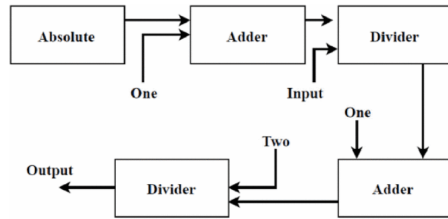
(c)



(d)

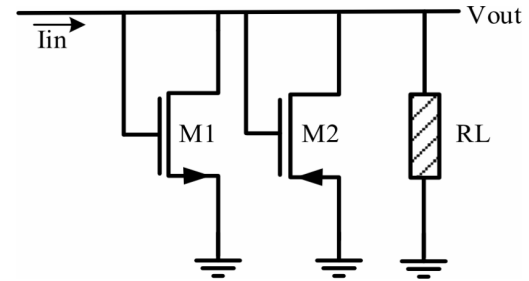


# Sigmoid, digital

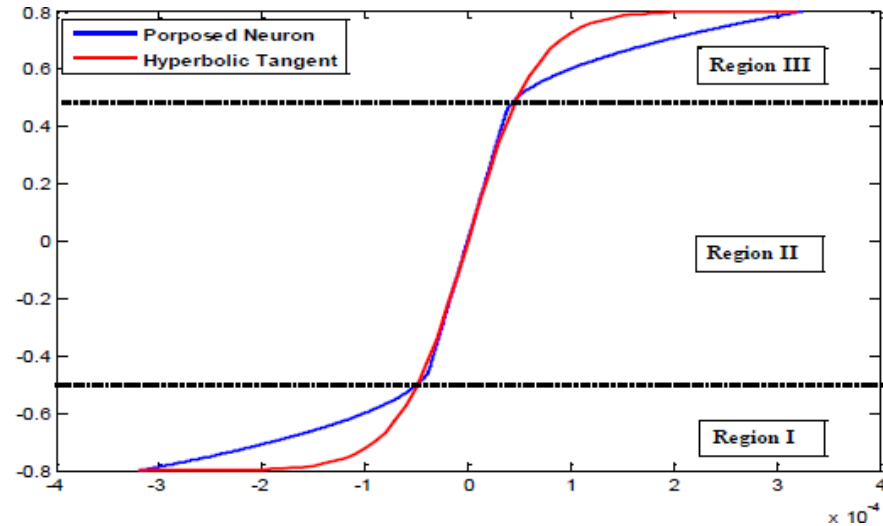


# Hyperbolic tangent

- **Passive resistive**
- **Max error 19.7%**
- **Average error 6.88%**

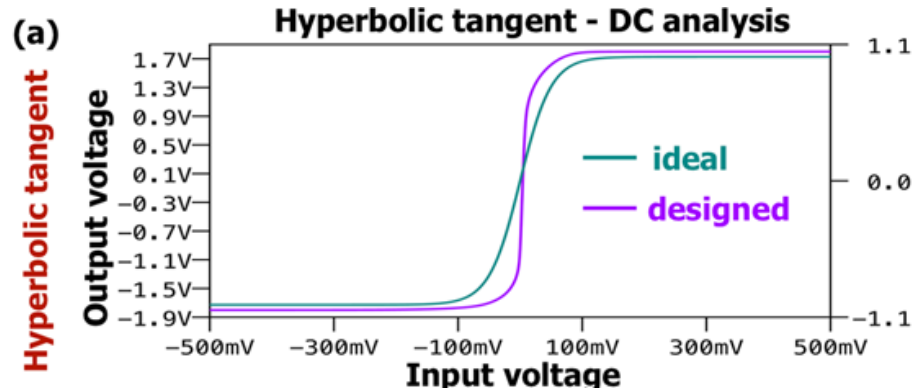
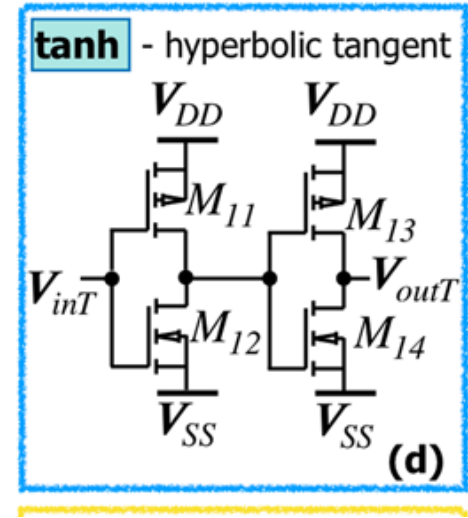


Region	$V_{out}$	$M_1$	$M_2$
I	$V_{out} < -V_{tp}$	OFF	Sat
II	$-V_{tp} < V_{out} < V_{tn}$	OFF	OFF
III	$V_{out} > V_{tn}$	Sat	OFF

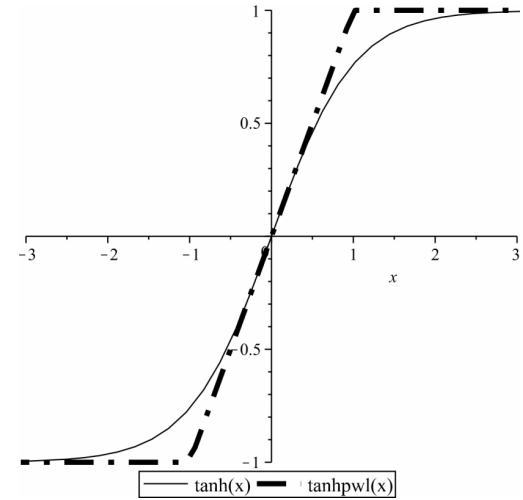
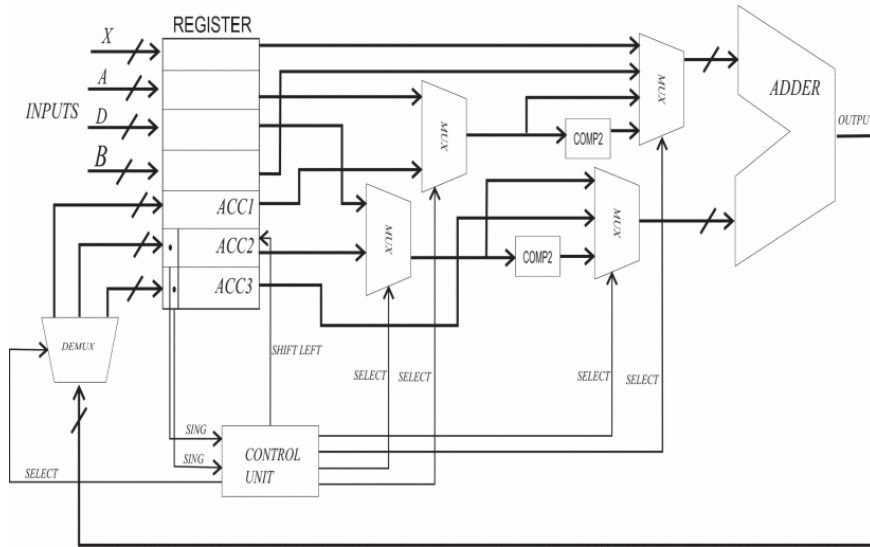


# Hyperbolic tangent

- “Hard” tanh
- Two adjusted inverters
- Small on-chip area and power consumption compared to other traditional tanh



# Hyperbolic tangent, PWL

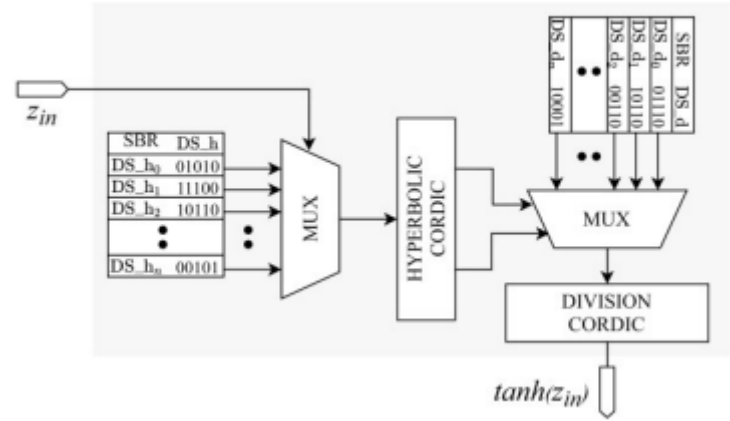


Format	$I$ -bits	$F$ -bits	Range	Max.error	Av.error
(2,6)	2	6	0,1.89750	0.53000	0.31623097
(3,5)	3	5	0,3.98675	0.238405844	0.08753644
(4,4)	4	4	0,7.93750	0.238405844	0.08649234

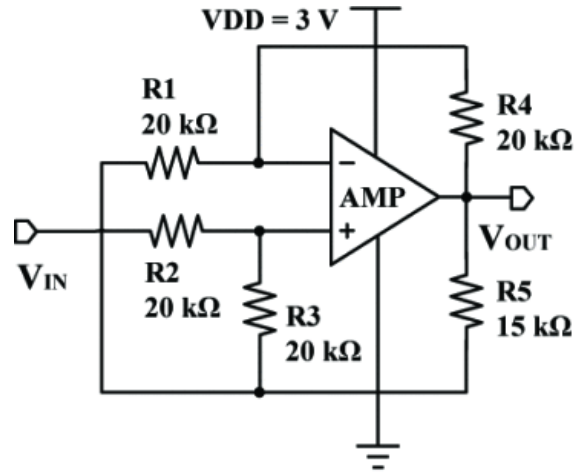


# Hyperbolic tangent, CORDIC

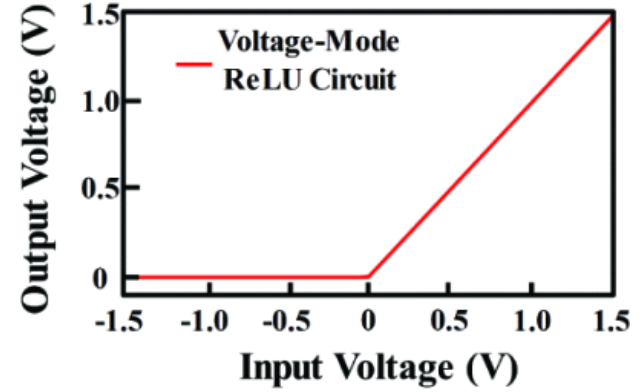
- Coordinate Rotation Digital Computer (CORDIC)
- CORDICs used for example in transmitters



# ReLU



(a)

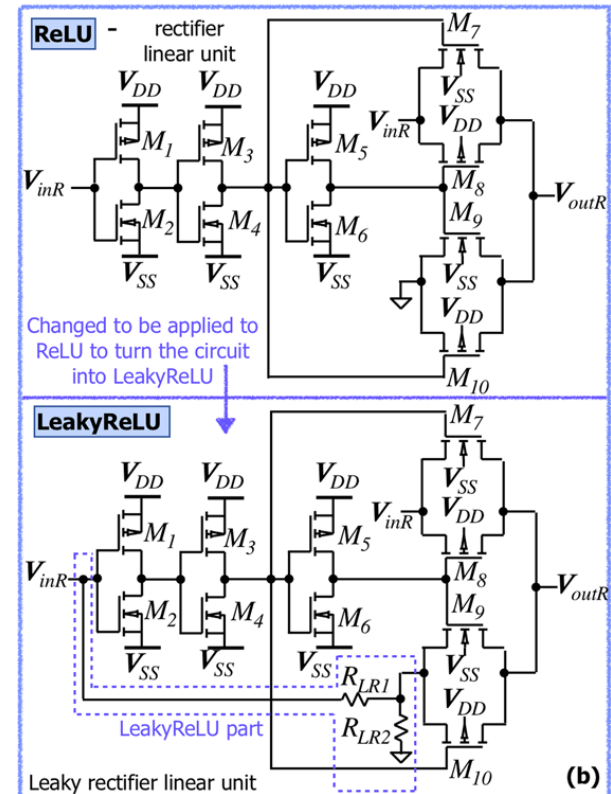
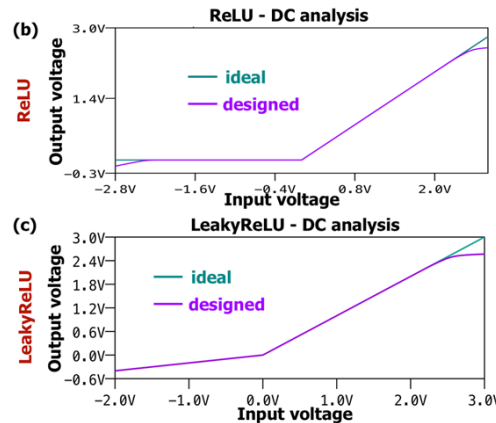


(b)

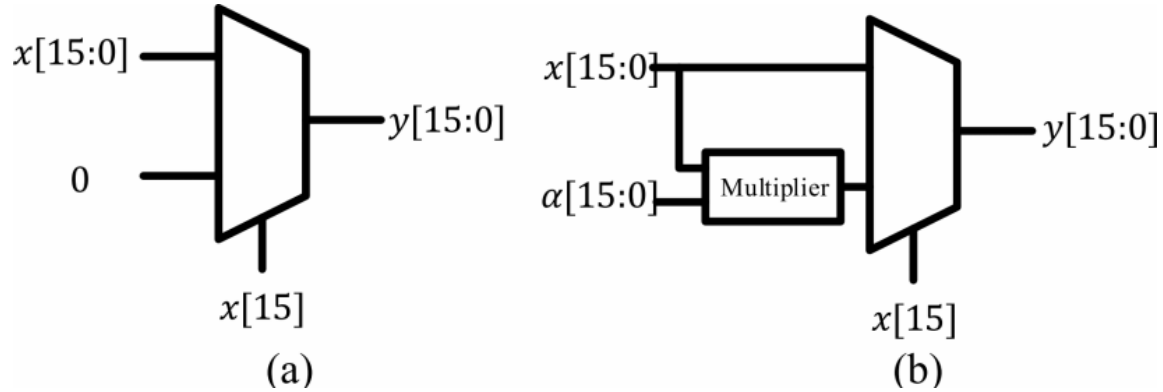
- Voltage-mode
- Due to op amp, good linearity and operating range

# ReLU

- Based on transmission gate (M7-M8 and M9-M10)
- Inverters' threshold voltage is zero
- Adding voltage divider to “negative” transmission gate makes ReLU leaky



# ReLU, digital

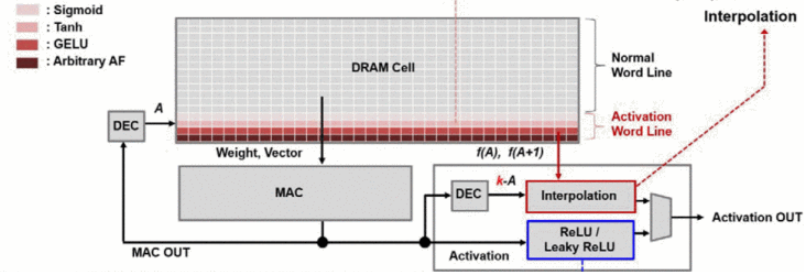
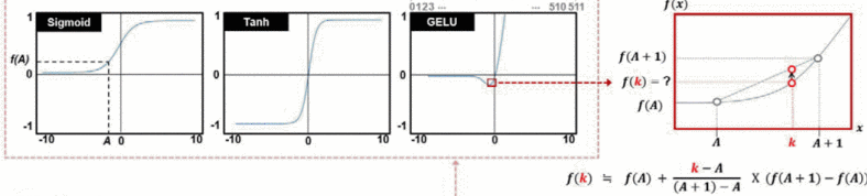


# LUT

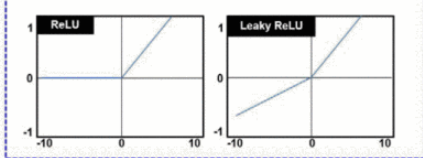
- LUT for every neuron present in the network
- Range addressable LUTs to reduce the LUT size
- A furthermore reduction in LUT is achieved by linearizing the activation function (Hybrid)
- Can have arbitrary activation function
- Simple, faster, and provide reasonable accuracy
- Only involves delay of one-memory access time to output the result, which is less than the usual computation time needed in arithmetic circuits

# LUT

Activation Functions w/ LUT



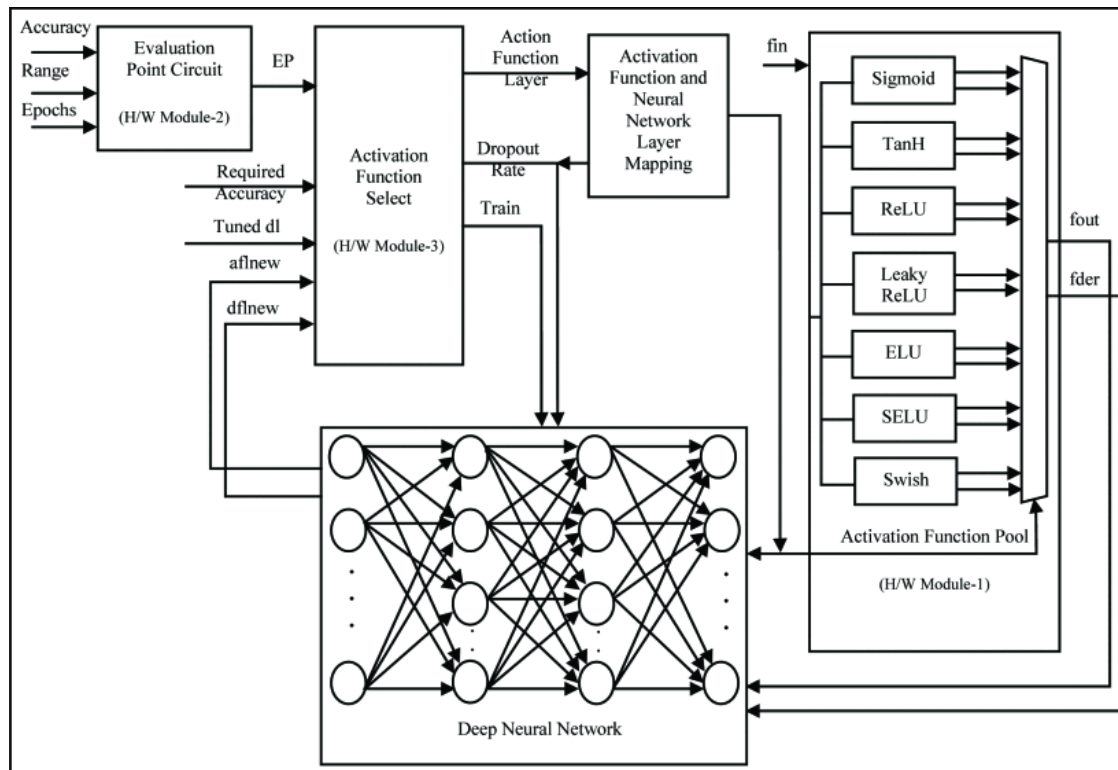
Activation Functions w/ Calculation



## Comparison

	[1]	[2]	[3],[7]	This work
DRAM Type	LPDDR4	DDR4	HBM2	GDDR6
Process	20 nm	2x nm	20 nm	1y nm
Memory Density	8GB/chip (8H 8Gb mono die)	8GB/DIMM	6GB/cube (Buffer die + 4H 4Gb core-die with PCU + 4H 8Gb core-die)	8Gb/chip (4Gb DDP)
Data Rate	3.2Gbps	2.4Gbps	2.4Gbps	16Gbps
Bandwidth	25.6GB/s per chip	19.2GB/s per DIMM	307GB/s per cube	64GB/s per chip
# of Channel	1 per chip	16 per DIMM	8 per cube	2 per chip
# of Processing Unit (PU)	2048 per chip (256 per die)	128 per DIMM (8 per chip)	128 per cube (32 per core-die)	32 per chip (16 per die)
Processing Operation Speed	250MHz	500MHz	300MHz	1GHz
1 PU Throughput	2 GOPS (250MHz x 8byte)	4 GOPS (500MHz x 8byte)	9.6 GFLOPS (300MHz x 32byte)	32 GFLOPS (1GHz x 32byte)
Total Throughput (1 PU Throughput x # of PU)	0.5 TOPS per chip (2 GOPS x 256)	0.5 TOPS per DIMM (4 GOPS x 128)	1.2 TFLOPS per cube (9.6 GFLOPS x 128)	1 TFLOPS per chip (32 GFLOPS x 32)
Operation precision	INT8	INT8	FP16	BF16
Supported Activation Functions	-	-	ReLU	Sigmoid, Tanh, GELU, ReLU, Leaky ReLU, and Arbitrary AF

# LUT



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# Assignment

For each the most used activation function(Sigmoid, tanh, ReLU), find on the Internet example of the application and why that function is chosen over others.