Phys-E0414: Advanced quantum mechanics

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  - 12 lectures (2x45min), Tuesdays 10.15-12.00, Nanotalo 228
  - 12 exercise classes, Thursdays 12.15-14.00
  - 11 homework exercises (hand in via MyCourses)
  - 1 exam (3 hours), December 8, 13.00-16.00

- Exercise classes: 1 exercise sheet w. ~3 problems
  The last exercise is the homework problem

- Grading: 1-5, Exam: 60%, Homework: 40%

- Course material:
  - Zettili: "Quantum Mechanics: Concepts & Applications"
  - other notes & hand-written lecture notes on MyCourses

  (- recorded lectures from 2020 on MyCourses)

- Required background:
  - some quantum mechanics
  - some linear algebra
  - some complex analysis

Course content:

Week 1: Intro to course, history of QM

Week 2: Math of QM, two-level systems,
time-evolution, time-independent
Schrödinger equation

Week 3: Two-level systems (continued)
Bloch sphere, measurements,
time-dependent Schrödinger equation

Week 4: Quantum harmonic oscillator,
coherent states, displacement operator

Week 5: Orbital angular momentum,
Stern-Gerlach experiment, spin

Week 6: Perturbation theory & examples
(WKB method)

Fall break

Week 7: Composite systems, addition of two spins, Pauli principle, exchange coupling

Week 8: Entanglement; creation & detection

Bell and CHSH inequality

Week 9: Qubits, quantum computing, Deutsch's algorithm, Grover's algorithm

Week 10: Quantum communication and quantum teleportation

Week 11: Open quantum systems, density matrices, decoherence

Week 12: Summary of course

History of quantum mechanics (some of the major steps)

1900: Max Planck; theory of black body radiation

1905: Albert Einstein; explanation of the photoelectric effect (+ brownian motion & relativity theory)

1913: Niels Bohr; quantum model of atoms

1918: Max Planck, Nobel Prize (NP)

1921: Albert Einstein, NP

1922: Niels Bohr, NP

1922: Stern-Gerlach experiment, spin of the electron

1924: Louis de Broglie; wave-particle duality of electrons

1925: Wolfgang Pauli; the exclusion principle

1925: Werner Heisenberg; matrix formulation of quantum mechanics

1926: Erwin Schrödinger; wave mechanics & Schrödinger equation

1927: Werner Heisenberg; uncertainty principle

1927: Max Born: probabilistic interpretation of QM

1929: de Broglie, NP

(1931): (Kurt Gödel: incompleteness theorem)

1932: Werner Heisenberg, NP

1933: Schrödinger, NP

1933: Otto Stern: measures the magnetic moment of the proton

1935: Einstein, Podolsky, and Rosen: EPR paradox

(1936: Alan Turing: Describes the Turing Machine of computation)

1940: Pauli: proves the spin-statistics theorem

1943: Otto Stern, NP

1944: Pauli, NP.

→ The QM that we know today was more-or-less developed by WWII.

1950

Nuclear Physics

Quantum optics

Condensed matter physics

High-energy physics

→ Lasers, semi-conductor technology, electronics, computers, ...

1964: Bell inequality

1969: CSHS — " —

1980

1982: Test of Bell inequality (Aspect)

Deutsch algorithm (1985)

Shor's algorithm (1994)

Grover's algorithm (1996)

2015 →

Quantum engineering and technology

IBM, Google, Microsoft, D-Wave, ...

~ 4-15 qubits

~ simple quantum algorithms have been implemented

↓

Quantum cryptography

Quantum communication

~ 2025? Quantum simulation

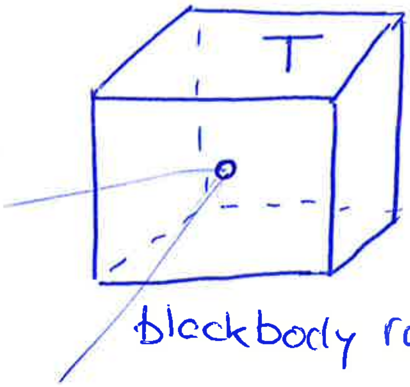
Quantum computation

Quantum sensing

↓

Quantum future!

Planck's radiation law:

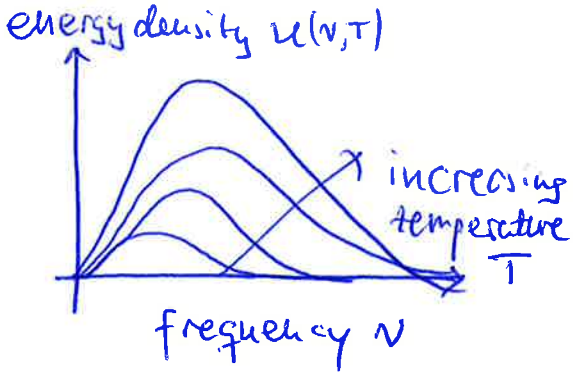


blackbody radiation

- Wien's energy distribution (1884):

$$u(\nu,T) = A\nu^3 e^{-B\nu/T}$$

- Fits well at large frequencies.



- Rayleigh-Jeans law (1900):

Electromagnetic radiation inside the cavity are standing waves that can be described as harmonic oscillators

Number of modes between ν and $\nu+d\nu$:
(Density)

$$N(\nu) = \frac{8\pi}{c^3}\nu^2$$ [← calculate in statistical mechanics]

Energy density:

$$u(\nu,T) = N(\nu)\langle E\rangle = \frac{8\pi}{c^3}\nu^2\langle E\rangle$$

Average energy:

$$\langle E\rangle = \int_0^\infty dE\, E\, p(E) = \int_0^\infty dE\, E\, \frac{e^{-\beta E}}{\int_0^\infty dE\, e^{-\beta E}}$$

with $\beta = 1/k_BT$ and k_B is Boltzmann's constant (determined by Planck!)

Notice that

$$\langle E \rangle = -\partial_\beta \ln Z,$$

where $Z = \int_0^\infty dE\, e^{-\beta E}$ is the partition function:

$$\langle E \rangle = -\partial_\beta \ln Z = \frac{-\partial_\beta Z}{Z} = \frac{\int_0^\infty dE\, E e^{-\beta E}}{\int_0^\infty dE\, e^{-\beta E}}$$

Now, $Z = \int_0^\infty dE\, e^{-\beta E} = \frac{1}{-\beta}\left[e^{-\beta E}\right]_0^\infty = \frac{1}{\beta} = k_B T$

and $\langle E \rangle = -\partial_\beta \ln \frac{1}{\beta} = \partial_\beta \ln \beta = \frac{1}{\beta} = k_B T$ //

$$\Rightarrow u(\nu, T) = \frac{8\pi}{c^3} \nu^2 k_B T \; //$$

Fits at low frequencies, but diverges at high frequencies $\rightsquigarrow$ the ultraviolet catastrophe

- Planck's distribution:

Planck postulated that the energy of each oscillator is quantized such that

$$E_n = n h \nu, \quad n = 0, 1, 2, \ldots$$

are the only allowed energies. Here, h is known as Planck's constant.

Planck's postulate changes the average energy:

$$\langle E \rangle = \sum_{n=0}^{\infty} E_n \, p(E_n) = \sum_{n=0}^{\infty} E_n \frac{e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}}$$

$$= -\partial_\beta \ln Z \quad \text{with } Z = \sum_{n=0}^{\infty} e^{-\beta E_n}$$

We find that

$$Z = \sum_{n=0}^{\infty} \left(e^{-\beta h\nu}\right)^n = \frac{1}{1-e^{-\beta h\nu}} //$$

thus,

$$\langle E \rangle = -\partial_\beta \ln Z = \partial_\beta \ln\left(1-e^{-\beta h\nu}\right)$$

$$= \frac{h\nu e^{-\beta h\nu}}{1-e^{-\beta h\nu}} = \frac{h\nu}{e^{\beta h\nu}-1}$$

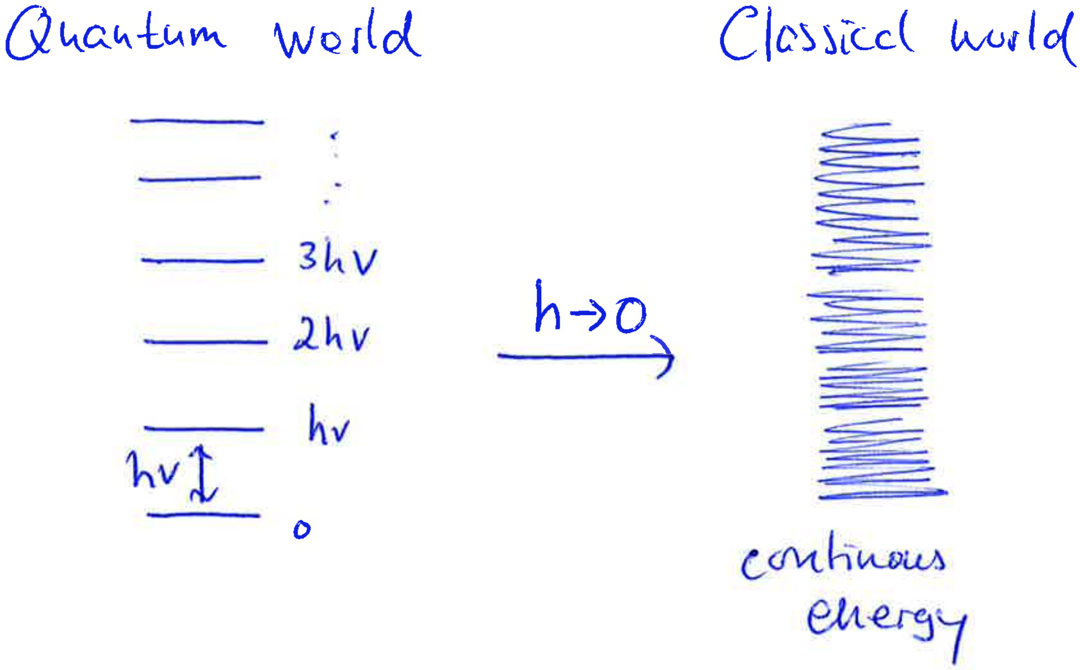
$$\Rightarrow \boxed{u(\nu,T) = \frac{8\pi}{c^3}\nu^2 \frac{h\nu}{e^{\beta h\nu}-1}}$$

Planck's radiation law works!

But why is energy quantized?

$\longrightarrow$ quantum mechanics

How do we recover classical physics?



$h = 6.626 \times 10^{-34}$ Js (Planck's constant)

$k_B = 1.3807 \times 10^{-23}$ J/K (Boltzmann's constant)

$S = k_B \ln \Omega$

Other natural constants: $G, e, \varepsilon_0, \dots$

Comparison of scales: L

$\frac{L}{d_{moon}} \ll 1$ $\qquad$ $\frac{L}{d_{atom}} \gg 1$

We recover the classical world, when the energy quantization is small compared to the thermal energy $k_B T$, i.e.

$$\varepsilon \equiv \frac{h\nu}{k_B T} = \beta h\nu \ll 1 \quad (\text{equivalent to } h \to 0!)$$

From Planck's law, we get

$$u(\nu,T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\beta h\nu}-1}$$

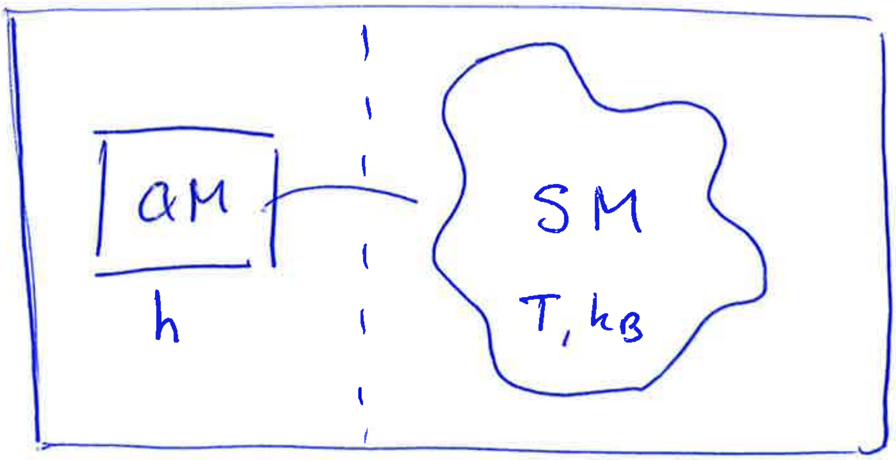
$$\overset{\beta h\nu \ll 1}{\simeq} \frac{8\pi\nu^2}{c^3}\frac{h\nu}{1+\beta h\nu-1}$$

$$= \frac{8\pi\nu^2}{c^3}\frac{1}{\beta} = \frac{8\pi\nu^2}{c^3}k_B T$$

→ Rayleigh-Jeans law

and h has disappeared!

→ classical limit



In quantum mechanics, we use $h \to 0$,

and we need statistical mechanics to take $\beta h\nu \ll 1$.