Problem 1: Zero sequence and the space vector

Consider the phase voltages

$$u_\mathrm{a} = u'_\mathrm{a} + u_0 \qquad u_\mathrm{b} = u'_\mathrm{b} + u_0 \qquad u_\mathrm{c} = u'_\mathrm{c} + u_0$$

where $u'_\mathrm{a} + u'_\mathrm{b} + u'_\mathrm{c} = 0$ holds and u_0 is the zero-sequence component. Show that the zero sequence disappears in the space-vector transformation.

Solution

Let us insert the phase voltages into the space-vector transformation:

$$\begin{aligned}
\boldsymbol{u}_\mathrm{s}^\mathrm{s} &= \frac{2}{3}\left(u_\mathrm{a} + u_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + u_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right) \\
&= \frac{2}{3}\left[(u'_\mathrm{a} + u_0) + (u'_\mathrm{b} + u_0)\mathrm{e}^{\mathrm{j}2\pi/3} + (u'_\mathrm{c} + u_0)\mathrm{e}^{\mathrm{j}4\pi/3}\right] \\
&= \frac{2}{3}\left(u'_\mathrm{a} + u'_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + u'_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right) + \frac{2}{3}\underbrace{\left(1 + \mathrm{e}^{\mathrm{j}2\pi/3} + \mathrm{e}^{\mathrm{j}4\pi/3}\right)}_{=0} u_0 \\
&= \frac{2}{3}\left(u'_\mathrm{a} + u'_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + u'_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right)
\end{aligned}$$

It is geometrically clear that $1+\mathrm{e}^{\mathrm{j}2\pi/3}+\mathrm{e}^{\mathrm{j}4\pi/3} = 0$ holds (draw these three unit vectors to see it). Hence, the zero sequence does not affect the space vector.

Problem 2: Synchronous machine model in rotor coordinates

(a) Equations for the stator voltage and stator flux linkage in stator coordinates are

$$\boldsymbol{u}_\mathrm{s}^\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}^\mathrm{s}}{\mathrm{d}t} \qquad \boldsymbol{\psi}_\mathrm{s}^\mathrm{s} = L_\mathrm{s}\boldsymbol{i}_\mathrm{s}^\mathrm{s} + \psi_\mathrm{f}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$$

Express these equations in rotor coordinates.

(b) Express the previous equations in rotor coordinates in steady state.

(c) Starting from

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\}$$

derive the torque expression in rotor coordinates as a function of i_d and i_q.

Solution

(a) Inserting $\boldsymbol{u}_\mathrm{s}^\mathrm{s} = \boldsymbol{u}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$, $\boldsymbol{i}_\mathrm{s}^\mathrm{s} = \boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$, and $\boldsymbol{\psi}_\mathrm{s}^\mathrm{s} = \boldsymbol{\psi}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$ into the stator voltage equation leads to

$$\begin{aligned}
\boldsymbol{u}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} &= R_\mathrm{s}\boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{\psi}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}\right) \\
&= R_\mathrm{s}\boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} + \boldsymbol{\psi}_\mathrm{s}\frac{\mathrm{d}\,\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}}{\mathrm{d}t} \\
&= R_\mathrm{s}\boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} + \boldsymbol{\psi}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}\frac{\mathrm{d}(\mathrm{j}\vartheta_\mathrm{m})}{\mathrm{d}t}
\end{aligned}$$

Dividing the both sides of the equation by $\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$ and noting that $\mathrm{d}\vartheta_\mathrm{m}/\mathrm{d}t = \omega_\mathrm{m}$, the voltage equation in rotor coordinates becomes

$$\boldsymbol{u}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \frac{\mathrm{d}\boldsymbol{\psi}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{m}\boldsymbol{\psi}_\mathrm{s} \tag{1}$$

Inserting $\boldsymbol{\psi}_\mathrm{s}^\mathrm{s} = \boldsymbol{\psi}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$ and $\boldsymbol{i}_\mathrm{s}^\mathrm{s} = \boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$ into the flux linkage equation gives

$$\boldsymbol{\psi}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} = L_\mathrm{s}\boldsymbol{i}_\mathrm{s}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}} + \psi_\mathrm{f}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$$

Dividing the both sides by $\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$ gives the equation in rotor coordinates,

$$\boldsymbol{\psi}_\mathrm{s} = L_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \psi_\mathrm{f} \tag{2}$$

(b) In rotor coordinates, the vectors are constant in steady state and $\mathrm{d}/\mathrm{d}t = 0$ holds. The voltage equation (1) reduces to

$$\boldsymbol{u}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}\boldsymbol{\psi}_\mathrm{s}$$

The flux equation (2) holds in steady state as it is.

(c) Inserting the stator flux linkage in (2) into the torque expressions yields

$$\begin{aligned}\tau_\mathrm{M} &= \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}\boldsymbol{\psi}_\mathrm{s}^*\right\} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\boldsymbol{i}_\mathrm{s}(L_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \psi_\mathrm{f})^*\right\} \\ &= \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{L_\mathrm{s}\boldsymbol{i}_\mathrm{s}\boldsymbol{i}_\mathrm{s}^* + \psi_\mathrm{f}\boldsymbol{i}_\mathrm{s}\right\} = \frac{3n_\mathrm{p}}{2}\,\mathrm{Im}\left\{\psi_\mathrm{f}(i_\mathrm{d} + \mathrm{j}i_\mathrm{q})\right\} \\ &= \frac{3n_\mathrm{p}}{2}\psi_\mathrm{f}i_\mathrm{q}\end{aligned}$$

It can be seen that the torque is proportional to i_q while i_d does not contribute to the torque at all.

Problem 3: Operating points of a permanent-magnet synchronous motor

The datasheet values for a three-phase permanent-magnet synchronous motor are:

maximum continuous torque	15 Nm @ 2 400 r/min
voltage constant	0.159 V/(r/min)
number of pole pairs	$n_\mathrm{p} = 4$
stator inductance	$L_\mathrm{s} = 4.86$ mH
stator resistance	$R_\mathrm{s} = 0.46\ \Omega$

(a) The motor rotates at the speed of 2 400 r/min. Calculate the mechanical angular speed, electrical angular speed, and supply frequency.

(b) Calculate the peak-valued phase-to-neutral back-emf induced by the permanent magnets, when the motor rotates at 2 400 r/min. Calculate also the permanent-magnet flux constant ψ_f.

(c) The torque is 15 Nm. Calculate the output power of the motor at the speed of 2 400 r/min and at zero speed.

(d) The control principle $i_\mathrm{d} = 0$ is used. Calculate the stator current $\boldsymbol{i}_\mathrm{s}$ and the stator voltage $\boldsymbol{u}_\mathrm{s}$ in the following operating points: 1) torque is 15 Nm at 2 400 r/min; 2) torque is 15 Nm at zero speed; and 3) no load at 2 400 r/min.

Solution

(a) The mechanical angular speed of the rotor is

$$\omega_\mathrm{M} = 2\pi n = 2\pi \cdot \frac{2\,400 \text{ r/min}}{60 \text{ s/min}} = 2\pi \cdot 40 \text{ rad/s}$$

The electrical angular speed is

$$\omega_\mathrm{m} = n_\mathrm{p}\omega_\mathrm{M} = 4 \cdot 2\pi \cdot 40 \text{ rad/s} = 2\pi \cdot 160 \text{ rad/s}$$

The supply frequency is

$$f = n_\mathrm{p} n = 4 \cdot \frac{2\,400 \text{ r/min}}{60 \text{ s/min}} = 160 \text{ Hz}$$

(b) The steady-state voltage equation can be represented as

$$\begin{aligned} \boldsymbol{u}_\mathrm{s} &= R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}\boldsymbol{\psi}_\mathrm{s} = R_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}(L_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \psi_\mathrm{f}) \\ &= (R_\mathrm{s} + \mathrm{j}\omega_\mathrm{m}L_\mathrm{s})\boldsymbol{i}_\mathrm{s} + \boldsymbol{e}_\mathrm{s} \end{aligned} \tag{3}$$

where the last term $\boldsymbol{e}_\mathrm{s} = \mathrm{j}\omega_\mathrm{m}\psi_\mathrm{f}$ is the back-emf induced by the permanent magnets. The back-emf depends only on the speed (but not on the current).

Unless otherwise noted, the voltage values given in the datasheets and name-plates refer to rms-valued line-to-line voltages. Taking this into account, the peak-valued phase-to-neutral back-emf at the given speed is

$$|\boldsymbol{e}_\mathrm{s}| = \sqrt{\frac{2}{3}} \cdot 0.159 \ \frac{\text{V}}{\text{r/min}} \cdot 2\,400 \text{ r/min} = 311.6 \text{ V}$$

The flux constant (or the permanent-magnet flux linkage) is

$$\psi_\mathrm{f} = \frac{|\boldsymbol{e}_\mathrm{s}|}{\omega_\mathrm{m}} = \frac{311.6 \text{ V}}{2\pi \cdot 160 \text{ rad/s}} = 0.31 \text{ Vs}$$

It is worth noticing that the flux constant does not depend on the speed (i.e., the same value for ψ_f would be obtained at any other speeds).

Remark: In order to be able to use the standard equations and equivalent circuits, it is a recommended practice to transform all parameter values to SI units and line-to-line voltages to line-to-neutral voltages (as we did here). The results of the calculations can then be transformed back to the required form.

(c) At the speed of 2 400 r/min, the output power (mechanical power) is

$$p_\mathrm{M} = \tau_\mathrm{M}\omega_\mathrm{M} = 15 \text{ Nm} \cdot 2\pi \cdot 40 \text{ rad/s} = 3.77 \text{ kW}$$

At zero speed, the mechanical power is zero (but the power fed to the stator is positive due to the losses).

(d) The q-axis current i_q can be solved from the torque expression:

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{f} i_\mathrm{q} \qquad \Rightarrow \qquad i_\mathrm{q} = \frac{2}{3n_\mathrm{p}\psi_\mathrm{f}}\tau_\mathrm{M}$$

Due to the control principle $i_\mathrm{d} = 0$, the stator current vector is $\boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = \mathrm{j}i_\mathrm{q}$. The stator current is proportional to the torque and independent of the speed.

The stator voltage $\boldsymbol{u}_\mathrm{s}$ can be calculated from (3) using the previously calculated values for ω_m and $\boldsymbol{i}_\mathrm{s}$. The results are collected in the table below.

	$\omega_\mathrm{m} = 0$	$\omega_\mathrm{m} = 2\pi \cdot 160$ rad/s
$\tau_\mathrm{M} = 0$	$\boldsymbol{i}_\mathrm{s} = 0 + \mathrm{j}0$ $\boldsymbol{u}_\mathrm{s} = 0 + \mathrm{j}0$	$\boldsymbol{i}_\mathrm{s} = 0 + \mathrm{j}0$ $\boldsymbol{u}_\mathrm{s} = 0 + \mathrm{j}311.6$ V
$\tau_\mathrm{M} = 15$ Nm	$\boldsymbol{i}_\mathrm{s} = 0 + \mathrm{j}8.1$ A $\boldsymbol{u}_\mathrm{s} = 0 + \mathrm{j}3.7$ V	$\boldsymbol{i}_\mathrm{s} = 0 + \mathrm{j}8.1$ A $\boldsymbol{u}_\mathrm{s} = -39.4 + \mathrm{j}315.4$ V

Remark: The vector diagram for a positive torque and a positive speed is illustrated in the figure below. The stator resistance $R_\mathrm{s} = 0$ is assumed for simplicity. Could you sketch the vectors in the previous operating points. What would happen, if the speed or the torque were reversed?