Problem 1: Space-vector components from line-to-line voltages

Line-to-line voltages u_{ab} and u_{bc} are known. Calculate u_α and u_β.

Solution

The voltage space vector is

$$\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}} = \frac{2}{3}\left(u_{\mathrm{a}} + u_{\mathrm{b}}\mathrm{e}^{\mathrm{j}2\pi/3} + u_{\mathrm{c}}\mathrm{e}^{\mathrm{j}4\pi/3}\right) = u_\alpha + \mathrm{j}u_\beta$$

where u_{a}, u_{b}, and u_{c} are the phase voltages Since the zero-sequence component does not affect the space vector, we can freely add an arbitrary voltage u_0 to each phase voltage without affecting the space vector. Let us denote the modified voltages as $u'_{\mathrm{a}} = u_{\mathrm{a}} + u_0$, $u'_{\mathrm{b}} = u_{\mathrm{b}} + u_0$, and $u'_{\mathrm{c}} = u_{\mathrm{c}} + u_0$. We can select $u_0 = -u_{\mathrm{b}}$ and then apply the space-vector transformation:

$$\begin{aligned}
\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}} &= \frac{2}{3}\left(u'_{\mathrm{a}} + u'_{\mathrm{b}}\mathrm{e}^{\mathrm{j}2\pi/3} + u'_{\mathrm{c}}\mathrm{e}^{\mathrm{j}4\pi/3}\right) \\
&= \frac{2}{3}\left[(u_{\mathrm{a}} - u_{\mathrm{b}}) + (u_{\mathrm{b}} - u_{\mathrm{b}})\mathrm{e}^{\mathrm{j}2\pi/3} + (u_{\mathrm{c}} - u_{\mathrm{b}})\mathrm{e}^{\mathrm{j}4\pi/3}\right] \\
&= \frac{2}{3}\left(u_{\mathrm{ab}} - u_{\mathrm{bc}}\mathrm{e}^{\mathrm{j}4\pi/3}\right) \\
&= \frac{2}{3}u_{\mathrm{ab}} + \frac{1}{3}u_{\mathrm{bc}} + \mathrm{j}\frac{1}{\sqrt{3}}u_{\mathrm{bc}}
\end{aligned}$$

Hence, the components are $u_\alpha = (2u_{\mathrm{ab}} + u_{\mathrm{bc}})/3$ and $u_\beta = u_{\mathrm{bc}}/\sqrt{3}$.

Problem 2: Inverse transformation

The inverse space-vector transformations are

$$u_{\mathrm{a}} = \mathrm{Re}\left\{\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}}\right\} \qquad u_{\mathrm{b}} = \mathrm{Re}\left\{\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}}\mathrm{e}^{-\mathrm{j}2\pi/3}\right\} \qquad u_{\mathrm{c}} = \mathrm{Re}\left\{\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}}\mathrm{e}^{-\mathrm{j}4\pi/3}\right\}$$

Let us consider the phase b as an example here. Show that the above expression for the phase voltage u_{b} holds.

Solution

The space vector is

$$\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}} = \frac{2}{3}\left(u_{\mathrm{a}} + u_{\mathrm{b}}\mathrm{e}^{\mathrm{j}2\pi/3} + u_{\mathrm{c}}\mathrm{e}^{\mathrm{j}4\pi/3}\right)$$

Multiplying both sides by $\mathrm{e}^{-\mathrm{j}2\pi/3}$ gives

$$\begin{aligned}
\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}}\mathrm{e}^{-\mathrm{j}2\pi/3} &= \frac{2}{3}\left(u_{\mathrm{b}} + u_{\mathrm{a}}\mathrm{e}^{-\mathrm{j}2\pi/3} + u_{\mathrm{c}}\mathrm{e}^{\mathrm{j}2\pi/3}\right) \\
&= \frac{2}{3}\left\{u_{\mathrm{b}} + u_{\mathrm{a}}\left[\cos\left(-\frac{2\pi}{3}\right) + \mathrm{j}\sin\left(-\frac{2\pi}{3}\right)\right] + u_{\mathrm{c}}\left[\cos\left(\frac{2\pi}{3}\right) + \mathrm{j}\sin\left(\frac{2\pi}{3}\right)\right]\right\}
\end{aligned}$$

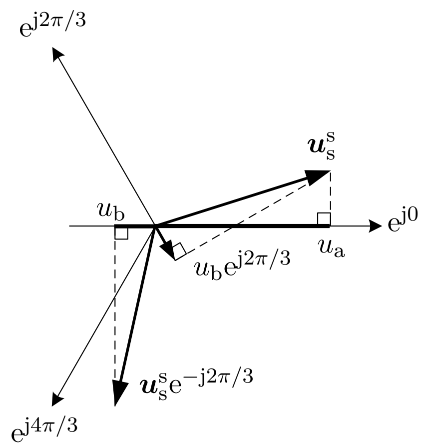
Taking the real part gives

$$\mathrm{Re}\left\{\boldsymbol{u}_{\mathrm{s}}^{\mathrm{s}}\mathrm{e}^{-\mathrm{j}2\pi/3}\right\} = \frac{2}{3}\left[u_{\mathrm{b}} + u_{\mathrm{a}}\cos\left(\frac{2\pi}{3}\right) + u_{\mathrm{c}}\cos\left(\frac{2\pi}{3}\right)\right] = \frac{2}{3}\left[u_{\mathrm{b}} - \frac{1}{2}(u_{\mathrm{a}} + u_{\mathrm{c}})\right]$$

since $\cos(2\pi/3) = -1/2$. Since the space vector does not include the zero-sequence component, we can use $u_\mathrm{a} + u_\mathrm{b} + u_\mathrm{c} = 0$, giving $u_\mathrm{a} + u_\mathrm{c} = -u_\mathrm{b}$. Hence, the inverse transformation is

$$u_\mathrm{b} = \mathrm{Re}\left\{\boldsymbol{u}_\mathrm{s}^\mathrm{s} \mathrm{e}^{-\mathrm{j}2\pi/3}\right\}$$

Remark: The projection of the vector on the direction of the phase b (having the angle of $2\pi/3$) is illustrated in the figure below. It is worth noticing that the voltage u_b is negative in the figure.



Problem 3: Field weakening

Consider a three-phase four-pole permanent-magnet synchronous motor. The stator inductance is $L_\mathrm{s} = 0.035$ H and the stator resistance can be assumed to be zero. The permanent magnets induce the rated voltage of 400 V at the rotational speed of 1 500 r/min. The rated current is 7.3 A.

(a) The control principle $i_\mathrm{d} = 0$ is used. The motor is operated at the rated voltage and current. Calculate the rotational speed, torque, and mechanical power.

(b) The motor is driven in the field-weakening region at the rated voltage and current. The speed is increased until the absolute values of i_d and i_q are equal. Calculate the rotational speed, torque, and mechanical power.

Draw also the vector diagrams.

Solution

The peak-valued quantities will be used. The rated current is $i_\mathrm{N} = \sqrt{2}\cdot 7.3\ \mathrm{A} = 10.3$ A and the rated line-to-neutral voltage is $u_\mathrm{N} = \sqrt{2/3}\cdot 400\ \mathrm{V} = 326.6$ V. It is known that the induced voltage is $|\boldsymbol{e}_\mathrm{s}| = u_\mathrm{N}$ at the electrical angular speed

$$\omega_\mathrm{m} = 2\pi n_\mathrm{p} n = 2\pi \cdot 2 \cdot \frac{1\,500\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 2\pi \cdot 50\ \mathrm{rad/s}$$

Hence, the permanent-magnet flux linkage can be solved as

$$\psi_\mathrm{f} = \frac{|\boldsymbol{e}_\mathrm{s}|}{\omega_\mathrm{m}} = \frac{326.6\ \mathrm{V}}{2\pi \cdot 50\ \mathrm{rad/s}} = 1.040\ \mathrm{Vs}$$

(a) Since $i_\mathrm{d} = 0$ and $i_\mathrm{q} = i_\mathrm{N}$, the current vector is

$$\boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = \mathrm{j}i_\mathrm{N} = \mathrm{j}10.3 \text{ A}$$

The stator flux linkage is

$$\boldsymbol{\psi}_\mathrm{s} = L_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \psi_\mathrm{f} = 0.035 \text{ H} \cdot \mathrm{j}10.3 \text{ A} + 1.040 \text{ Vs} = 1.040 + \mathrm{j}0.361 \text{ Vs}$$

and its magnitude is

$$|\boldsymbol{\psi}_\mathrm{s}| = \sqrt{\psi_\mathrm{d}^2 + \psi_\mathrm{q}^2} = \sqrt{1.040^2 + 0.361^2} \text{ Vs} = 1.10 \text{ Vs}$$

Omitting the stator resistance, the steady-state voltage equation is

$$\boldsymbol{u}_\mathrm{s} = \mathrm{j}\omega_\mathrm{m}\boldsymbol{\psi}_\mathrm{s}$$

Hence, the electrical angular speed of the rotor becomes

$$\omega_\mathrm{m} = \frac{|\boldsymbol{u}_\mathrm{s}|}{|\boldsymbol{\psi}_\mathrm{s}|} = \frac{326.6 \text{ V}}{1.10 \text{ Vs}} = 296.9 \text{ rad/s}$$

and the corresponding rotational speed is

$$n = \frac{\omega_\mathrm{m}}{2\pi n_\mathrm{p}} = \frac{296.9 \text{ rad/s}}{2\pi \cdot 2} \cdot 60 \text{ s/min} = 1\,418 \text{ r/min}$$

The torque is

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{f} i_\mathrm{q} = \frac{3 \cdot 2}{2} \cdot 1.040 \text{ Vs} \cdot 10.3 \text{ A} = 32.1 \text{ Nm}$$

and the mechanical power is

$$p_\mathrm{M} = \tau_\mathrm{M}\omega_\mathrm{M} = \tau_\mathrm{M}\frac{\omega_\mathrm{m}}{p} = 32.1 \text{ Nm} \cdot \frac{296.9 \text{ rad/s}}{2} = 4.77 \text{ kW}$$

The vector diagram is shown at the end of the solution.

(b) Now $|i_\mathrm{d}| = |i_\mathrm{q}|$ and $|\boldsymbol{i}_\mathrm{s}| = \sqrt{i_\mathrm{d}^2 + i_\mathrm{q}^2} = i_\mathrm{N}$. Hence, the absolute values of the current components are

$$|i_\mathrm{d}| = |i_\mathrm{q}| = i_\mathrm{N}/\sqrt{2} = 7.3 \text{ A}$$

The component i_d is negative in the field-weakening region and the component i_q is positive at positive torque:

$$\boldsymbol{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = -7.3 + \mathrm{j}7.3 \text{ A}$$

The stator flux linkage is

$$\begin{aligned}\boldsymbol{\psi}_\mathrm{s} &= L_\mathrm{s}\boldsymbol{i}_\mathrm{s} + \psi_\mathrm{f} \\ &= 0.035 \text{ H} \cdot (-7.3 + \mathrm{j}7.3) \text{ A} + 1.040 \text{ Vs} = 0.785 + \mathrm{j}0.256 \text{ Vs}\end{aligned}$$

and its magnitude is

$$|\boldsymbol{\psi}_\mathrm{s}| = \sqrt{\psi_\mathrm{d}^2 + \psi_\mathrm{q}^2} = \sqrt{0.785^2 + 0.256^2}\ \mathrm{Vs} = 0.825\ \mathrm{Vs}$$

Hence, the electrical angular speed of the rotor becomes

$$\omega_\mathrm{m} = \frac{|\boldsymbol{u}_\mathrm{s}|}{|\boldsymbol{\psi}_\mathrm{s}|} = \frac{326.6\ \mathrm{V}}{0.825\ \mathrm{Vs}} = 395.9\ \mathrm{rad/s}$$

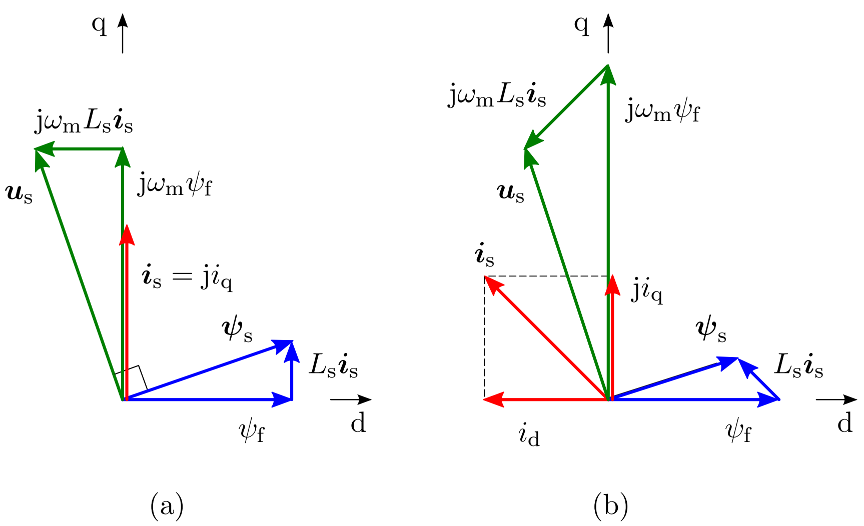
and the corresponding rotational speed is

$$n = \frac{\omega_\mathrm{m}}{2\pi n_\mathrm{p}} = \frac{395.9\ \mathrm{rad/s}}{2\pi \cdot 2} \cdot 60\ \mathrm{s/min} = 1\,890\ \mathrm{r/min}$$

The torque and mechanical power are

$$\tau_\mathrm{M} = \frac{3n_\mathrm{p}}{2}\psi_\mathrm{f} i_\mathrm{q} = \frac{3 \cdot 2}{2} \cdot 1.040\ \mathrm{Vs} \cdot 7.3\ \mathrm{A} = 22.8\ \mathrm{Nm}$$
$$p_\mathrm{M} = \tau_\mathrm{M}\frac{\omega_\mathrm{m}}{n_\mathrm{p}} = 22.8\ \mathrm{Nm} \cdot \frac{396\ \mathrm{rad/s}}{2} = 4.5\ \mathrm{kW}$$

The vector diagrams are shown below.



Remark: It can be noticed that the torque decreases more than inversely proportionally to the speed in the field-weakening region and the mechanical power decreases. In surface-mounted permanent-magnet machines, the d component of the current produces no torque; it only magnetises against the permanent magnets in order to decrease the stator flux magnitude.