Name:
Student no:

Answer all five questions (in English, Finnish, or Swedish). Using a calculator is allowed, but all memory must be cleared! **Please return also this problem paper.** Remember to write your name and student number above.

1. Describe the field-oriented control system for permanent-magnet synchronous motors. Draw also the block diagram of the control system, label the signals in the diagram, and describe the tasks of the blocks.

   Solution:

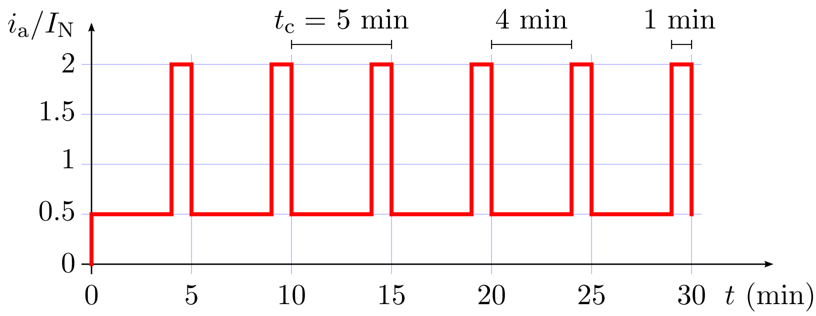
   See lectures, exercises, and readings.

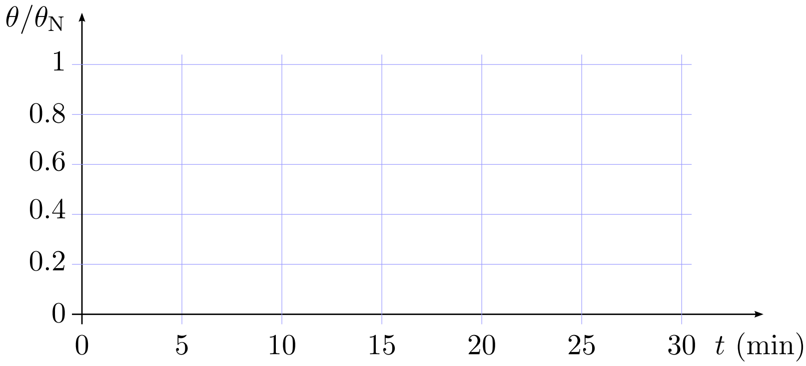
2. Answer briefly to the following questions:
   (a) Why three-phase machines are preferred to single-phase AC machines?
   (b) How the physical size of the motor approximately depends on the rated values of the motor?
   (c) Why the antiwindup is used in PI controllers?

   Solution:

   See lectures, exercises, and readings.

3. (a) A DC motor is used in a periodic duty, whose cycle length is $t_c = 5$ min. As shown in the figure below, the armature current is $0.5I_N$ for 4 min and $2I_N$ for 1 min during each cycle. Calculate the rms current $I_{a,rms}$.
   (b) The thermal time constant of the motor is $\tau_{th} = 15$ min. The motor is cold in the beginning, i.e., the initial temperature rise $\theta = 0$ at $t = 0$. In the graph below, draw the waveform for the average temperature rise as a function of time corresponding to the rms current $I_{a,rms}$. What is the value of the average temperature rise at $t = 30$ min? Sketch also the waveform for the instantaneous temperature rise.





Solution:

(a) Using the given current waveform, the rms current is

$$I_{a,rms} = \sqrt{\frac{1}{t_c}\int_0^{t_c} i_a^2 dt} = \sqrt{\frac{(0.5I_N)^2 \cdot 4 \text{ min} + (2I_N)^2 \cdot 1 \text{ min}}{5 \text{ min}}} = I_N$$

Therefore, the motor is dimensioned such that the steady-state average temperature rise matches the rated temperature rise of the motor.

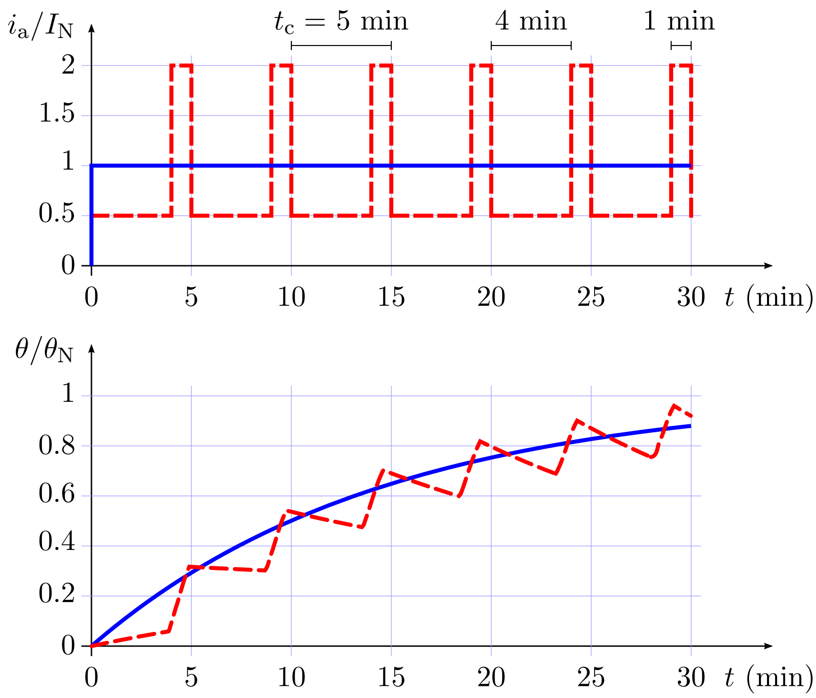
(b) The thermal model can be assumed to be a first-order system. Therefore, the average temperature rise as a function of time corresponding to the rms current $I_{a,rms}$ is

$$\theta(t) = \theta_0 + (\theta_\infty - \theta_0)\left(1 - e^{-t/\tau_{th}}\right) = \theta_N\left(1 - e^{-t/\tau_{th}}\right)$$

where $\theta_0 = 0$ and $\theta_\infty = \theta_N$ in this case. This waveform is drawn in the graph below using the blue solid curve.

The average temperature rise at $t = 30$ min is $\theta = (1 - e^{-30/15})\theta_N = 0.86\theta_N$.

The waveform for the instantaneous temperature rise is also sketched in the graph using the red dashed curve.


i_a/I_N
t_c = 5 min
4 min
1 min
2
1.5
1
0.5
0
0
5
10
15
20
25
30
t (min)
θ/θ_N
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
t (min)

4. Consider a three-phase four-pole permanent-magnet synchronous motor. The stator inductance is $L_\mathrm{s} = 0.035$ H and the stator resistance can be assumed to be zero. The permanent magnets induce the rated voltage of 400 V at the rotational speed of 1 500 r/min. The rated current is 7.3 A.

   (a) The control principle $i_\mathrm{d} = 0$ is used. The motor is operated at the rated voltage and current. Calculate the rotational speed, torque, and mechanical power.

   (b) The motor is driven in the field-weakening region at the rated voltage and current. The speed is increased until the absolute values of i_d and i_q are equal. Calculate the rotational speed, torque, and mechanical power.

   (c) Draw also the vector diagrams corresponding to Parts (a) and (b).

Solution:

The peak-valued quantities will be used. The rated current is $i_\mathrm{N} = \sqrt{2}\cdot 7.3\ \mathrm{A} = 10.3$ A and the rated line-to-neutral voltage is $u_\mathrm{N} = \sqrt{2/3}\cdot 400\ \mathrm{V} = 326.6$ V. It is known that the induced voltage is $|\underline{e}_\mathrm{s}| = u_\mathrm{N}$ at the electrical angular speed

$$\omega_\mathrm{m} = 2\pi p n = 2\pi\cdot 2\cdot\frac{1500\ \mathrm{r/min}}{60\ \mathrm{s/min}} = 2\pi\cdot 50\ \mathrm{rad/s}$$

Hence, the permanent-magnet flux linkage can be solved as

$$\psi_\mathrm{f} = \frac{|\underline{e}_\mathrm{s}|}{\omega_\mathrm{m}} = \frac{326.6\ \mathrm{V}}{2\pi\cdot 50\ \mathrm{rad/s}} = 1.040\ \mathrm{Vs}$$

(a) Since $i_\mathrm{d} = 0$ and $i_\mathrm{q} = i_\mathrm{N}$, the current vector is

$$\underline{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = \mathrm{j}i_\mathrm{N} = \mathrm{j}10.3\ \mathrm{A}$$

The stator flux linkage is

$$\underline{\psi}_\mathrm{s} = L_\mathrm{s}\underline{i}_\mathrm{s} + \psi_\mathrm{f} = 0.035\ \mathrm{H}\cdot\mathrm{j}10.3\ \mathrm{A} + 1.040\ \mathrm{Vs} = 1.040 + \mathrm{j}0.361\ \mathrm{Vs}$$

and its magnitude is

$$|\underline{\psi}_\mathrm{s}| = \sqrt{\psi_\mathrm{d}^2 + \psi_\mathrm{q}^2} = \sqrt{1.040^2 + 0.361^2}\ \mathrm{Vs} = 1.10\ \mathrm{Vs}$$

Omitting the stator resistance, the steady-state voltage equation is

$$\underline{u}_\mathrm{s} = \mathrm{j}\omega_\mathrm{m}\underline{\psi}_\mathrm{s}$$

Hence, the electrical angular speed of the rotor becomes

$$\omega_\mathrm{m} = \frac{|\underline{u}_\mathrm{s}|}{|\underline{\psi}_\mathrm{s}|} = \frac{326.6\ \mathrm{V}}{1.10\ \mathrm{Vs}} = 296.9\ \mathrm{rad/s}$$

and the corresponding rotational speed is

$$n = \frac{\omega_\mathrm{m}}{2\pi p} = \frac{296.9\ \mathrm{rad/s}}{2\pi\cdot 2}\cdot 60\ \mathrm{s/min} = 1\,418\ \mathrm{r/min}$$

The torque is

$$T_\mathrm{M} = \frac{3p}{2}\psi_\mathrm{f} i_\mathrm{q} = \frac{3\cdot 2}{2}\cdot 1.040\ \mathrm{Vs}\cdot 10.3\ \mathrm{A} = 32.1\ \mathrm{Nm}$$

and the mechanical power is

$$P_\mathrm{M} = T_\mathrm{M}\omega_\mathrm{M} = T_\mathrm{M}\frac{\omega_\mathrm{m}}{p} = 32.1\ \mathrm{Nm}\cdot\frac{296.9\ \mathrm{rad/s}}{2} = 4.77\ \mathrm{kW}$$

The vector diagram is shown at the end of the solution.

(b) Now $|i_\mathrm{d}| = |i_\mathrm{q}|$ and $|\underline{i}_\mathrm{s}| = \sqrt{i_\mathrm{d}^2 + i_\mathrm{q}^2} = i_\mathrm{N}$. Hence, the absolute values of the current components are

$$|i_\mathrm{d}| = |i_\mathrm{q}| = i_\mathrm{N}/\sqrt{2} = 7.3\ \mathrm{A}$$

The component i_d is negative in the field-weakening region and the component i_q is positive at positive torque:

$$\underline{i}_\mathrm{s} = i_\mathrm{d} + \mathrm{j}i_\mathrm{q} = -7.3 + \mathrm{j}7.3\ \mathrm{A}$$

The stator flux linkage is

$$\begin{aligned}\underline{\psi}_\mathrm{s} &= L_\mathrm{s}\underline{i}_\mathrm{s} + \psi_\mathrm{f}\\ &= 0.035\ \mathrm{H}\cdot(-7.3 + \mathrm{j}7.3)\ \mathrm{A} + 1.040\ \mathrm{Vs} = 0.785 + \mathrm{j}0.256\ \mathrm{Vs}\end{aligned}$$

and its magnitude is

$$|\underline{\psi}_\mathrm{s}| = \sqrt{\psi_\mathrm{d}^2 + \psi_\mathrm{q}^2} = \sqrt{0.785^2 + 0.256^2}\ \mathrm{Vs} = 0.825\ \mathrm{Vs}$$

Hence, the electrical angular speed of the rotor becomes

$$\omega_\mathrm{m} = \frac{|\underline{u}_\mathrm{s}|}{|\underline{\psi}_\mathrm{s}|} = \frac{326.6\ \mathrm{V}}{0.825\ \mathrm{Vs}} = 395.9\ \mathrm{rad/s}$$

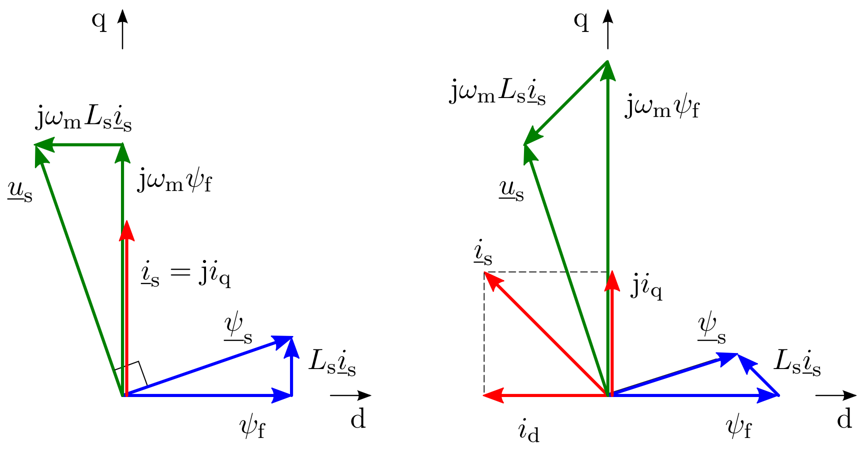
and the corresponding rotational speed is

$$n = \frac{\omega_\mathrm{m}}{2\pi p} = \frac{395.9\ \mathrm{rad/s}}{2\pi\cdot 2}\cdot 60\ \mathrm{s/min} = 1\,890\ \mathrm{r/min}$$

The torque and mechanical power are

$$\begin{aligned}T_\mathrm{M} &= \frac{3p}{2}\psi_\mathrm{f} i_\mathrm{q} = \frac{3\cdot 2}{2}\cdot 1.040\ \mathrm{Vs}\cdot 7.3\ \mathrm{A} = 22.8\ \mathrm{Nm}\\ P_\mathrm{M} &= T_\mathrm{M}\frac{\omega_\mathrm{m}}{p} = 22.8\ \mathrm{Nm}\cdot\frac{396\ \mathrm{rad/s}}{2} = 4.5\ \mathrm{kW}\end{aligned}$$

The vector diagrams are shown below.

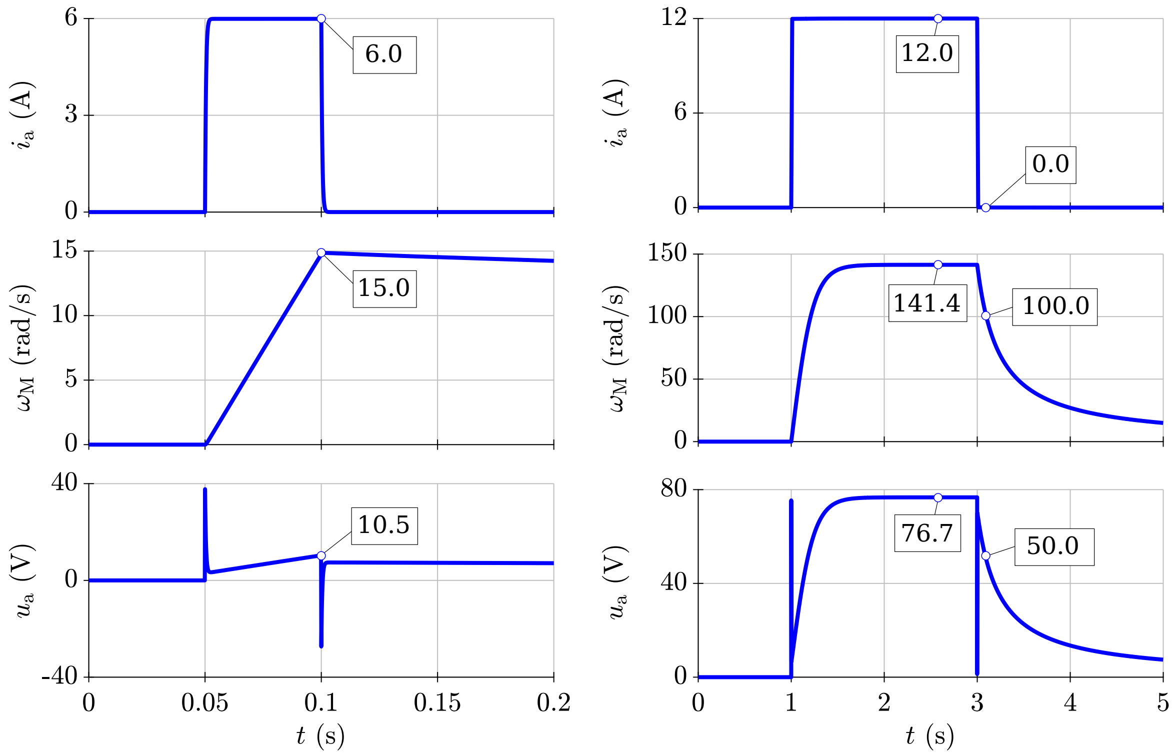


(a) (b)

Remark: It can be noticed that the torque decreases more than inversely proportionally to the speed in the field-weakening region and the mechanical power decreases. In surface-mounted permanent-magnet machines, the d component of the current produces no torque; it only magnetises against the permanent magnets in order to decrease the stator flux magnitude.

5. A DC motor is fed from a current-controlled converter. The shaft of the rotor is connected to a fan, whose load-torque profile $|T_\mathrm{L}| = k\omega_\mathrm{M}^2$ is quadratic. Two different armature current pulses are applied, as shown in the figures below. Based on the waveforms, determine the total inertia J and the load-torque coefficient k. You may use some assumptions, but justify them briefly.

   [Hint: Determine first the flux factor k_f.]



Solution:
The armature voltage of the DC motor is

$$u_a = R_a i_a + L_a \frac{di_a}{dt} + k_f \omega_M$$

The flux factor k_f can be solved using the waveforms on the right-hand side. After $t = 3$ s, the armature current i_a is zero, and the armature voltage equals the back-emf, i.e., $u_a = k_f \omega_M$. Therefore, the flux factor is

$$k_f = u_a/\omega_M = 50.0 \text{ V}/100 \text{ rad/s} = 0.5 \text{ Vs}$$

The equation of motion is

$$J \frac{d\omega_M}{dt} = T_M - T_L$$

where the electromagnetic torque is $T_M = k_f i_a$ and the load torque is $T_L = k\omega_M^2$. The load-torque coefficient k can be calculated using the waveforms on the right-hand side. Based on the waveforms, the motor operates at constant speed at $t = 2 \ldots 3$ s, i.e., $T_L = T_M = k_f i_a = 0.5 \text{ Vs} \cdot 12 \text{ A} = 6$ Nm. Therefore,

$$k = T_L/\omega_M^2 = 6 \text{ Nm}/(141.4 \text{ rad/s})^2 = 0.0003 \text{ Nm} \cdot \text{s}^2$$

The total inertia can be solved from the waveforms on the left-hand side. The load torque is very low at low speeds due to the quadratic load-torque profile (less than 0.07 Nm at 15 rad/s based on the previous result). Therefore, $T_L = 0$ can be assumed during the acceleration at $t = 0.05 \ldots 0.1$ s, resulting in the total inertia

$$J = \frac{T_M}{\Delta\omega_M/\Delta t} = \frac{3 \text{ Nm}}{(15 \text{ rad/s})/0.05 \text{ s}} = 0.01 \text{ kgm}^2$$

Remark 1: The total inertia could also be solved from the test shown on the right-hand side, if the data at low speeds (zoom after $t = 1$ s) were given.

Remark 2: If the flux factor k_f were calculated using the data at $t = 2\ldots3$, an (unnecessary) error of 8% would appear in k_f (since this approach assumes $R_\mathrm{a} = 0$).

Remark 3: The armature resistance R_a could also be solved using the right-hand side waveforms. Since the current is constant at at $t = 2\ldots3$, the resistance is

$$R_\mathrm{a} = \frac{u_\mathrm{a} - k_\mathrm{f}\omega_\mathrm{M}}{i_\mathrm{a}} = \frac{76.7\ \mathrm{V} - 0.5 \cdot 141.4\ \mathrm{V}}{12.0\ \mathrm{A}} = 0.5\ \Omega$$

This is not a practical test for determining the resistance, however.