

Some properties of the sum operator, expected value, variance and covariance

In all equations below, x , y and z are random variables. a , b , c are constants.

Sum operator:

$$\begin{aligned}\sum_{t=1}^T x_t &= x_1 + x_2 + x_3 + \dots + x_{T-1} + x_T \\ \sum_{t=1}^T (x_t + y_t) &= \sum_{t=1}^T x_t + \sum_{t=1}^T y_t \\ \sum_{t=1}^T ax_t &= a \sum_{t=1}^T x_t \\ \sum_{t=1}^T (x_t + b) &= \sum_{t=1}^T x_t + bT\end{aligned}$$

Expected value (Measuring the mean of a random variable):

$$\begin{aligned}E(x + y) &= E(x) + E(y) \\ E(ax) &= aE(x) \\ E(x + b) &= E(x) + b \\ E(ax + by) &= aE(x) + bE(y)\end{aligned}$$

Note that the expected value of a constant is the constant itself.

Variance: (Measuring the uncertainty of a random variable)

$$\begin{aligned}Var(x) &= E[(x - E(x))^2] \\ Var(ax) &= a^2Var(x) \\ Var(x + b) &= Var(x) \\ Var(ax + by) &= a^2Var(x) + b^2Var(y) + 2abCov(x, y)\end{aligned}$$

Note that the variance of a constant is zero.

Covariance: (Measuring the comovement of two random variables)

$$Cov(x, y) = E[(x - E(x))(y - E(y))]$$

$$Cov(x, x) = E[(x - E(x))(x - E(x))] = E[(x - E(x))^2] = Var(x)$$

$$Cov(ax, by) = abCov(x, y)$$

$$Cov(x + a, y + b) = Cov(x, y)$$

$$Cov(x + y, z) = Cov(x, z) + Cov(y, z)$$

$$Cov(ax + by, cz) = acCov(x, z) + bcCov(y, z)$$