1) We are given :

$N = 400$, $R = 5\Omega$

Magnetic core area: $5cm \times 5cm = 25\times10^{-4} m^2$

Length of airgap: $g = 1mm$ (each)

Average force: $F = 550N$

a) Using ampere's law:

$$H_g \cdot 2g = N \cdot I \quad \text{(2 for airgap at the both sides)}$$

$$I = \frac{H_g \cdot 2g}{N}, \qquad \left(H = \frac{B}{\mu}\right)$$

$$\Rightarrow I = \frac{B_g \cdot 2g}{\mu_0 \cdot N} \qquad (1)$$

Magnetic pressure is:

$$F_m = \frac{F}{A_g} \qquad (2)$$

Also

$$F_m = \frac{B_g^2}{2\mu_0} \qquad (3)$$

Combining (2) and (3):

$$\frac{B_g^2}{2\mu_0} = \frac{F}{A_g}$$

$$B_g = \sqrt{\frac{2\mu_0 \cdot F}{A_g}}$$

Where $A_g = 2\times25\times10^{-4}\times = 50\times10^{-4} m^2$

$$B_g = \sqrt{\frac{2\mu_0 \cdot F}{A_g}} = \sqrt{\frac{2\cdot4\pi\cdot10^{-7}\cdot550}{50\cdot10^{-4}}}$$

$$B_g = 0.525T$$

Placing it into the equation (1):

$$I = \frac{(0.525)\cdot(2)\cdot(0.001)}{(4\pi\cdot10^{-7})\cdot(400)} = 2.08A$$

Now we can proceed to find the required DC voltage to produce this current.

$$U = I \cdot R \Rightarrow 5\cdot2.08 = 10.44V \text{ (DC)}$$

Now the stored energy

$$w_m = F_m = \frac{F}{A_g}$$

$$w_m = \frac{550}{50 \cdot 10^{-4}} = 110 \cdot 10^3 J / m^3$$

$$W_m = w_m \times V_g$$

$$W_m = 110 \cdot 10^3 \cdot 1 \cdot 10^{-3} \cdot 50 \cdot 10^{-4} \qquad \left(V_g = g \times A_g\right)$$

$$W_m = 0.55J$$

b)

$$U_{RMS} = Z \cdot I_{RMS}$$

$$Z = \sqrt{R^2 + (wL)^2}$$

$$L = \frac{N^2}{\Re}$$

$$\Re_g = \frac{2g}{\mu_0 \cdot A_g} = \frac{2 \cdot 1 \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 5 \cdot 10^{-2}}$$

$$L = \frac{400^2 \cdot 4 \cdot \pi \cdot 10^{-7} \cdot 5 \cdot 10^{-2} \cdot 5 \cdot 10^{-2}}{2 \cdot 1 \cdot 10^{-3}} = 0.251H$$

Therefore,

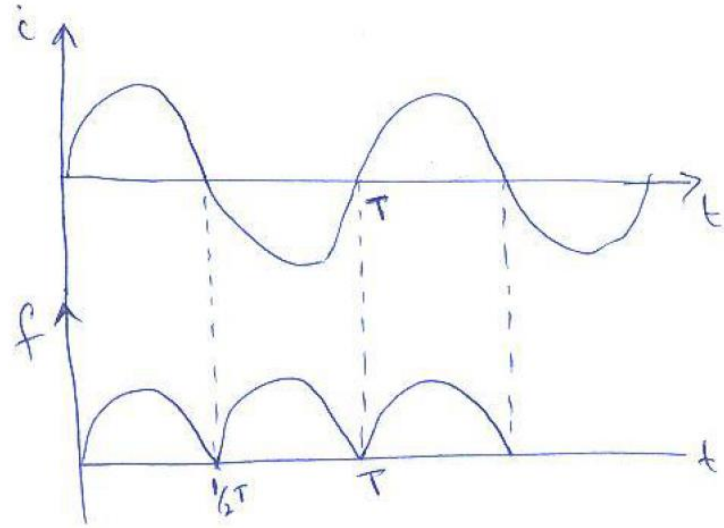
$$Z = \sqrt{5^2 + (2\pi \cdot 60 \cdot 0.251)^2} = 94.76\Omega$$

Now the force:

$$f = f_m \cdot A_g = \frac{b_g}{2\mu_0} A_g$$

$$b_g = \frac{i \cdot \mu_0 \cdot N}{2g}$$

$$f = \frac{N^2 \cdot \mu_0 \cdot A_g}{8g^2} \cdot i^2 = (125.66) \cdot i^2$$



Average is found by the integration:

$$F_{av} = \frac{1}{T/2} \cdot \int_0^{T/2} f \cdot dt$$

$$F_{av} = \frac{1}{T/2} \cdot \int_0^{T/2} (125.66) \cdot i^2 \cdot dt$$

Since $\quad I_{RMS} = \frac{1}{T/2} \cdot \int_0^{T/2} i \cdot dt$:

$$F_{av} = 125.66 \cdot I_{RMS}^2$$

$$I_{RMS} = \sqrt{\frac{F_{av}}{125.66}} = \sqrt{\frac{550}{125.66}} = 2.09A$$

$$U_{RMS} = I \cdot Z = (2.09) \cdot (94.76) = 198.04V$$

2) a)

$$i_s = \hat{i}_s \cos(wt + x)$$

$$L_{SR} = \hat{L}_{SR} \cos(\theta)$$

$$\psi_R = L_{SR} \cdot i_s + L_{RR} \cdot i_R \quad \text{(last term is constant)}$$

$$U_R = \frac{d\psi_R}{dt} = \frac{d(L_{SR} \cdot i_s)}{dt} = \frac{d(\hat{i}_s \hat{L}_{SR} \cos(\theta) \cos(wt + x))}{dt}$$

Let us choose $\theta = wt + \beta$

$$U_R = \frac{d(\hat{i}_s \hat{L}_{SR} \cos(wt + \beta) \cos(wt + x))}{dt}$$

Since $\quad \cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$:

$$U_R = \frac{d}{dt}\left[\frac{\hat{i}_S \hat{L}_{SR}}{2}(\cos(wt+\alpha+\beta)+\cos(\beta-\alpha))\right]$$

$$U_R = -\hat{i}_S \hat{L}_{SR}\cdot w\cdot \sin(2wt+\alpha+\beta)$$

$$\hat{i}_S = \sqrt{2}\cdot i_S = \sqrt{2}\cdot 5 = 7.07A$$

$$\hat{L}_{SR} = 0.08H$$

$$w = 2\pi f = 2\pi\frac{n}{60} = 377 rad/\mathrm{s}$$

$$U_R = -7.07\cdot 0.08\cdot 377\cdot \sin(2\cdot 377\cdot t+\alpha+\beta)$$

$$U_R = -213.23\cdot \sin(754+\alpha+\beta)$$

$$\hat{U}_R = 213.23V \qquad U_R = \frac{213.23}{\sqrt{2}} = 150.77V$$

$$f_r = \frac{w}{2\pi} = \frac{754}{2\pi} = 120Hz$$

b)

$$|w_m| = |w_S \pm w_R|$$

- $w_s + w_r = (60+60)\cdot 2\pi = 120\cdot 2\pi$ (1)
- $w_s - w_r = (60-60)\cdot 2\pi = 0$ (2)
- $-w_s - w_r = (-60-60)\cdot 2\pi = -120\cdot 2\pi$ (3)
- $-w_s + w_r = (-60+60)\cdot 2\pi = 0$ (4)

Using the Torque equation (eq 3.5 on page 113 of the course text book)

- ❖ when $w_m = 0$ (from 2 and 4)

$$T = -\frac{I_{SM}\cdot I_{RM}\cdot M}{4}\left[\begin{array}{l}\sin(120\cdot 2\pi t+\alpha+\delta)+\sin(-120\cdot 2\pi t-\alpha+\delta)\\ +\sin(-\alpha+\delta)+\sin(\alpha+\delta)\end{array}\right]$$

$$\alpha = 0 \Rightarrow T_{AVG} = -\frac{I_{SM}\cdot I_{RM}\cdot M}{2}\sin\delta = -\frac{\sqrt{2}\cdot 5\cdot\sqrt{2}\cdot 5\cdot 0.08}{2}\sin\delta = -2\sin\delta$$

$$T_{AVG,\max} = 2Nm \ \text{ for } \ \delta = 90°$$

- ❖ When $w_m = 120\cdot 2\pi$ (from 1) , $\alpha = 0$

$$T = -\frac{I_{SM}\cdot I_{RM}\cdot M}{4}\left[\begin{array}{l}\sin(240\cdot 2\pi t+\alpha+\delta)+\sin(-\alpha+\delta)+\sin(120\cdot 2\pi t-\alpha+\delta)\\ +\sin(120\cdot 2\pi t+\alpha+\delta)\end{array}\right]$$

$$T_{AVG} = -\sin\delta$$

$$T_{AVG,\max} = 1Nm \ \text{ for } \ \delta = 90°$$

- When $w_m = -120 \cdot 2\pi$ (from 3) , $\alpha = 0$

$$T = -\frac{I_{SM} \cdot I_{RM} \cdot M}{4}\begin{bmatrix} \sin(\alpha+\delta)+\sin(-240\cdot 2\pi t-\alpha+\delta)+\sin(-120\cdot 2\pi t-\alpha+\delta) \\ +\sin(-120\cdot 2\pi t+\alpha+\delta) \end{bmatrix}$$

$$T_{AVG} = -\sin\delta$$

$$T_{AVG,\max} = 1Nm \text{ for } \delta = 90^\circ$$