ELEC-E8422 An Introduction to Electric Energy

Exercise Session 2: Solar Energy

EX1 Solar Panels

Under some solar conditions, an ideal PV cell produces 3 W and its output voltage is 0,5 V. What is the output power, current and voltage of the system if there are 10 cells connected in series and three of these 10-cell units are connected in parallel.

EX2 Solar Panel Model

The current of a solar cell Is = 1,1 A and its reverse saturation current is Io = 1 nA. Calculate the voltage and current of the solar cell at the maximum power point. The model of the solar cell is shown below and the diode current can be calculated from $I_d = I_o\left(e^{V_d/V_T} - 1\right)$, where the thermal voltage VT = kT/q, the Boltzman's constant k = 1,380*10 -23 J/K, T is the temperature in Kelvins, and the electrons electric charge is e = 1,602*10-19 C.

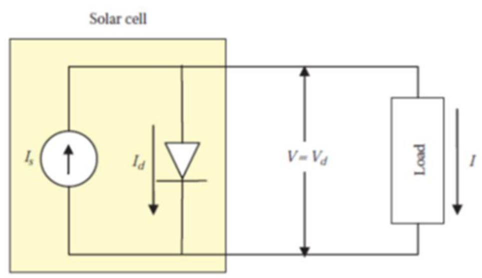

Figure: Model of the solar cell

EX3 Solar Panel Temperature

The cell of the previous question operates at the temperature of 35 °C (308,15 K). Calculate the value of the output voltage and load resistance at the maximum power point.

EX4 Solar Panel Efficiency

The surface area of the previous solar cell is 80 cm^2, the series resistance is 10 mΩ, the parallel resistance 500 Ω, and the load voltage is 0,5 V. At the given point, the solar power density is 300 W/m^2. Calculate the irradiance efficiency $\eta_{irradiance}$, i.e. the ratio of the power of the cell and power in radiation. What is η, the efficiency of the whole panel, i.e. the ratio of the output power and power in radiation?

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EX1 Solar Panels

Under some solar conditions, an ideal PV cell produces 3 W and its output voltage is 0,5 V. What is the output power, current and voltage of the system if there are 10 cells connected in series and three of these 10-cell units are connected in parallel.

The voltage of the panel is 10*0,5 V = 5 V.

The power of the panel is 3*10*3=90 W

The total current is 90/5 A = 18 A

EX2 Solar Panel Model

The current of a solar cell Is = 1,1 A and its reverse saturation current is Io = 1 nA. Calculate the voltage and current of the solar cell at the maximum power point. The model of the solar cell is shown below and the diode current can be calculated from $I_d = I_o\left(e^{V_d/V_T} - 1\right)$, where the thermal voltage VT = kT/q, the Boltzman's constant k = 1,380*10 -23 J/K, T is the temperature in Kelvins, and the electrons electric charge is e = 1,602*10-19 C.

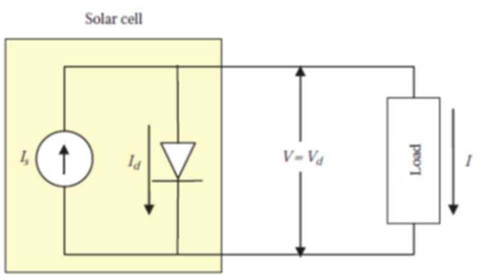


Figure: Model of the solar cell

Output power of the cell

$$P = VI$$

Maximum power point is obtained by derivation.

$$\frac{\partial P}{\partial V} = V\frac{\partial I}{\partial V} + I = 0$$

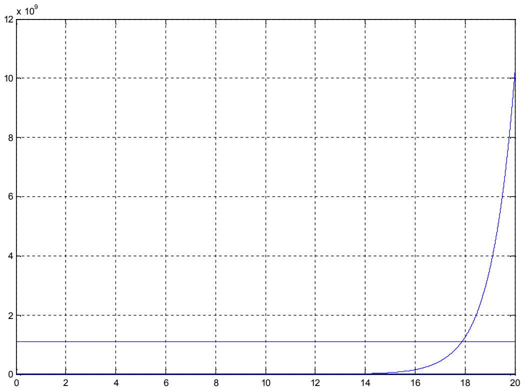
Derivative of the current is $\frac{\partial I}{\partial V} = -\frac{I_o}{V_T}\mathrm{e}^{\frac{V}{V_T}}$ and because load current $I = I_S - I_d$, derivative of power is

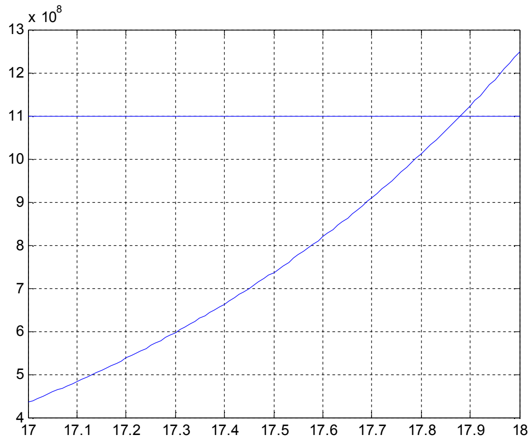
$$\frac{\partial P}{\partial V} = (I_S + I_o) - \left(1 + \frac{V}{V_T}\right) I_o \mathrm{e}^{\frac{V}{V_T}}$$

Marking this as zero, we get

$$\left(1 + \frac{V_{mp}}{V_T}\right) \mathrm{e}^{\frac{V_{mp}}{V_T}} = \frac{I_S + I_o}{I_o} = \frac{1{,}1 + 1 \cdot 10^{-9}}{1 \cdot 10^{-9}} \approx 1{,}1 \cdot 10^{9}$$

We should now solve V_{mp}/V_T from this but the equation is nonlinear. We could use either iterative or graphical methods to solve it. The following figures are showing the waveforms of $\left(1 + \frac{V_{mp}}{V_T}\right) \mathrm{e}^{\frac{V_{mp}}{V_T}}$ when $\frac{V_{mp}}{V_T}$ has values from 0 to 20 (left) and 17 to 18 (right). Both figures are also showing the line $1{,}1 \cdot 10^{9}$.





Based on the drawings, it can be concluded that at maximum power point $\frac{V_{mp}}{V_T} \approx 17{,}88$. Then the current of the maximum point is

$$I_{mp} = I_s - I_o\left(\mathrm{e}^{\frac{V_{mp}}{V_T}} - 1\right) = 1{,}1 - 10^{-9}(\mathrm{e}^{17{,}88} - 1)\ \mathrm{A} \approx 1{,}0417\ \mathrm{A}$$

EX3 Solar Panel Temperature

The cell of the previous question operates at the temperature of 35 °C (308,15 K). Calculate the value of the output voltage and load resistance at the maximum power point.

Thermal voltage is

$$V_T = \frac{kT}{q} = \frac{1{,}380 \cdot 10^{-23} \cdot 308{,}15}{1{,}602 \cdot 10^{-19}} \approx 26{,}54\ \mathrm{mV}$$

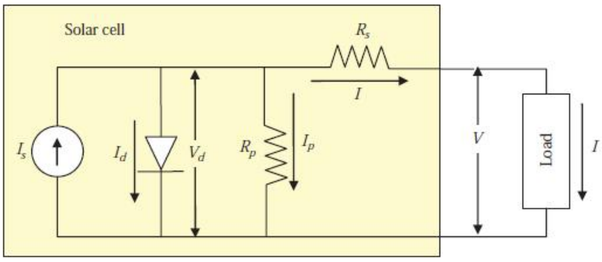
And voltage at maximum power point is $V_{mp} \approx 17{,}88 \cdot 26{,}54\ \mathrm{mV} \approx 0{,}4745\ \mathrm{V}$

$$R_{mp} = \frac{V_{mp}}{I_{mp}} \approx \frac{0{,}4745}{1{,}0417}\,\Omega \approx 0{,}456\,\Omega$$

EX4 Solar Panel Efficiency

The surface area of the previous solar cell is 80 cm^2, the series resistance is 10 mΩ, the parallel resistance 500 Ω, and the load voltage is 0,5 V. At the given point, the solar power density is 300 W/m^2. Calculate the irradiance efficiency $\eta_{irradiance}$, i.e. the ratio of the power of the cell and power in radiation. What is η, the efficiency of the whole panel, i.e. the ratio of the output power and power in radiation?

The model of a solar cell including the losses is below.



With the given values, voltage of the diode is

$$V_d = V + IR_S = 0{,}5\ \mathrm{V} + 1{,}1 \cdot 0{,}01\ \mathrm{V} = 0{,}511\ \mathrm{V}$$

Diode current is

$$I_d = I_o\left(e^{V_d/V_T} - 1\right) = 10^{-9} * \left(e^{\frac{0.511}{0.0265}} - 1\right) \approx 0.237\ \mathrm{A}$$

Current through the parallel resistance $I_d = \frac{V_d}{R_p} = \frac{0.511}{500} \approx 1.02\ \mathrm{mA}$, and thus cell current is

$$I_s = I + I_d + I_p = 1.1 + 0.237 + 0.001022 \approx 1.338\ A$$

Irradiance efficiency $\eta_{irradiance} = \frac{V_d I_s}{\rho A} = \frac{0.511*1.338}{300*0.008} \approx 0.285$

Part of the power is lost in the resistances of the cell and this part is

$$P_{e-loss} = I^2 R_s + I_p^2 R_p = 1.1^2 * 0.01 + 0.001022^2 * 500 \approx 12.622\ \mathrm{mW}$$

The electrical efficiency is

$$\eta_e = \frac{P_{out}}{P_{se}} = \frac{VI}{V_d I_s} = \frac{0.5 * 1.1}{0.511 * 1.338} \approx 0.805$$

And the efficiency of the whole panel is

$$\eta = \frac{VI}{\rho A} = \frac{P_{out}}{P_s} = \frac{P_{se}}{P_s}\frac{P_{out}}{P_{se}} = \eta_{irradiance}\eta_e \approx 0.285 * 0.805 \approx 0.229$$

In this solution we have used 1,1 A as the cell current, i.e. the current given in previous exercises where the panel model was assumed to be ideal. As we saw here, when including the real model, the current of the cell should actually be 1,338 A. This means that we should re-calculate previous operating points and even do several iterations.