ELEC-E8422 An Introduction to Electric Energy

Exercise Session 3: Wind Energy and Fuel Cells

EX 1 Wind Power

Speed of the wind is 15 m/s and the air density can be approximated to be 1,2 kg/m^3. What is the power density of the air?

In a wind turbine the length of the blades is 60 m. What is the power of the wind within this area? What is the maximum theoretical power of the blades?

EX 2 Tip Speed Ratio

In a wind turbine the gear ratio is 200 and its cut in speed is 910 rpm. The length of the blades is 5 m and the tip speed ratio is variable. Calculate TSR when wind speed is 10 m/s.

EX 3 Hypothetical Water Power

What is the power of the turbine of Exercise 2 if it is in water, the coefficient of performance of the turbine is 0,3 and other values are as in Exercise 2. Compare the result to Exercise one.

EX 4 Fuel Cells

Polarization curve of a fuel cell can be presented as an equation

$$V = 0.9 - 0.128\tan(1.2 - I)$$

where V is voltage and I current of the fuel cell. Draw polarization and power curves as function of current. Derive the maximum power point. In the activation area current increase 10 %, what is the drop in the voltage. What are these voltage drops in the ohmic and mass transportation areas?

ELEC-E8422 An Introduction to Electric Energy

Exercise Session 3: Wind Energy and Fuel Cells

EX 1 **Wind Power**

Speed of the wind is 15 m/s and the air density can be approximated to be 1,2 kg/m^3. What is the power density of the air?

In a wind turbine the length of the blades is 60 m. What is the power of the wind within this area? What is the maximum theoretical power of the blades?

Solution

Kinetic energy of a moving object is

$$W = \frac{1}{2}mv^2 = \frac{1}{2}Av\delta tv^2 = \frac{1}{2}A\delta tv^3 = \frac{1}{2}\pi r^2\delta tv^3$$

Where *m* is mass of air,*v* speed of air, δ density of air, *t* is time, *A* area and *r* radius. Power in the air can be calculated form the energy

$$P_{wind} = \frac{W}{t} = \frac{1}{2}A\delta v^3$$

And the asked power density is

$$\frac{P_{wind}}{A} = \frac{1}{2}\delta v^3 \approx 2025\ \mathrm{W/m^2}$$

and the power of the air in the area of the blades.

$$P_{wind} = \frac{1}{2}\pi r^2\delta v^3 \approx 22{,}9\ \mathrm{MW}$$

The power transferred to the blades depends on blade geometry and tip speed ratio. Even in theory this cannot be higher than 0,5926, which is so called Betz limit. Therefore, the maximum possible power in the blades is

$$P_{blade,max} \approx 0{,}5926 * \frac{1}{2}\pi r^2\delta v^3 \approx 13{,}57\ \mathrm{MW}$$

EX 2 **Tip Speed Ratio**

In a wind turbine the gear ratio is 200 and its cut in speed is 910 rpm. The length of the blades is 5 m and the tip speed ratio is variable. Calculate TSR when wind speed is 10 m/s.

Solution

Angular velocity of the rotor at cut in speed (lowest) is

$$n = \frac{910}{200}\text{rpm} \approx 4{,}55 \text{ rpm}$$

Speed of the blade tip is then

$$v_{tip} = \omega r = 2\pi \frac{4{,}55}{60} 5 \text{ m/s} \approx 2{,}382 \text{ m/s}$$

And tip speed ratio is

$$TSR = \frac{v_{tip}}{v} = \frac{2{,}382}{10} \approx 0{,}2382 \sim 23{,}82\%$$

EX 3 Hypothetical Water Power

What is the power of the turbine of Exercise 2 if it is in water, the coefficient of performance of the turbine is 0,3 and other values are as in Exercise 2. Compare the result to Exercise one.

Solution

$$P_{stream} = \frac{W}{t} = \frac{1}{2} A\delta v^3 \approx 39{,}3 \text{ MW}$$

And the power of the turbine is

$$P_{blades} = 0{,}3 * P_{stream} \approx 11{,}8 \text{ MW}$$

The power of the blades is nearly the same as in EX1 although blades are only 5 meters and water speed 10 m/s. The simple reason is that water density is nearly 1000 times that of the air.

EX 4 Fuel Cells

Polarization curve of a fuel cell can be presented as an equation

$$V = 0.9 - 0.128 \tan(1.2 - I)$$

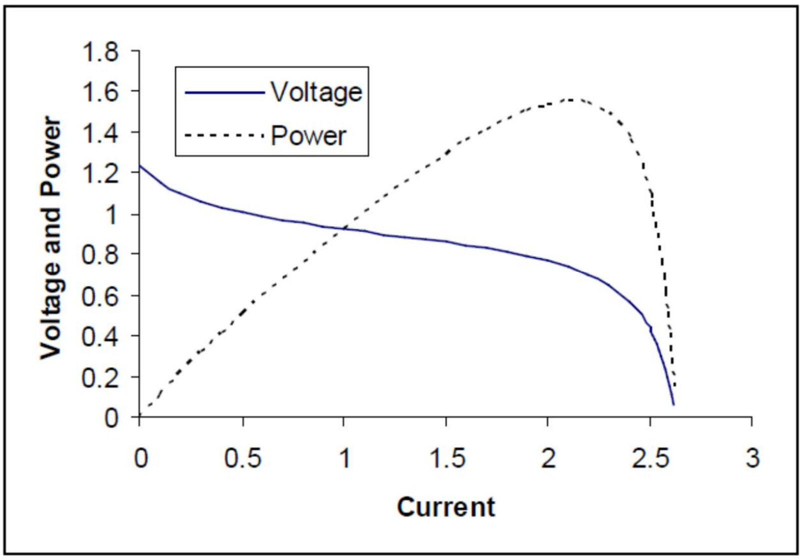
where *V* is voltage and *I* current of the fuel cell. Draw polarization and power curves as function of current. Derive the maximum power point. In the activation area current increase 10 %, what is the drop in the voltage. What are these voltage drops in the ohmic and mass transportation areas?

Solution

Power is obtained by multiplication

$$)1.2 - I(\tan * VI = 0.9 * I - 0.128 * I = P$$

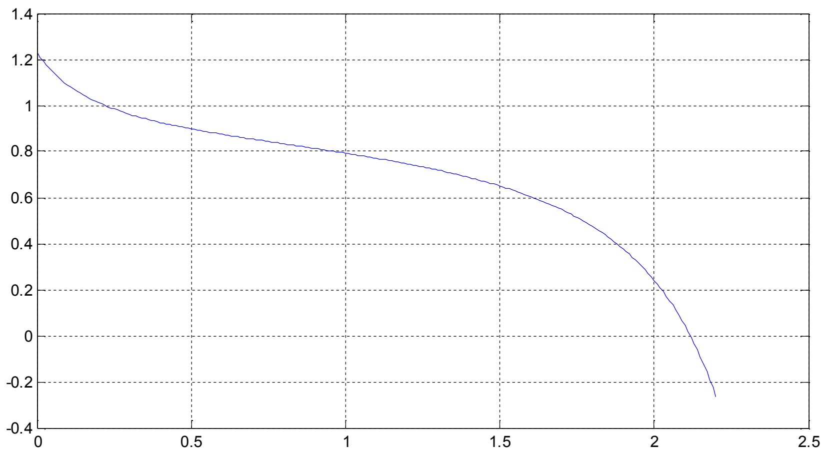
And both voltage and current are drawn in the figure below. Note that voltage becomes negative when current exceeds around 2,6 A, i.e. the given equation is valid only until this point.



Maximum power point is obtained by derivation

$$\frac{\partial P}{\partial I} = 0.9 - 0.128 * \tan(I - 1.2) - 0.128 * I * \frac{1}{\cos^2(I - 1.2)} = 0$$

The result is non-linear. One simple approach is to draw the waveform as shown in the next figure.



From the derivative and also from the power curve one can conclude that the maximum power point is around 2,1 A and voltage is then

$$V = 0.9 - 0.128 \tan(2.1 - 1.2)\ \mathrm{V} \approx 0.898\ \mathrm{V}$$

In active area, if current increases 10 % e.g. form 0,1 A to 0,11 A, voltage of the cell changes

$$\frac{\Delta V}{\Delta V_1} = \frac{V_1 - V_2}{V_1} = \frac{0.128 * [-\tan(0.1 - 1.2) + \tan(0.11 - 1.2)]}{0.9 - 0.128 * \tan(0.1 - 1.2)} \approx 0.0053$$

In the ohmic are e.g. change from 1 A to 1,1 A changes voltage

$$\frac{\Delta V}{\Delta V_1} = \frac{V_1 - V_2}{V_1} = \frac{0.128 * [-\tan(1 - 1.2) + \tan(1.1 - 1.2)]}{0.9 - 0.128 * \tan(1 - 1.2)} \approx 0.0142$$

And in mass transportation area e.g. change from 2,2 a to 2,42 changes voltage

$$\frac{\Delta V}{\Delta V_1} = \frac{V_1 - V_2}{V_1} = \frac{0.128 * [-\tan(2.2 - 1.2) + \tan(2.42 - 1.2)]}{0.9 - 0.128 * \tan(2.2 - 1.2)} \approx 0.2147$$

which is the largest relative change.