ELEC-E8422 An Introduction to Electric Energy

Exercise Session 5: Converters

EX 1: Buck-Converter

The dc input voltage of a buck (step-down) converter is 24 V and the desired dc output voltage is 16.8 V. Assume the converter lossless and omit the ripple in the output voltage (i.e., large filter capacitance)

a) Draw the circuit diagram of the converter
b) Draw the waveforms of the inductor voltage, the inductor current, and the input current below each other
c) Calculate the IGBT duty ratio, the average input current, the average output current, and the peak-to-peak ripple in the inductor current. The inductance is 0.1 mH, the output power is 28 W, and the switching frequency is 200 kHz.

EX 2: Boost-Converter

The supply voltage of a step-up converter is 25 V and the output voltage is 40 V. The switching frequency is 1 kHz and the load resistance 100 Ω. Calculate

a) The peak-to-peak change in the inductor current when L = 30 mH
b) The average of the load current
c) The power of the load

EX 3 & 4 DC-AC-Converter

A three-phase dc/ac converter is used to supply a star-connected load, where each phase has a 10 Ω resistance. The dc-voltage is 540 V and the inverter works under full control producing the maximum output voltage. This means that each phase is connected half of the time to the positive dc-bus and half of the time to the negative bus. There is 120 degrees phase shift between the phases

a) Draw the status of the switches in the converter and under these the waveforms of the line currents and phase voltages
b) Draw the waveform of the line-to-line voltage
c) The value of the current taken from the dc-bus
d) The rms value of the phase current
e) The rms values of the phase and line-to-line voltages

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Exercise Session 5: Converters

EX 1: Buck-Converter

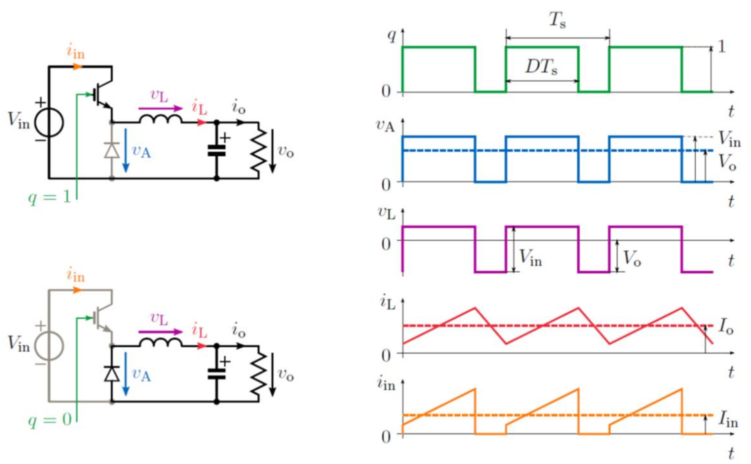
The dc input voltage of a buck (step-down) converter is 24 V and the desired dc output voltage is 16.8 V. Assume the converter to be lossless and omit the ripple in the output voltage (i.e., large filter capacitance)

a) Draw the circuit diagram of the converter
b) Draw the waveforms of the inductor voltage, the inductor current, and the input current below each other
c) Calculate the IGBT duty ratio, the average input current, the average output current, and the peak-to-peak ripple in the inductor current. The inductance is 0.1 mH, the output power is 28 W, and the switching frequency is 200 kHz.

Solution

a) and b) The circuit diagram and the waveforms are shown in the figure. The control signal q of the IGBT is also shown. Since the output voltage $v_o = V_o$ is assumed to be constant, the inductor voltage is $v_L = v_A - V_o$. The inductor current i_L is obtained as an integral of the inductor voltage:

$$v_L = L\frac{di_L}{dt} \quad \Rightarrow \quad i_L = \frac{1}{L}\int v_L dt$$



The average of the inductor voltage has to be zero in dc steady state,

$$(V_{in} - V_o)DT_s = V_o(1 - D)T_s$$

Hence, the duty ratio of the IGBT is

$$D = \frac{V_o}{V_{in}} = \frac{16.8}{24} = 0.7$$

The average output current is

$$I_o = \frac{P_o}{V_o} = \frac{28 \text{ W}}{16.8 \text{ V}} = 1.67 \text{ A}$$

Since the converter is assumed lossless, the average input current is

$$I_{in} = \frac{P_o}{V_{in}} = \frac{28 \text{ W}}{24 \text{ V}} = 1.17 \text{ A}$$

Using the first equation, the peak-to-peak ripple in the inductor current is

$$\Delta i_L = \frac{V_{in} - V_o}{L} DT_s = \frac{V_o}{L}(1-D)T_s$$
$$= \frac{16.8 \text{ V}}{0.1 \text{ mH}} \cdot (1-0.7) \cdot 5 \; \mu\text{s} = 0.25 \text{ A}$$

EX 2: Boost-Converter

The supply voltage of a step-up converter is 25 V and output voltage 40 V. Switching frequency is 1 kHz and load resistance 100 Ω. Calculate

a) The peak to peak change in the inductor current when *L* = 30 mH
b) The average of the load current
c) The power of the load

Solution

The circuit diagram of the step-up converter, voltage over the inductance and current are shown in the next figure.

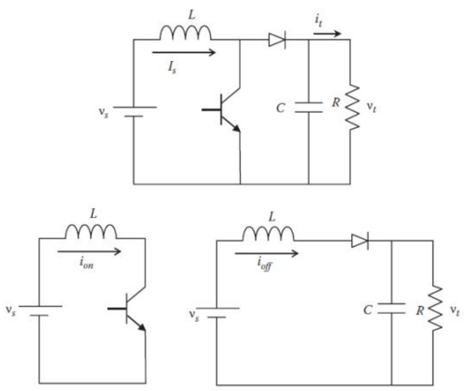


Figure 10.35 A simple boost converter.

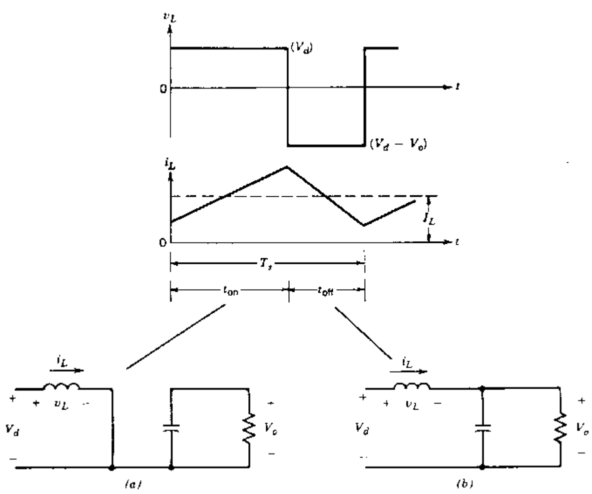


Figure 7-12 Continuous-conduction mode: (a) switch on; (b) switch off.

a) In steady state, the voltage integrals over the inductance are the same when the switch is on and off.

$$V_d t_{ON} + (V_d - V_o)t_{OFF} = 0 => t_{OFF} = -\frac{V_d}{(V_d - V_o)} t_{ON} = \frac{25}{40-25} t_{ON} = 1{,}667 t_{ON}$$

When the switching frequency is 1 kHz then $t_{ON} + t_{OFF} = 1 \text{ ms} => t_{ON} = \frac{1}{2{,}667} t_{OFF} \approx 0{,}375 \text{ ms}$

The voltage over the inductance is $v = L\, di/dt$ and thus the change in the current is

$$\Delta i_{ON} = \frac{V_d}{L} t_{ON} = \frac{25}{0{,}03} 0{,}375\ A \approx 312{,}5 \text{ mA}$$

b) Average of the load current

$$I_{ave} = \frac{V_o}{R} = \frac{40}{100} \text{A} = 0{,}4 \text{ A}$$

c) Output power

$$P = V_o I_{ave} = 40 \cdot 0{,}4\text{W} = 16 \text{ W}$$

EX 3 & 4 DC-AC-Converter

Three-phase dc/ac converter is used to supply a star connected load, where each phase has 10 resistance. The dc-voltage is 540 V and the inverter works under full control producing the maximum output voltage. This means that each phase is connected half of the time to the positive dc-bus and half of the time to the negative bus. There is 120 degrees phase shift between the phases

a) Draw the status of the switches in the converter and under these the waveforms of the line currents and phase voltages
b) Draw the waveform of the line-to-line voltage
c) The value of the current taken from the dc-bus
d) The rms value of the phase current

The rms values of the phase and line-to-line voltages

Solution

a) The 0peration of the dc/ac converter is demonstrated in the following figure with two different states. In the first one the switches Q1 and Q5 are conducting and connecting phases a and c to the positive bus and the conducting switch Q6 connects phase b to the negative bus. Therefore, the current from the dc-bus is equally divided between phases a and c and it flows through the star point to the minus bus. This means that the current in phase b is twice compared to a and c but its sign is negative. In the second state Q5 is turned of but Q2 connects the phase c to the minus bus. Thus the current in phase a is two times higher than in b and c and their current is also negative. The other four states can be deduced in the same way.

As the load is purely resistive, the voltage waveform is the same as that of the current but the amplitude is different.

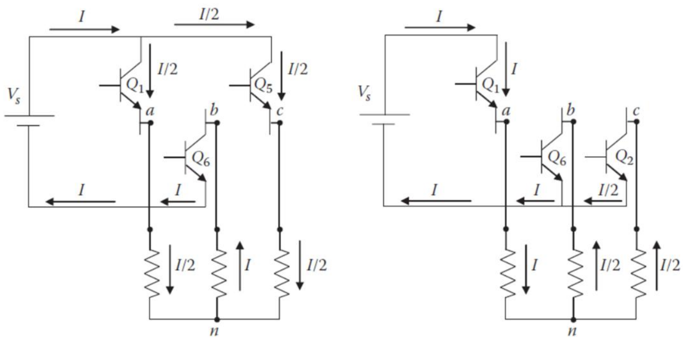


Figure 10.41 Active transistors and current flow during the first two intervals.

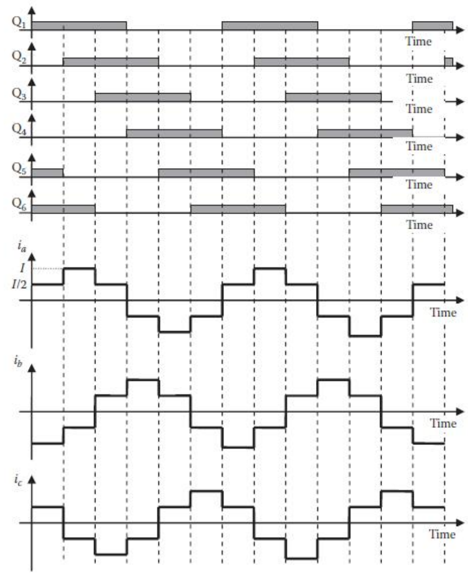
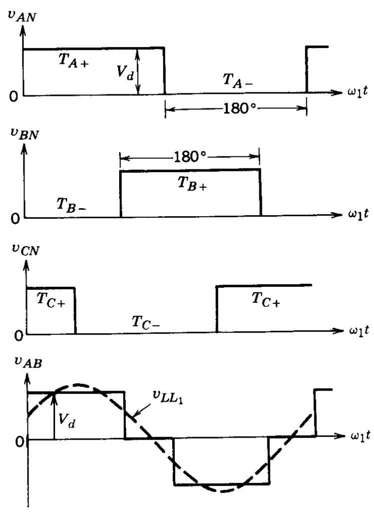


Figure 10.40 Timing of transistors and the phase currents.

b) The line-to-line voltage is the voltage between two phases. The next figure shows the voltages between the phase and the minus bus. The line-to-line voltage is obtained by subtracting two of these voltages from each other.



(b)

c) The current of the dc-bus can be calculated with the load resistances. E.g., in the cases shown in above, two phases are connected in parallel and the third is in series with them. Then the resistance of the load is

$$R_{total} = R_{phase} + \frac{R_{phase}R_{phase}}{R_{phase} + R_{phase}} = 10\ \Omega + \frac{10 \cdot 10}{10 + 10}\ \Omega = 15\ \Omega$$

and the current from the dc-bus is

$$I = \frac{V_s}{R_{total}} = \frac{540}{15}\,\mathrm{A} = 36\ \mathrm{A}$$

This is also the maximum value of the phase current. Sometimes the phase current is also half of this, i.e. 18 A and both values are negative, i.e. -18 A and -36 A.

d) The rms value of the phase current is calculated directly from the current waveform. One output cycle is composed of six 60 degrees long periods. In two of these, the absolute value of the current is 38 A and in four it is 16 A. Thus

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_a^2 dx} = \sqrt{\frac{1}{2\pi}\left(2 \cdot I^2 \frac{\pi}{3} + 4 \cdot \left(\frac{I}{2}\right)^2 \frac{\pi}{3}\right)} = \sqrt{\frac{1}{2}\left(I^2 \frac{2}{3} + I^2 \frac{1}{3}\right)} = \frac{I}{\sqrt{2}} \approx 25{,}45\ \mathrm{A}$$

e) The rms value of the phase voltage is $V_a = RI_{rms}$ ≈254,5 A and the rms value of line-to-line voltage is

$$V_{ab} = \sqrt{3}V_{an} \approx 440{,}9\ \mathrm{V}.$$

Additionally, the rms value of the line-to-line voltage can be calculated from the waveform shown in above. It is equal to the positive dc-bus voltage 120 degrees, 120 degrees negative and two times 60 degrees zero. The rms value is then:

$$V_{ab} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_{ab}^2 dx} = \sqrt{\frac{1}{2\pi}\, 2 \cdot V_s^2 \frac{2\pi}{3}} = \sqrt{\frac{2}{3}}V_s = \sqrt{\frac{2}{3}}\, 540\ \mathrm{V} \approx 440{,}9\ \mathrm{V}$$

and of course it is the same as calculated with the current and resistance.