Exercise Session 6: Basic Components of Power System

## EX1 Three Phase Power

How large real power can be transmitted in a three phase system at different voltages, when power factor $\cos \phi=0.9$ and current is 100 A per phase. The voltage is: a) 400 V , b) 20 kV , c) 110 kV and d) 400 kV .

## EX2 Capacity of a Transmission Line

How large real power can be transmitted by 795 kV line over 1000 km?. The conductor cross section is 1095 $\mathrm{mm}^{2}$ (radius $r=21,5 \mathrm{~mm}$ ). Spacing between phases is $\mathrm{d}=17 \mathrm{~m}$. How the power is changed if a bundle conductor is used, radius $r=30 \mathrm{~cm}$. The power angle (phase angle between voltages in line ends) is limited to $\delta=30$ degrees. The power transmitted in a transmission line is obtained from the power-angle equation:

$$
P=\frac{\mathrm{U}_{1} U_{2}}{X} \sin \delta
$$

Line inductance is:
$L=\frac{\mu_{0}}{2 \pi}\left[\frac{1}{4}+\ln \frac{d}{r}\right]$
$\mu_{0}=4 \pi \cdot 10^{-7} \mathrm{H} / \mathrm{m}$

And reactance $X=\omega L=2 \pi f L-\omega$ is angular frequency $(2 \pi f), f$ is frequency $(50 \mathrm{~Hz})$

## EX3 Medium Voltage Cable Capacity

Cables are usually short and the capacity is mostly limited by current. Current causes power losses which heat the cable. Heating must be limited due to maximum temperatures accepted to the insulation materials. Let us consider a 20 kV cable $3 * 240 \mathrm{~mm}^{2}$, maximum current Imax $=385 \mathrm{~A}$. What is the corresponding maximum power?. And what is the maximum power of $3 * 50 \mathrm{~mm}^{2}$ cable, $\operatorname{Imax}=155 \mathrm{~A}$ ?

## EX4 Capacity of a Medium Voltage Overhead Line

20 kV MV line capacity is mostly limited by voltage drop, due to long distances. Let us take a line with resistance and reactance $r=0.3 \mathrm{ohm} / \mathrm{km}, x=0.3 \mathrm{ohm} / \mathrm{km}$, maximum voltage drop $\Delta U \max =5 \%$, and length 40 km .

What is the maximum power that can be transmitted if power factor is =1 $(P=S)$

Voltage drop longitudinal component: $\Delta U p=$ RIp + XIq
Voltage drop transverse component: $\Delta \mathrm{Uq}=\mathrm{RIq}+\mathrm{XIp}$

ELEC-E8422 An Introduction to Electric Energy

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a) 400 V
b) 20 kV
c) 110 kV
d) 400 kV

Three phase power: $P=\operatorname{Sqrt}(3) U I \cos \phi$
a) $\mathrm{P}=1.732 * 400 * 100 * 0,9 \mathrm{~W}=62 \mathrm{~kW}$
b) $\mathrm{P}=1.732 * 20$ * 100 * $0,9 \mathrm{~kW}=3,1 \mathrm{MW}$
c) $P=1.732 * 110 * 100 * 0,9 \mathrm{~kW}=17,1 \mathrm{MW}$
d) $\mathrm{P}=1.732 * 400$ * 100 * $0,9 \mathrm{~kW}=62,3 \mathrm{MW}$

## EX2 Capacity of a Transmission Line

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And reactance $X=\omega L=2 \pi f L-\omega$ is angular frequency $(2 \pi f)$, $f$ is frequency ( 50 Hz )
$\mathrm{L}=210^{-7}(0.25+\ln (17 / 0.0215)) \mathrm{H} / \mathrm{m} \sim \mathrm{X}=0.43 \mathrm{ohm} / \mathrm{km}$
$\mathrm{P}=\left[(795 \mathrm{kV})^{2} /(1000 \mathrm{~km} * 0.43 \mathrm{ohm} / \mathrm{km})\right] * \sin 30^{\circ}=735 \mathrm{MVA}(\mathrm{MW})$
If bundle conductor $r=30 \mathrm{~cm}$
$\mathrm{X}=0.27$ ohm $/ \mathrm{km}$
$\mathrm{P}=1170 \mathrm{MVA}(\mathrm{MW})$

## EX3 Medium Voltage Cable Capacity

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Power factor $\cos \phi=1 \Leftrightarrow S=P$
Power: $\mathrm{P}=\mathrm{S}=\operatorname{Sqrt}(3) * \mathrm{U}$ *

U is line voltage ( 20 kV ), I is phase current (385 A)
$P=\operatorname{Sqrt}(3) * 20 \mathrm{kV} * 385 \mathrm{~A}=13,3 \mathrm{MW}$
For 3 * $50 \mathrm{~mm}^{2}$ cable, $\operatorname{Imax}=155 \mathrm{~A}$
$P=\operatorname{Sqrt}(3) * 20 \mathrm{kV} * 155 \mathrm{~A}=5,37 \mathrm{MW}$

## EX4 Capacity of a Medium Voltage Overhead Line

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What is the maximum power that can be transmitted if power factor is =1 $(P=S)$

Voltage drop longitudinal component: $\Delta \mathrm{Up}=\mathrm{RIp}+\mathrm{XIq}$
Voltage drop transverse component: $\Delta \mathrm{Uq}=\mathrm{RIq}+\mathrm{XIp}$
$(I p=I \cos \phi \quad \& I q=I \sin \phi)$. With resistive load only real power is transmitted and $\phi=0^{\circ}$.

Power factor $=1 \Leftrightarrow I=I p \& I q=0 \Leftrightarrow \Delta U p=\operatorname{RIp} \& \Delta U q=X I p$
Solution using longitudinal component:
$\Delta U p=\operatorname{RIp}=0.05 * 20 \mathrm{kV} / \operatorname{Sqrt}(3) \Leftrightarrow(0.3 \mathrm{ohm} / \mathrm{km} * 40 \mathrm{~km}) \mathrm{Ip}=0.05 * 20 \mathrm{kV} / \operatorname{Sqrt}(3)$
$\mathrm{Ip}=48 \mathrm{~A}$

And the corresponding maximum power is: $\mathrm{P}=3 \mathrm{Uv}$ * $\mathrm{Iv}=3$ * $11,5 \mathrm{kV}$ * $48 \mathrm{~A}=1,65 \mathrm{MW}$

The effect of transverse component:
$U=\operatorname{Sqrt}\left[(1-0.05)^{2}+0.05^{2}\right]=\operatorname{Sqrt}[0.9025+0.0025]=$ Sqrt $[0.905]=0.9513$ (vrt 0.95)

## THE LESSONS LEARNED:

In case of short lines, the capacity is limited by load current and the heating in conductors and insulation. This is the case for instance in city networks with cables.

In longer lines, like medium voltage overhead distribution lines, voltage drop is the dominating limiting factor for capacity.

In case of high voltage transmission, the power capacity depends on phase angle between line ends, together to line reactance and voltage magnitude.

