Aalto University

Shapes in Action Sept 21st

Orbifolds and topology

Program schedule for Sept 21st

13:15 Where are we ?

13:30 Orbifolds: How to relate topology to patterns ?

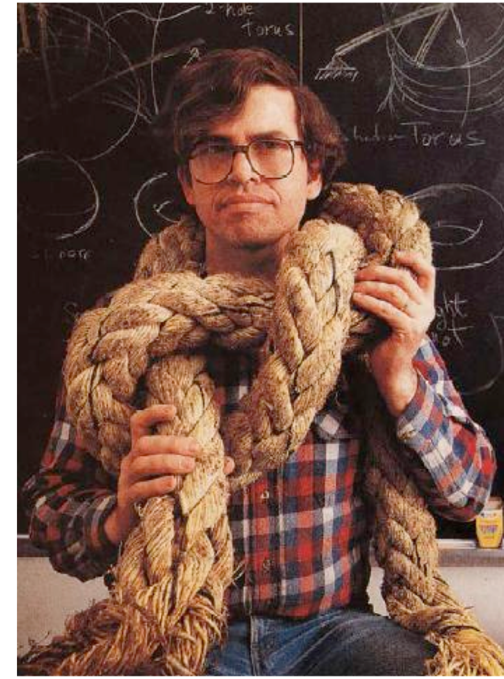
14:00 Magic theorem consequences

14:30 Break

14:50 Textile analysis in groups

15:30 Find orbifolds of your patterns

Where are we ?



Goal : Understanding

Signature/Orbifold notation due to B. Thurston and J.H. Conway (90´)

Done:

- Basic ideas on symmetries and signatures for planar patterns
- Miracle theorem stated: All planar patterns cost 2 euros (free to choose your favorite unit !)

Next: Based on Miracle theorem, what are the possible symmetries? Towards the proof for the ‘Miracle Theorem’ and its *applicability*, more topology

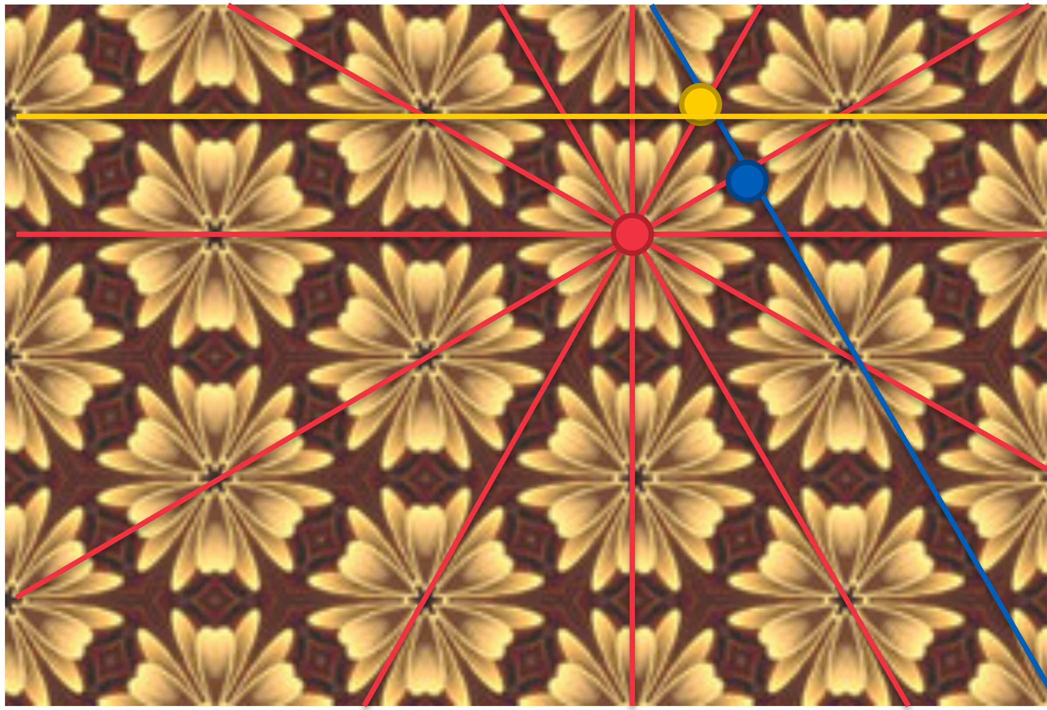
Pattern analysis steps

1. Draw all mirror lines (=lines of reflection)
2. Find the fundamental domain of the kaleidoscope
3. How many lines meet on each vertex? => Local symmetries of form *N
4. Find rotationally symmetric points (non-kaleidoscopic)
5. Are there mirror images without mirrors ? Then there must be at least one miracle x.
6. Helpful to look at the price list during the analysis and take the miracle theorem into account (Be patient: the proof will come a bit later…)
7. If there is there is only repletion into two directions (nothing from above) then the pattern is 'wandering' o

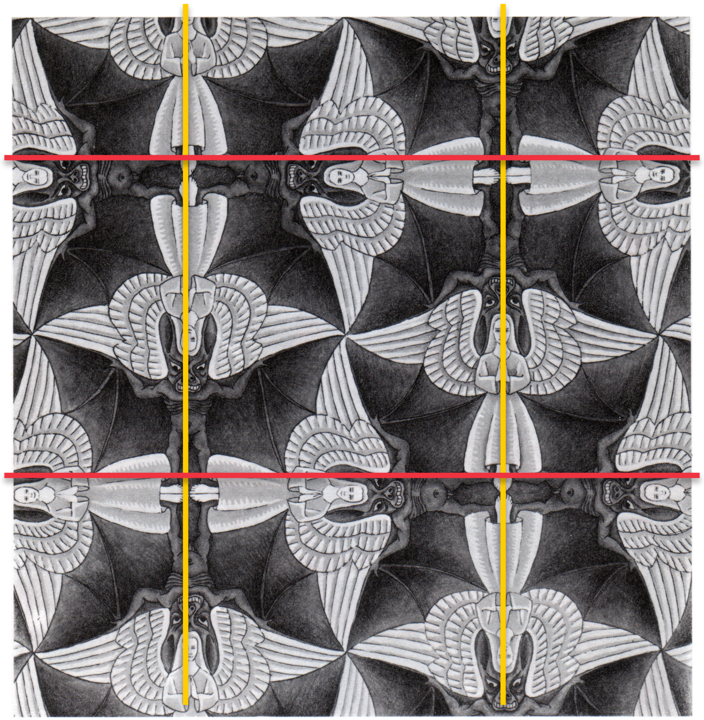
Signatures for plane patterns through *local* symmetries: *632

Cost=1+½(⅚+⅔+½)=2euro

- 6 lines of reflection
- 3 lines of reflection
- 2 lines of reflection



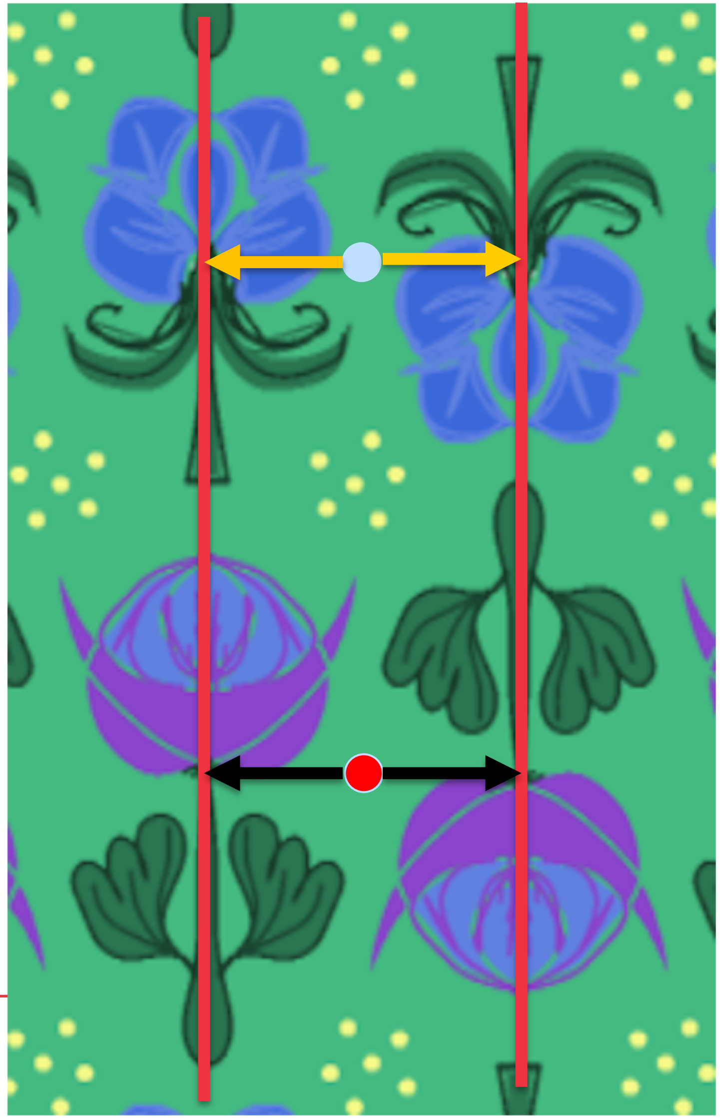
Only one type of kaleidoscopic vertex (*2) and one type of rotational vertex (4) => signature 4*2



Cost = ¾+1+¼= 2euro

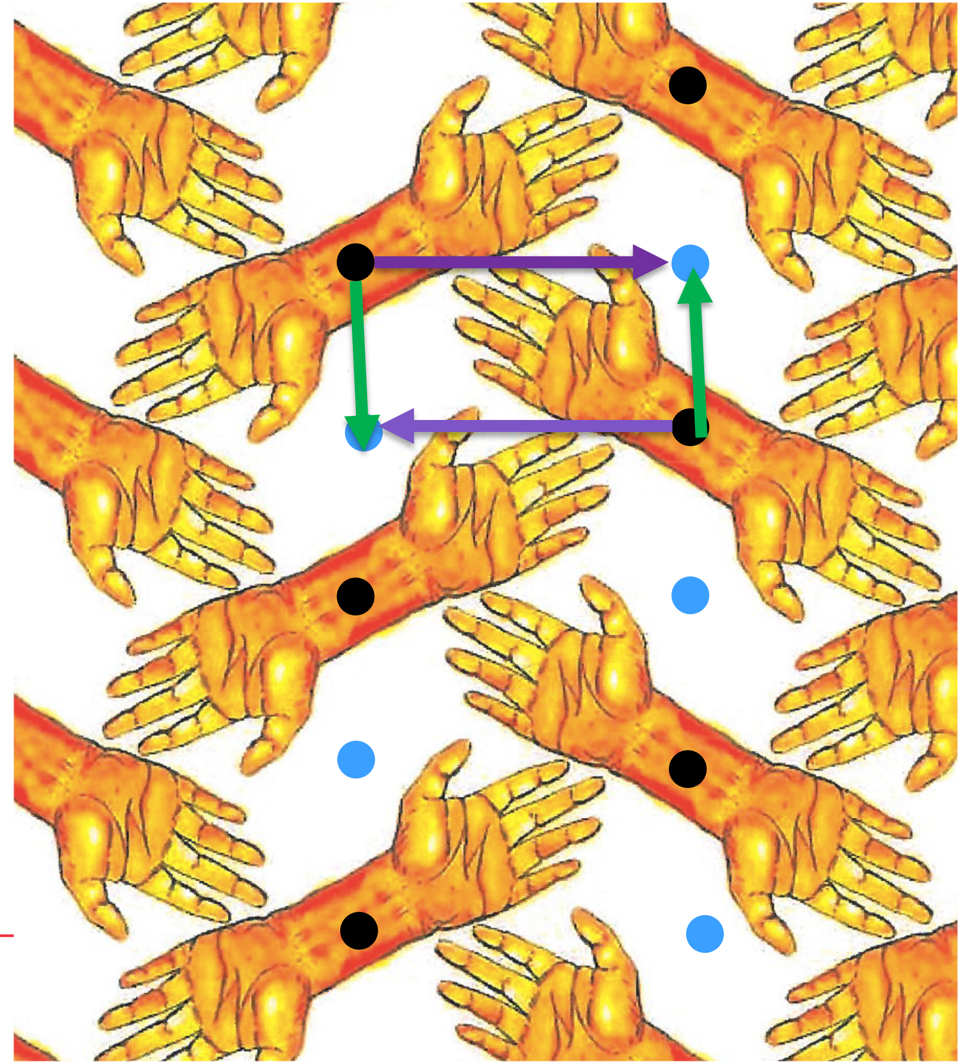
22*

- One reflection line
- Two rotation points

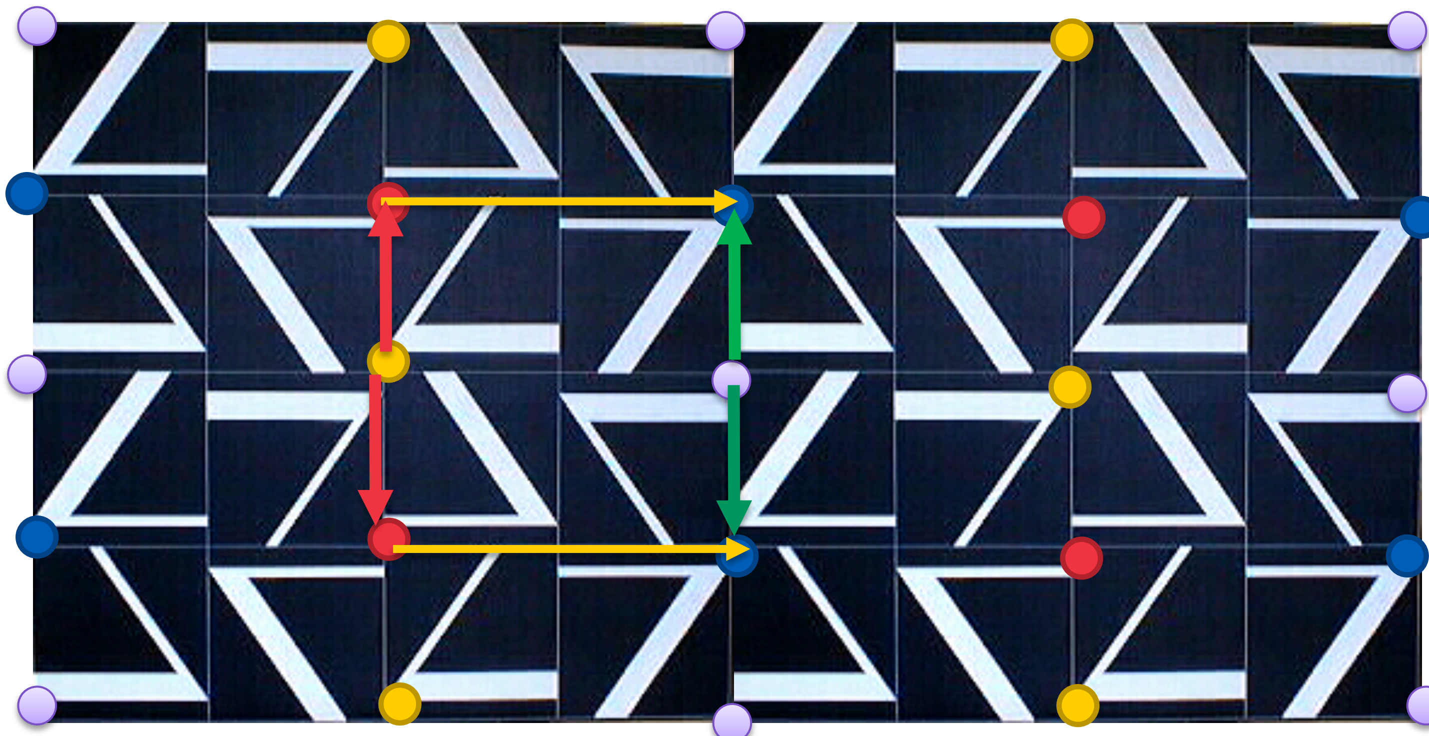


22x

- No reflection lines
- Two different rotation points of order 2 (price ½ € each)
- One miracle (mirror image without a mirror, x 1€)
- => Total cost 1+ ½ + ½ = 2 €

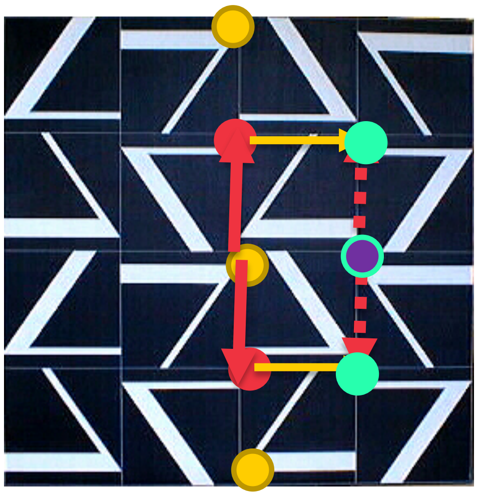


2222 vs. 22x

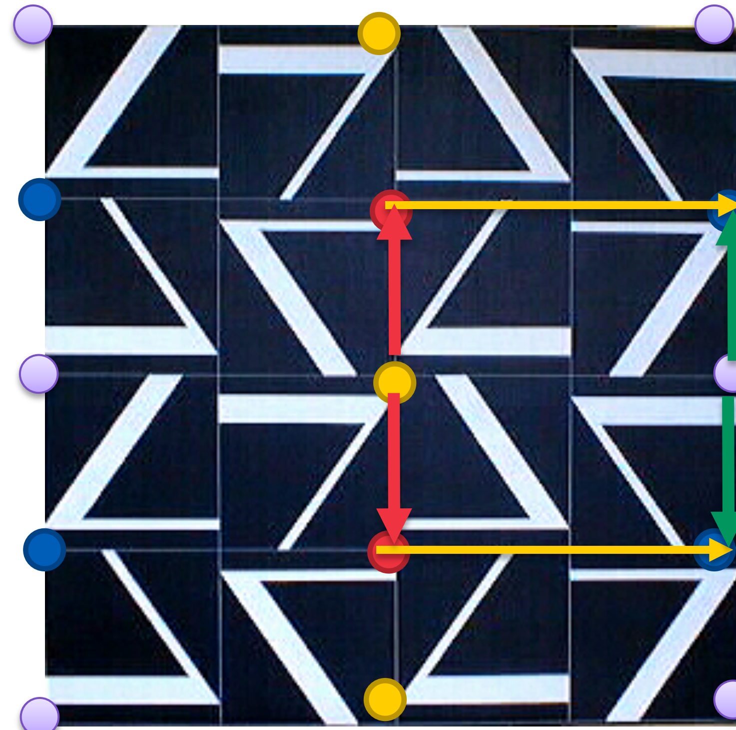


Cannot split a fundamental domain in 2222 into two in 22x

- **Which two rotation points to choose instead of 4?**



- On the left **NOT** rotation points
- contradicting info on the red edges



- *Note: horizontal reflection (+ translation) in the middle does not induce a global glide reflection !*

Every property has its cost (in euros)

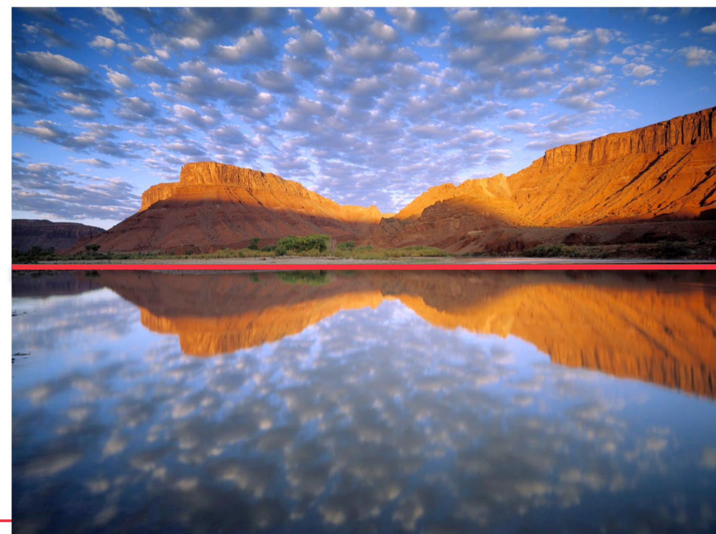
Symbol	Price	Symbol	Price
o	2	* or x	1
2	½	2	¼
3	⅔	3	⅓
4	¾	4	⅜
5	⅘	5	⅖
6	⅚	6	5/12
n	(n-1)/n	n	(n-1)/2n

Note: Blue symbols refer to operations that preserve orientation, red ones reverse orientation

Star * *Cost =1euro*

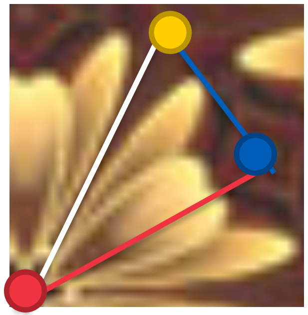
Star * (in the signature notation) denotes a *mirror* or *kaleidoscopic symmetry* = reflection wit respect to a line.

Star alone means: there is one (and only one) single line of mirror symmetry.

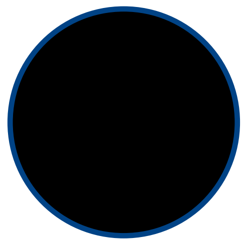


What kind of fundamental domains have we found ?

Triangle with no identifications on the boundary (different parts coming from reflection lines)



Topologically (= deformations that do not produce new holes are allowed)**: Disk**



Signatures for plane patterns through *local* symmetries: *632

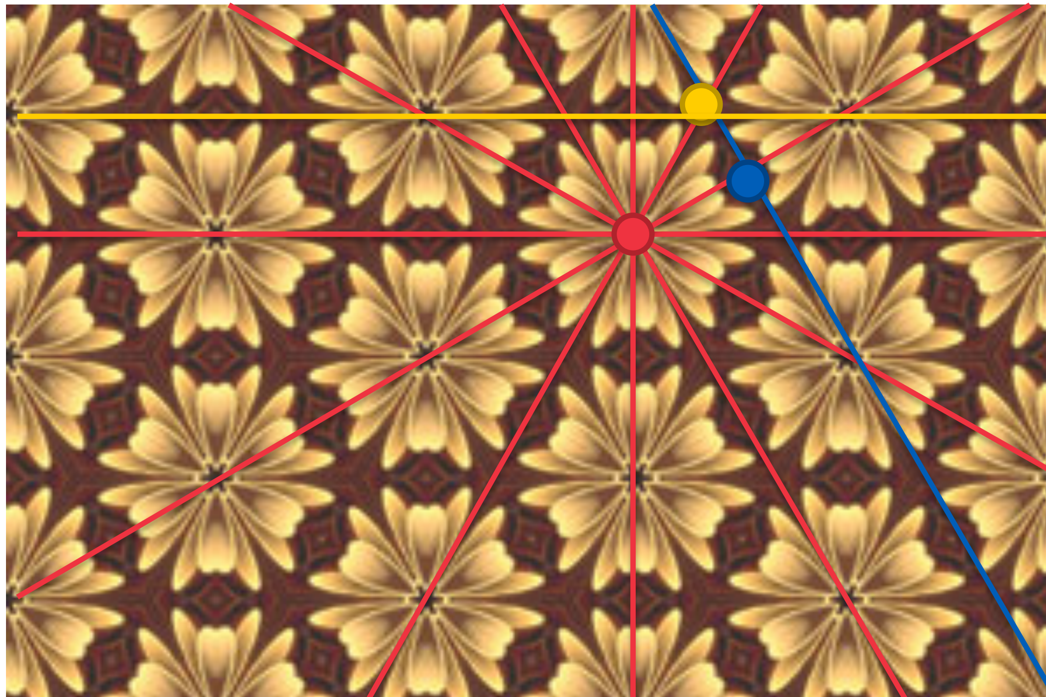
Cost$=1+\frac{1}{2}(\frac{5}{6}+\frac{2}{3}+\frac{1}{2})=2$euro

6 lines of reflection

3 lines of reflection

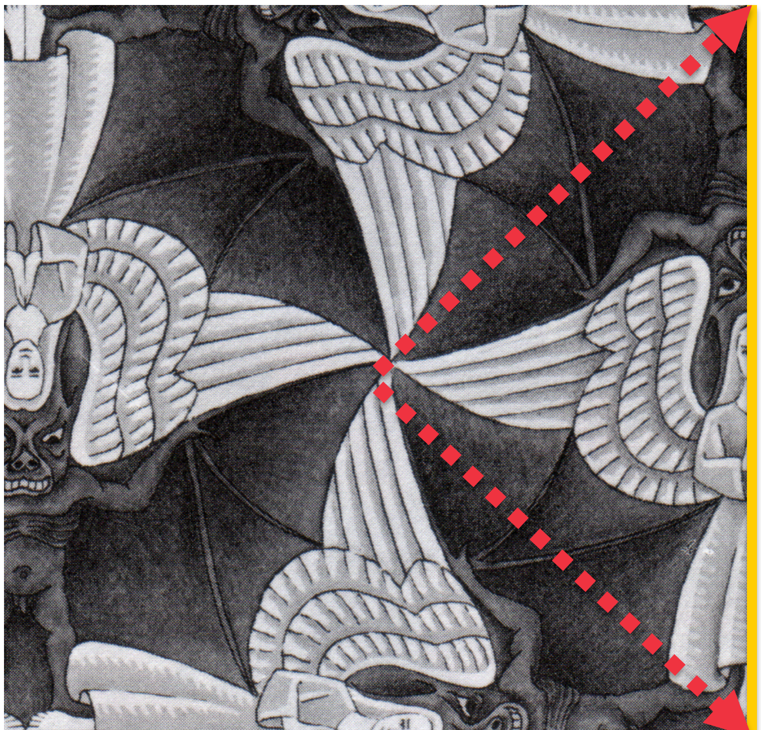
2 lines of reflection

Orbifold: Topological disk



Note: reflections equate same type of points (**orbits**) whose representatives in the chosen triangle give the **orbifold** (after identification)

Combination of rotation points and reflection lines
Ex: 4*2



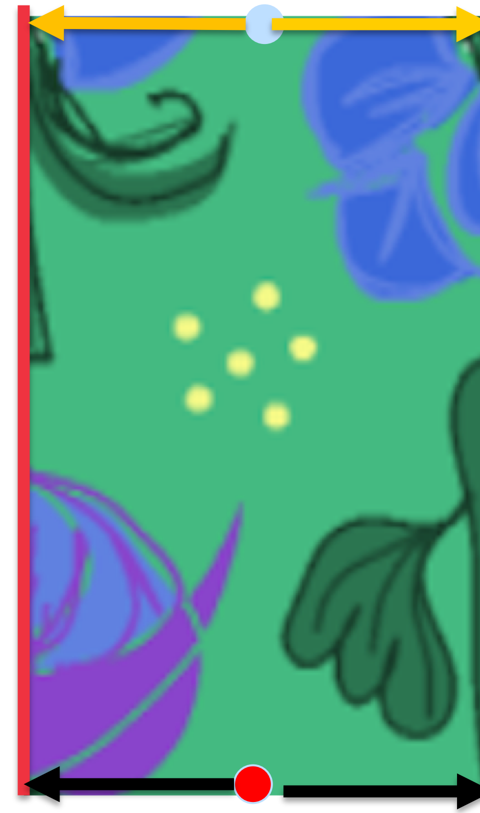
Fundamental domain: **A triangle with some identifications on the boundary** (red arrows due to the presence of a rotation point in the middle)

What is the **topological shape** of the piece **after the identification** (= gluing the red boundary arrows) ?

22*

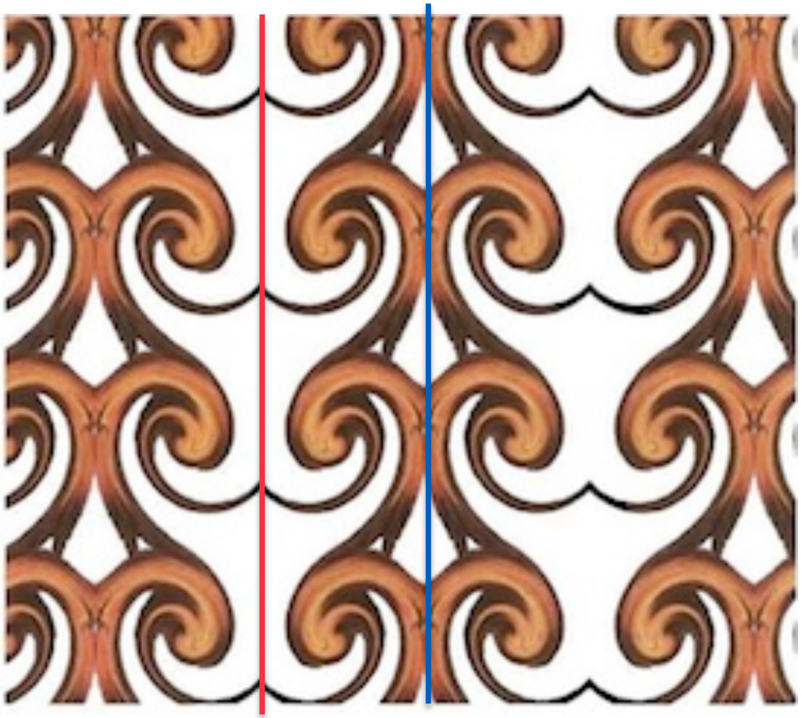
Disk orbifold again ?

Are there other types ?



Cost of a miracle (x) = 1euro

Signature ** Annulus orbifold

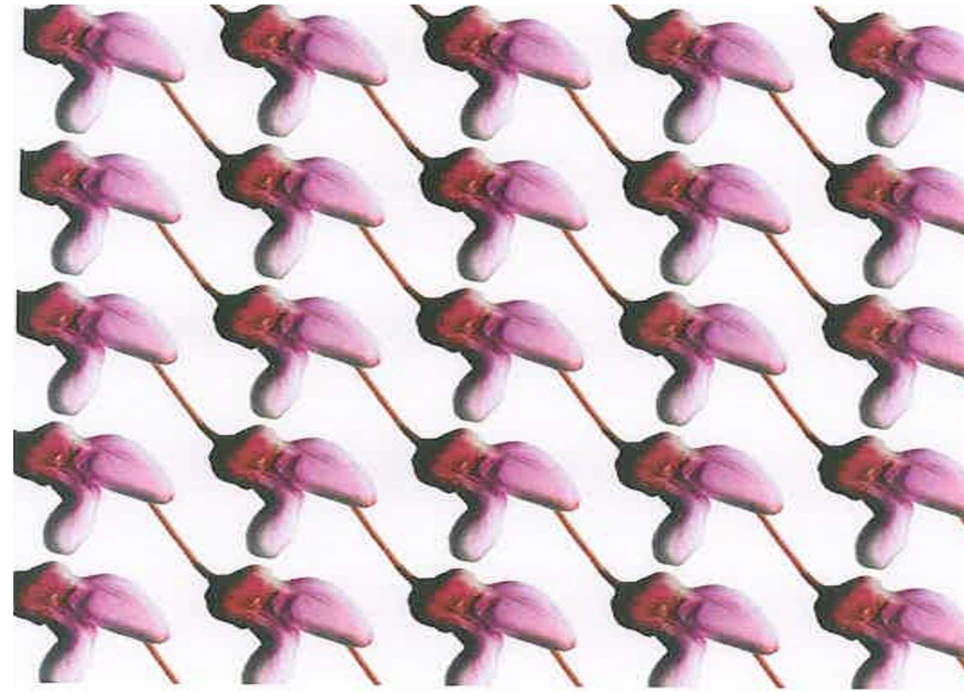


1+1= 2euro

Signature *x Möbius band orbifold

1+1=2euro

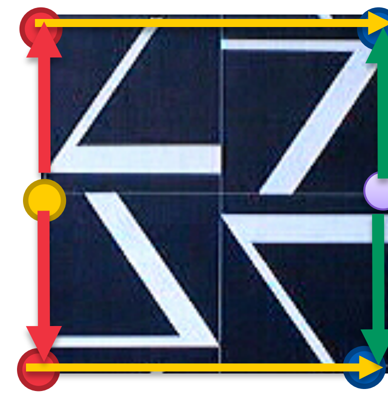
Wanderings O



Torus orbifold !

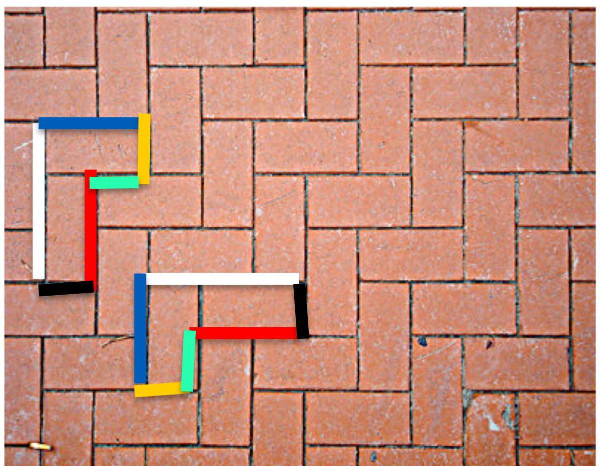
Rotation points only
Ex 2222

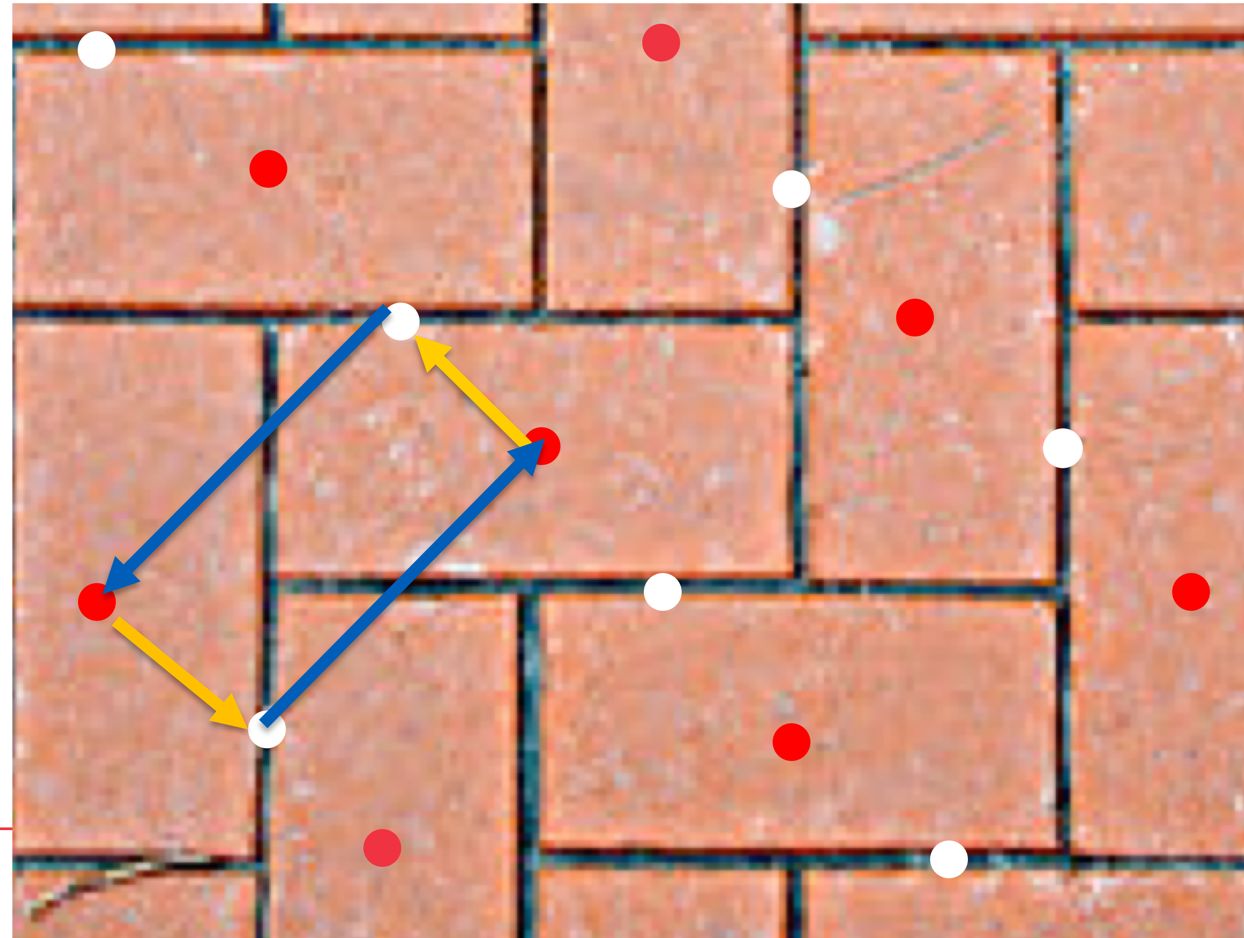
What is this shape after the boundary identifications ?



Ex: Brick walls/pavements

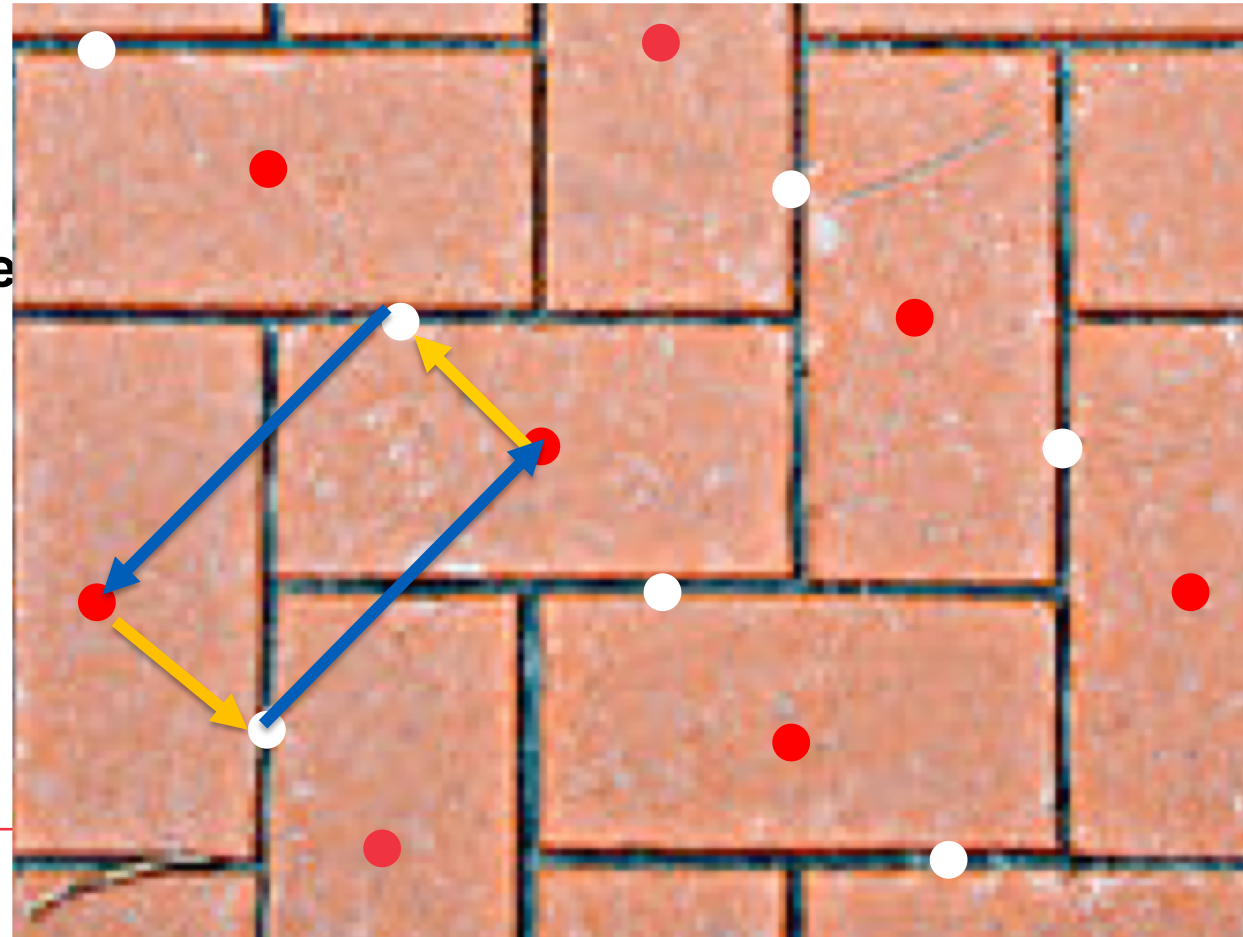
- 2 rotation points
- Mirror images without a reflection line
- => 22x



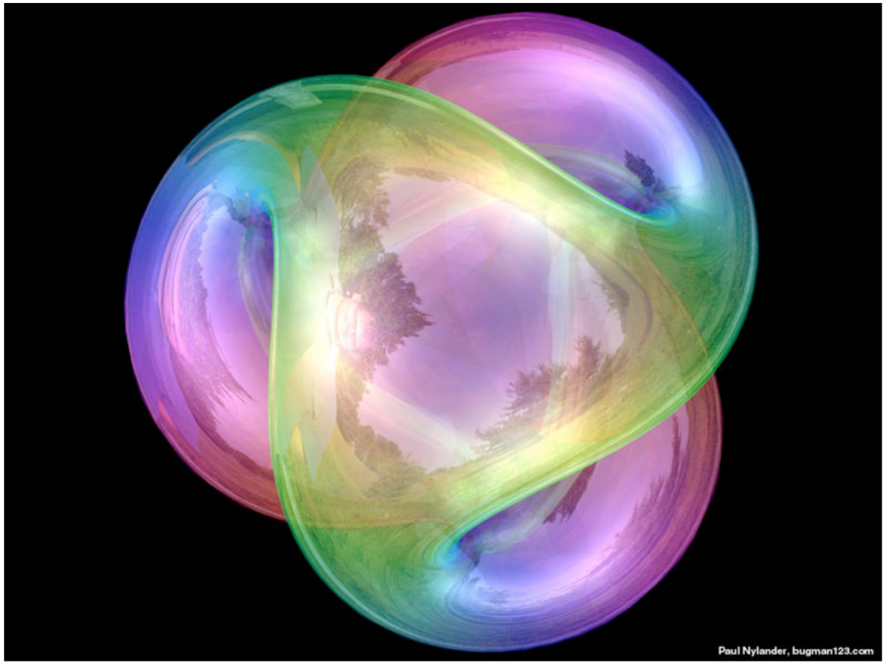


What is the orbifold of 22x symmetry?

What shape do you get when you do the identifications on the boundary ?

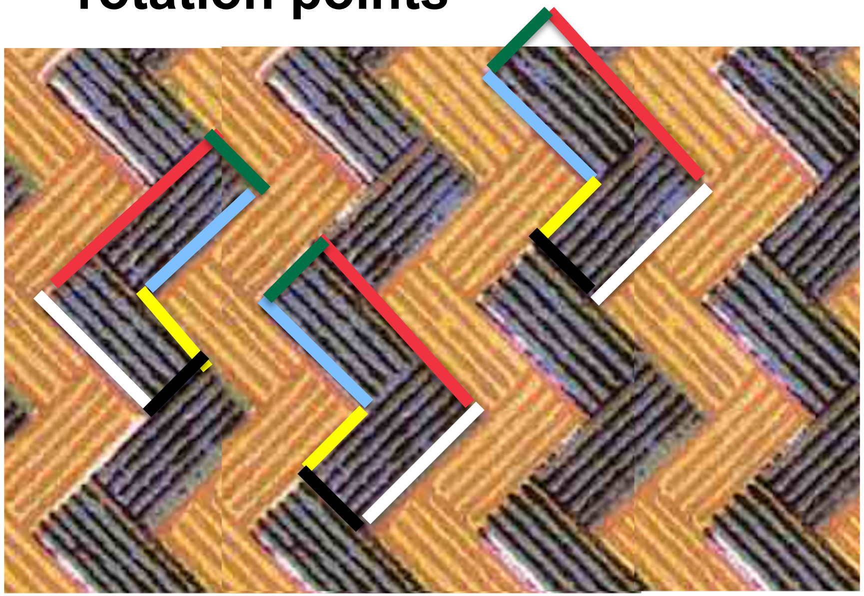


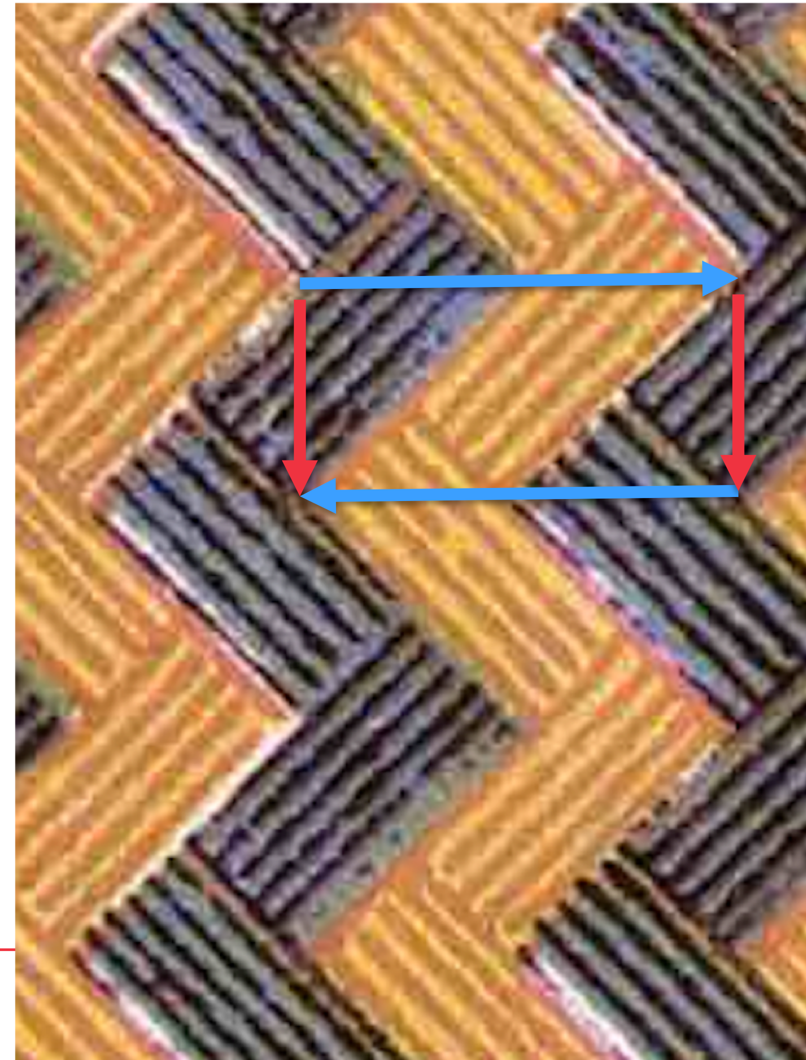
Real projective plane !



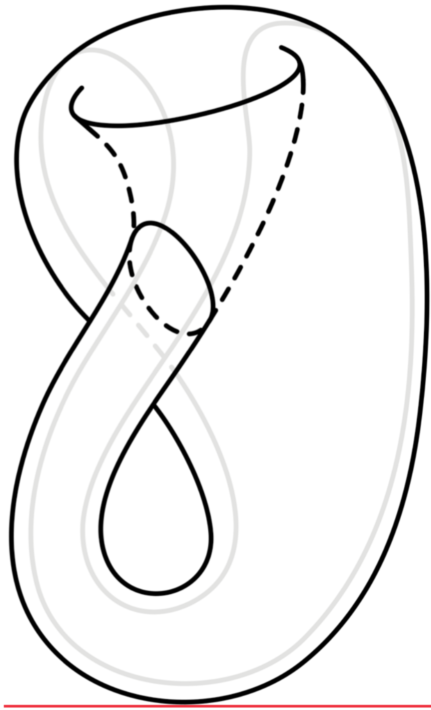
What about xx ?

- Two miracles (mirror images without reflection lines) no rotation points

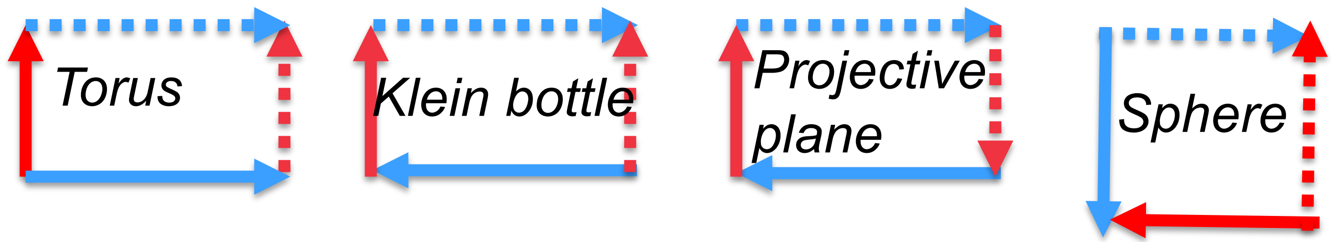




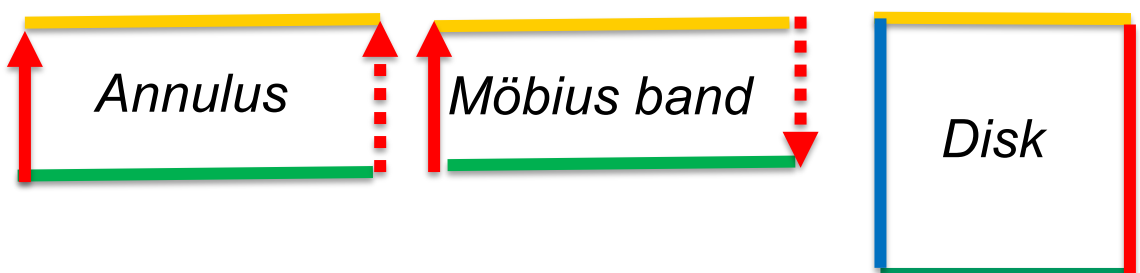
Klein bottle !



Surfaces via identifying boundary components of polyhedrons



(no boundary after gluing)



(non-empty boundary)

Are there other orbifolds coming from planar patterns ?

How many different signatures exist for plane patterns?

Assuming Magic Theorem to hold, this is similar question as asking:

How many different ways can I make change for one euro if I can use only 50, 20, 10 and 5 cents?

- Find all blue types
- Find all red types
- Find all hybrids

Blue types (orientation preserving)

Price for one n-fold rotational point is (n-1)/n <1 => need more than Two to cost 2 euros:

- 333, 442, 632
- 2222
- Wonder O

What is the orbifold of the given signatures?

Red types without miracles

Observation: If no miracles x then *AB….N corresponds to ABC…N since

1+(A-1)/2A+(B-1)/2B+….+(N-1)/2N =2 ⇔

(A-1)/A+(B-1)/B+….+(N-1)/N =2

=> Only types *333, *442, *632, *2222

can occur in addition to ** .

What is the orbifold of these?

Hybrids: mixture of blue and red or involve x

Observations :

- switching between **n*** and ***nn** does not change the total cost
- replacing **x** with * -”-
- replacing **final** * with **x** -”-
- **=>**
- **cannot be changed to a hybrid: *632**
- ***442 => 4*2**
- ***333 => 3*3**
- ***2222 => 2*22 => 22* => 22x**
- **** => *x => xx**

Orbifolds of the above?

Conclusions

Only 17 possible signatures = 17 symmetry types for repeating patterns in the plane:

*632	*442	*333	*2222	**
			2*22	*x
	4*2	3*3	22*	xx
			22x	
632	442	333	2222	o

Possible orbifolds for planar patterns

Orientable

Sphere (**632** **442** **333** **2222**)

Torus **o**

Annulus **

Disk (***632** ***442** ***333** ***2222**
2*22 **4*2** **3*3** **22***)

Non-orientable

Projective plane **22x**

Klein bottle xx

Möbius band *x

Groupwork with textiles

1) Choose 6 different patterns in your group to be presented for Laura next Fri

2) Take pictures and upload (as a group) to MyCourses asap or latest next Tue

3) Presentations on Fri only 5min/group

4) Give criteria/justification (either artistic or mathematical) for your choice.

5) For the repeated patterns, find the signature and orbifold (ignore ‘mistakes’ and minor details in the prints)

Q: How to benefit from the classification in (flat) surface design in practise?