Aalto University

Shapes in Action
Sept 25th

Spherical patterns

Program schedule for Sept 25th

15:15 Spherical symmetries

15:45 Folding activity

16:30 Break

16:50 Symmetries of Archimedean solids

17:45 Ideas for essays

Possible orbifolds for planar patterns

Orientable

Sphere (632 442 333 2222)

Torus o

Annulus **

Disk (*632 *442 *333 *2222
2*22 4*2 3*3 22*)

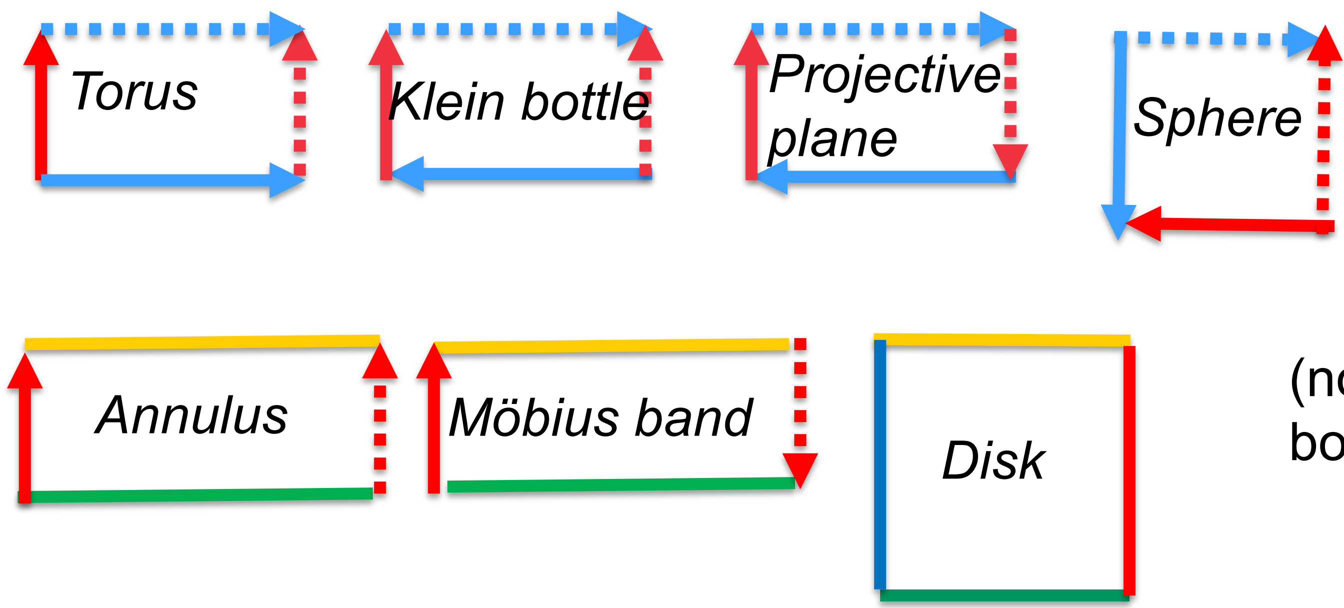
Non-orientable

Projective plane 22x

Klein bottle xx

Möbius band *x

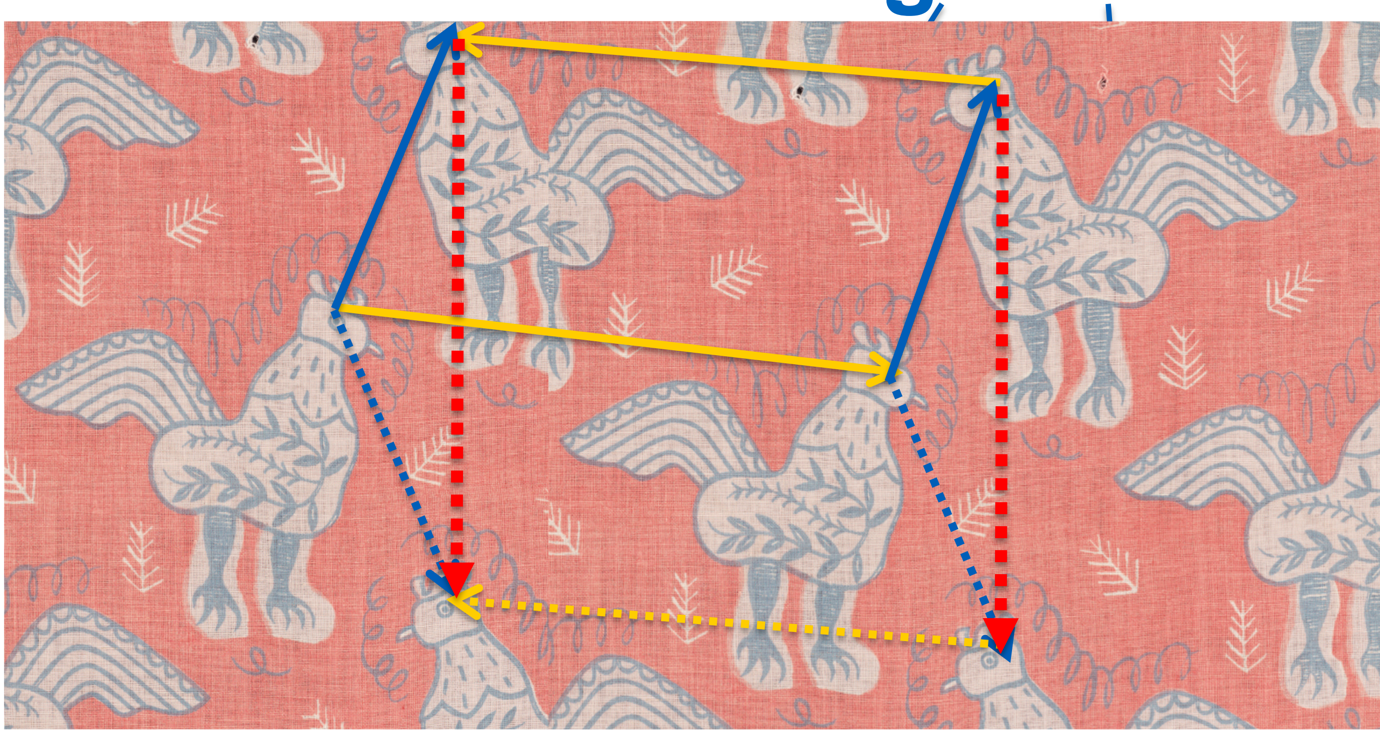
Orbifolds (of planar patterns) through boundary identifications



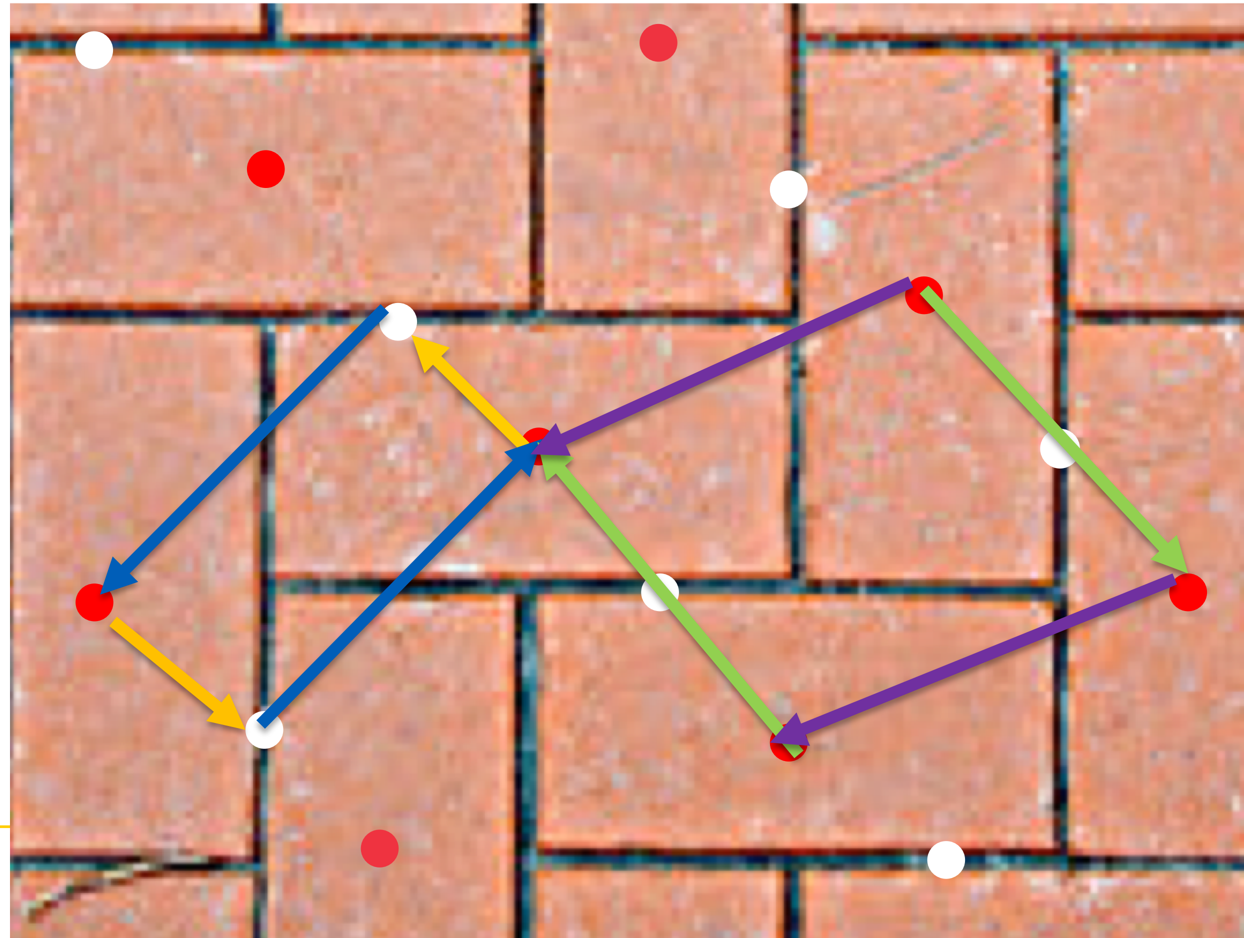
(no boundary after gluing)

(non-empty boundary)

Note: Two Klein bottles give a torus



Two projective spaces give a Klein bottle



Every property has its cost (in euros)

Symbol	Price	Symbol	Price
o	2	* or x	1
2	½	2	¼
3	⅔	3	⅓
4	¾	4	⅜
5	⅘	5	⅖
6	⅚	6	5/12
n	(n-1)/n	n	(n-1)/2n

Note: Blue symbols refer to operations that preserve orientation, red ones reverse orientation

What about spherical symmetries?

Rotation lines (vs points) and reflection planes (vs lines)

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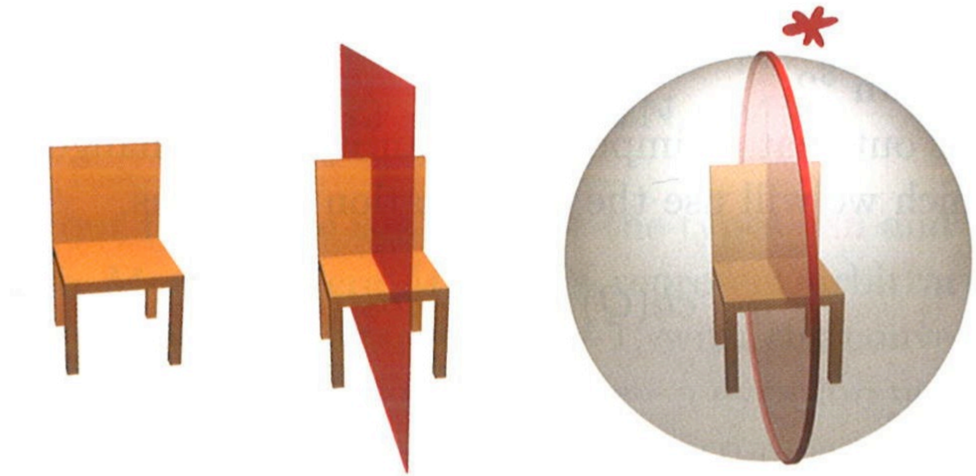
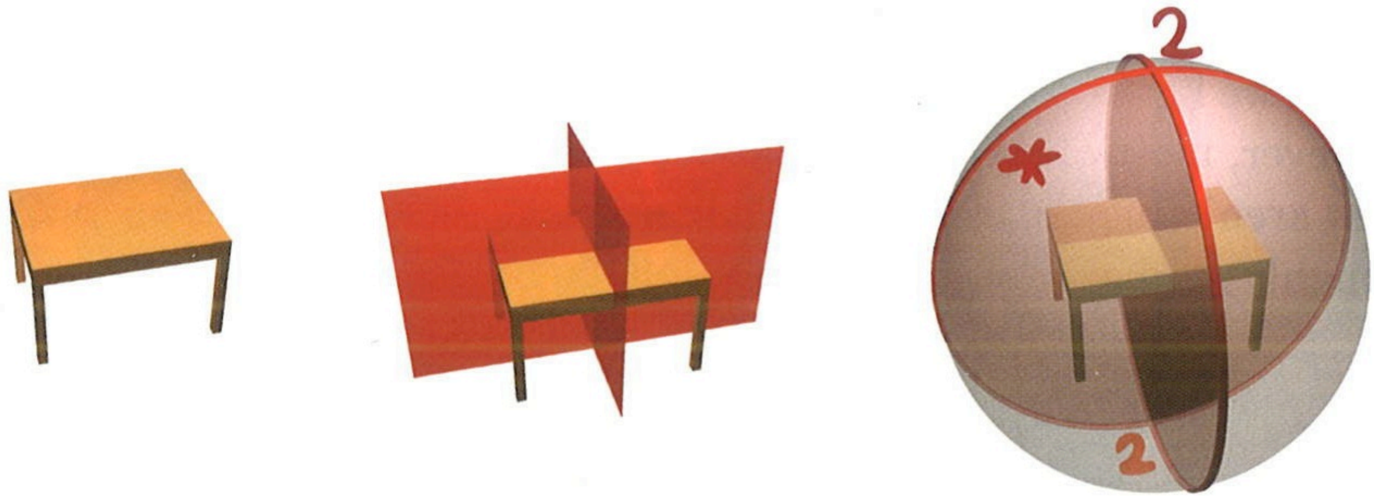
Temari balls, Bathsheba sculptures, ….

Spherical patterns are *cheaper* than planar patterns. (Will see….)

Ex: Bilateral symmetry = * interpreted as a reflection wrt to plane cost only 1 euro



Price of a rectangular table



Two intersecting reflection planes give signature *22, which cost **1+1/4+1/4=3/2** euro **=>** spherical patterns can have **different total prices.**

An important quantity ch=change (in euros)

Change from signature Q: **ch**(Q) = 2-**cost**(Q) euro

Above:

- **For the chair: ch(*)=2-cost(*)=2-1=1 euro**
- **For the table: ch(*22)=2-cost(*22)=2-3/2=1/2 euro**

The Magic Theorem for spherical patterns

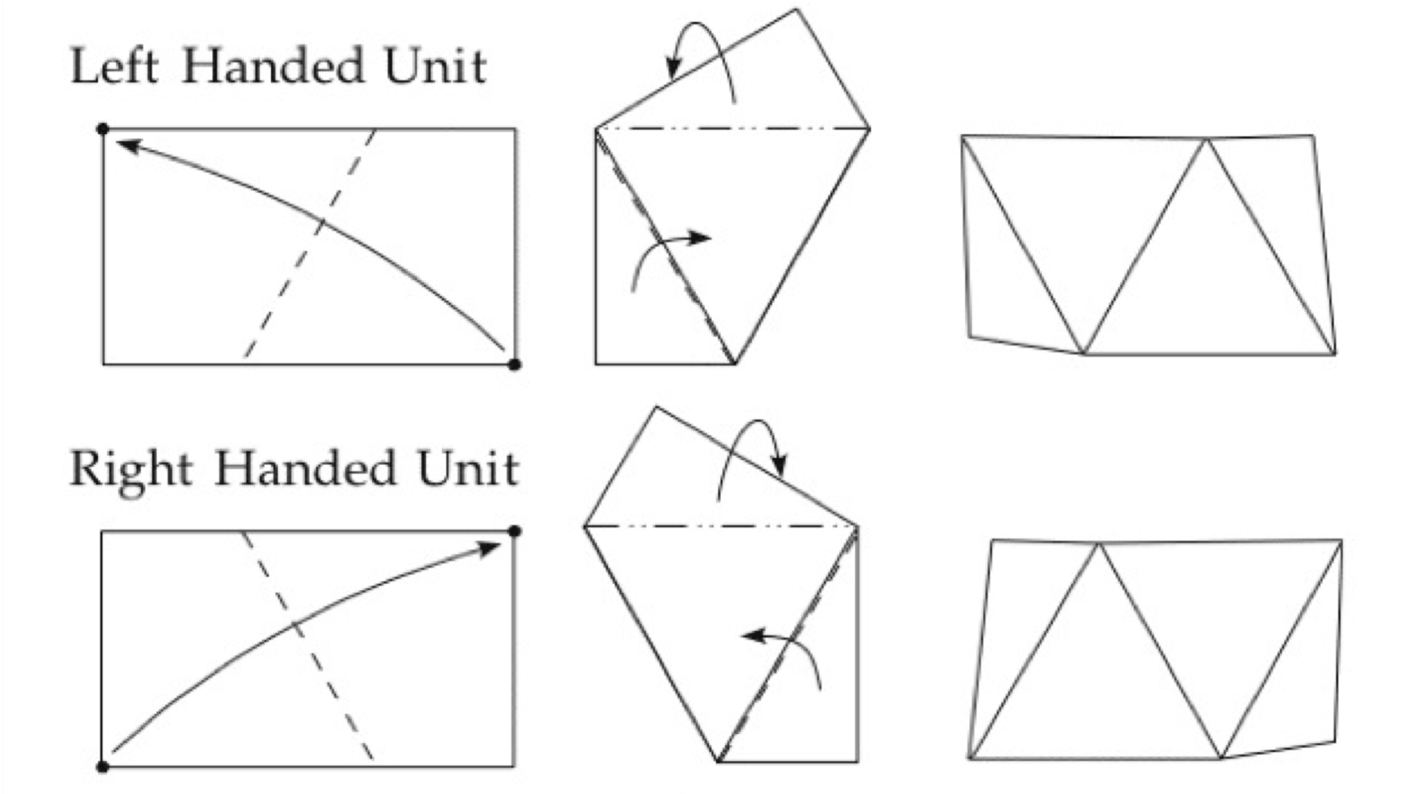
The signature of a spherical pattern costs exactly 2-2/d euros, where d is the total number of symmetries of the pattern.

Note:

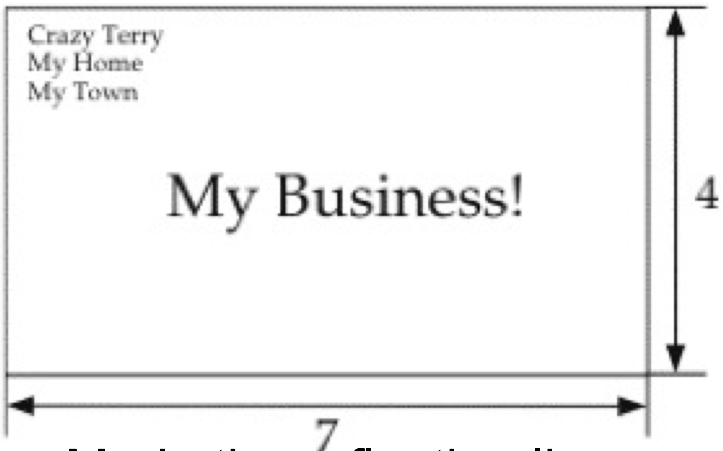
- **ch = 2/d**
- **for the chair d=2, for the table d=4**
- **In the plane case: d=∞ => only *one* Magic Theorem**

Lets produce some objects for analysis via folding …

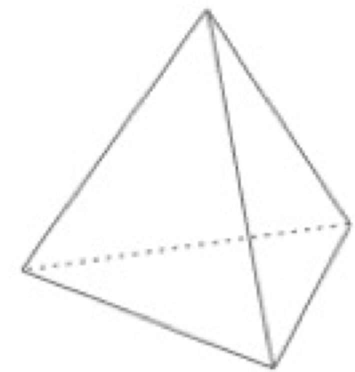
Business card modules (T. Hull, J. Mosely, K. Kawamura)



Are triangles equilateral ? Why ?

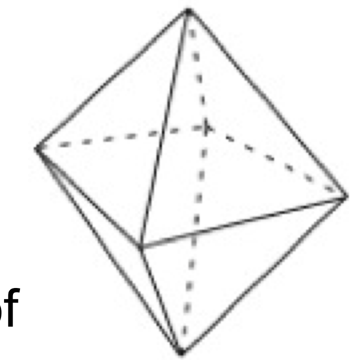


1) Make one left handed and one right handed module and try to **lock** them to a tetrahedron



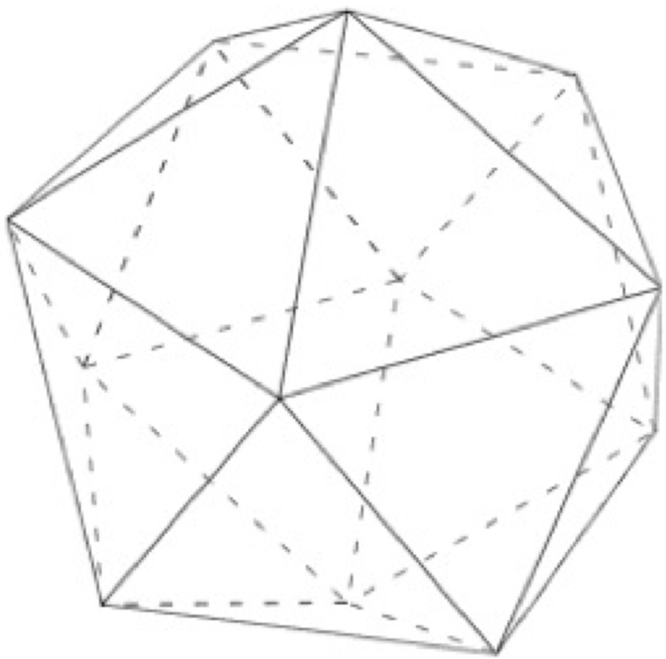
- Mark the reflection lines on your module
- What is the fundamental domain/orbifold ?
- How many reflection lines (=reflection plane intersection with the module) meet on the vertices of the fundamental domain?
- What is the number of symmetries ?
- Check that the Magic theorem holds

2) Construct an octahedron from 4 units



- Same questions as for the tetrahedron above
- Calculate V-E+F, V=number of vertices, E=number of edges, F= number of faces (also for the tetrahedron)

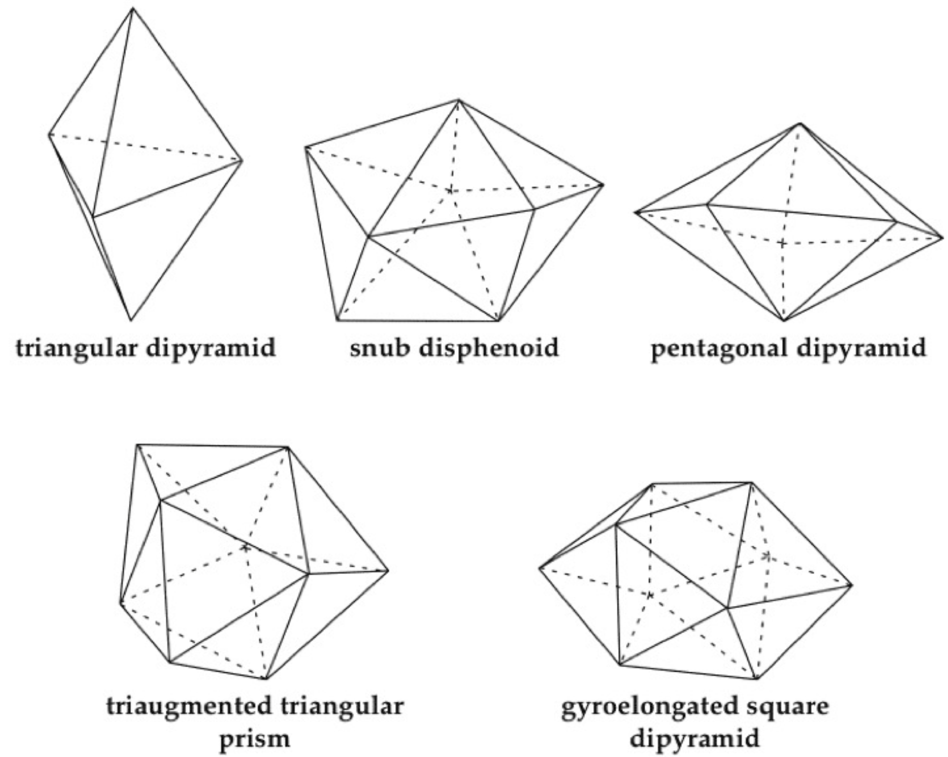
Possible to construct also an icosahedron from these modules



Hint: Use tape in construction
What other polyhedrons can be constructed from these modules ?

Same questions as for previous polyhedrons

Johnson solids with triangular faces



14 different spherical symmetry classes

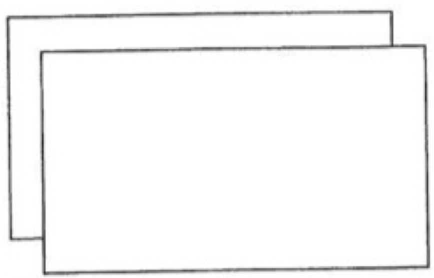
*532	*432	*332	*22N	*NN
				N*
		3*2	2*N	
				Nx
532	432	332	22N	NN

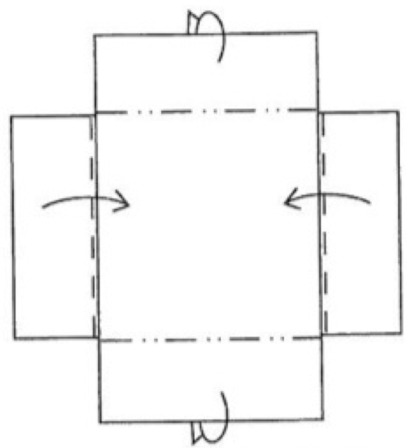
Note:

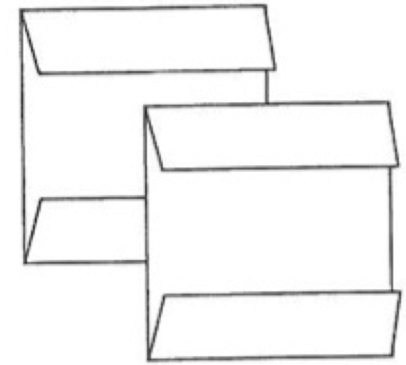
- **N=** 1,2,3… **but** digits 1 are omitted
- 1*=*11=*
- **However:** For example 11 11 = two rotation points of order 11

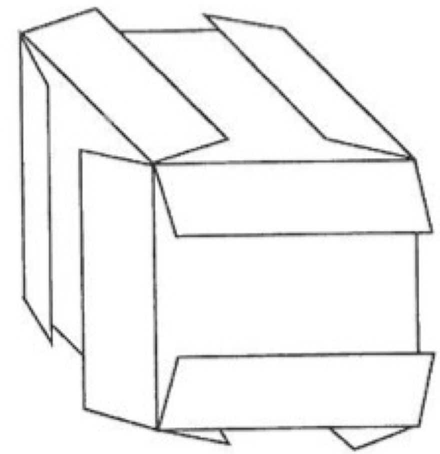
Business card cube

6 modules (one/face) constitute a (‘unpaneled’) cube, that can be joined together with flaps that remain outside.





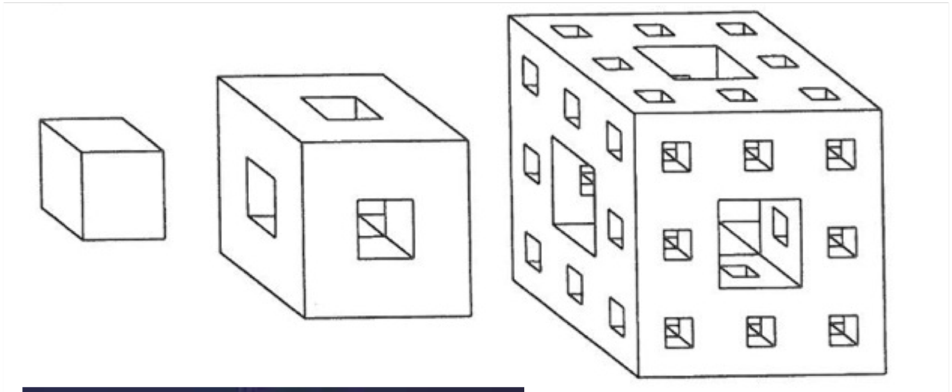


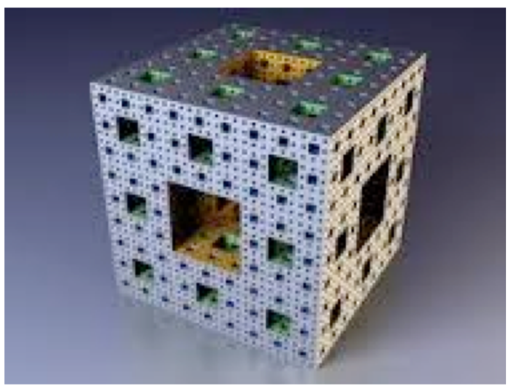


How do you ‘panel’ a cube **?**

Same questions as for earlier polyhedrons !

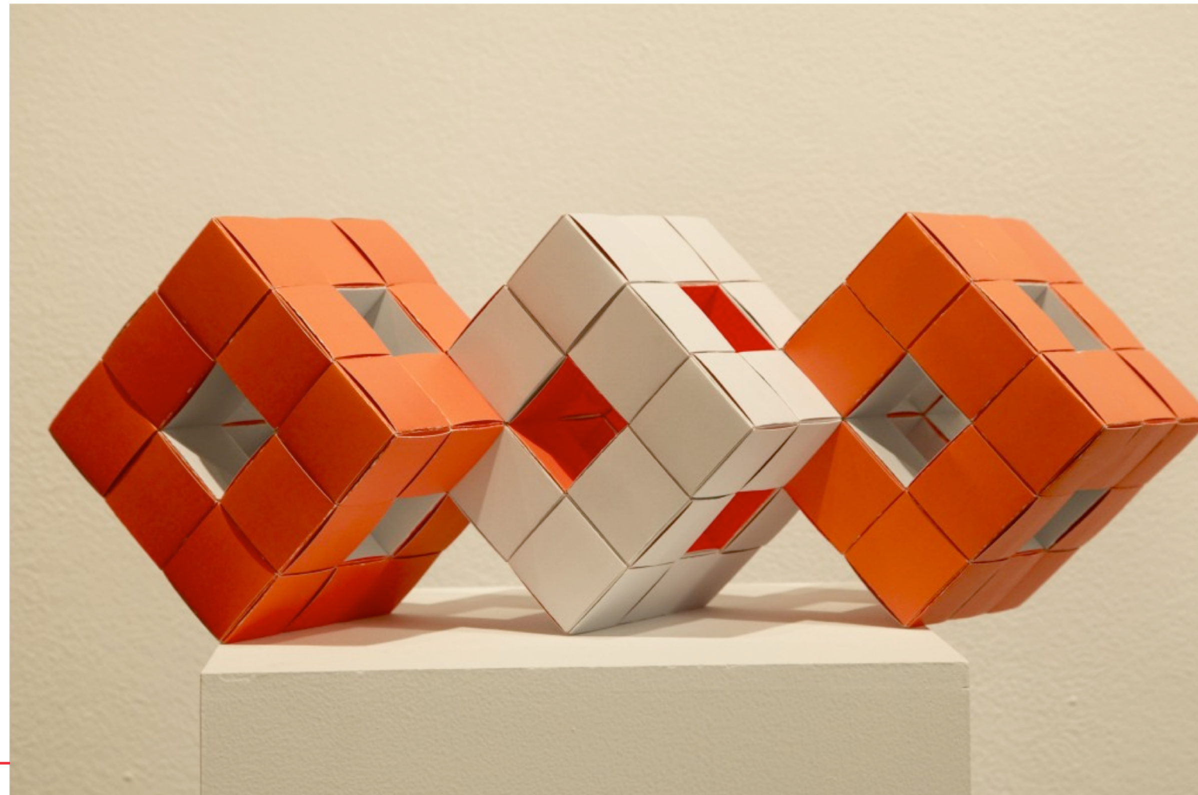
Building idea: Menger's Sponge





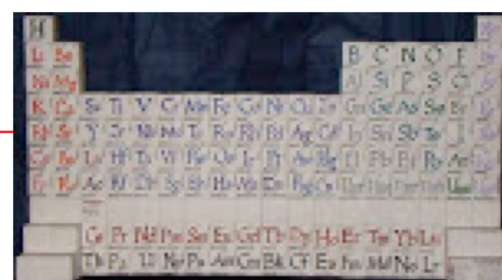
Jeannine Mosely
66048 business cards

Three interlinked Level One Menger Sponges, by Margaret Wertheim.



Union Station 2014, Worcester
more thand 60 000 business cards,
500 assistants

Mosely snowflake sponge 2012
49 000 business cards



James Lucas 2011,
periodic table
1414 business cards

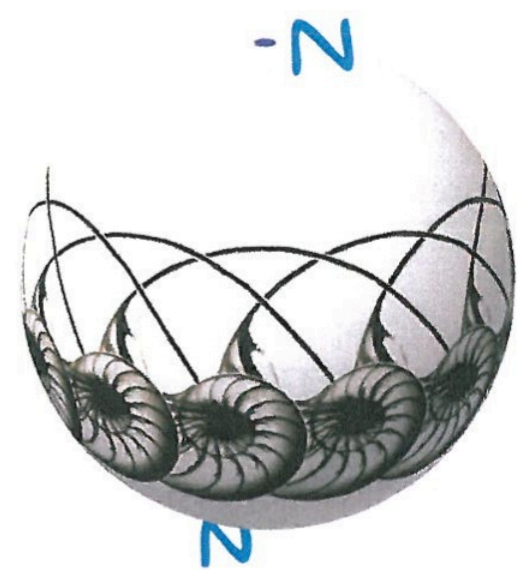
The five ‘true blue’ types (first one)

Total cost = 2-2/d <2 for every **d=** 1, 2, 3, …. =>

- no wonder rings O
- no more than 3 digits (distinct to 1): $(N-1)/N \geq \frac{1}{2}$ for all, N=2,3,…
- if three digits, then **at least one** must be 2 ($\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=2$, $(N-1)/N \geq \frac{2}{3}$ for all $N \geq 3$)

Two digit case: MN

(In fact only case M = N occurs)



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Case two 2's: 22N (second)

1+(N-1)/N<2 for all N=2,3,4,5,…

Last 3 of the five ‘true blue’ types

Three digits, one 2:

- one digit must be 3 ($\frac{1}{2}+\frac{3}{4}+\frac{3}{4}=2$)
- the remaining digit must be 3, 4 or 5 ($\frac{1}{2}+\frac{2}{3}+\frac{5}{6}=2$)

$\Rightarrow$ 332, 432, 532

Note: ch(332)=2-($\frac{2}{3}+\frac{2}{3}+\frac{1}{2}$)=$\frac{1}{6}$**=2/12**

ch(432)= 2/24

ch(532)=2/60

The five ‘true red’ types

No **, *x, xx signatures, all of type *AB…N

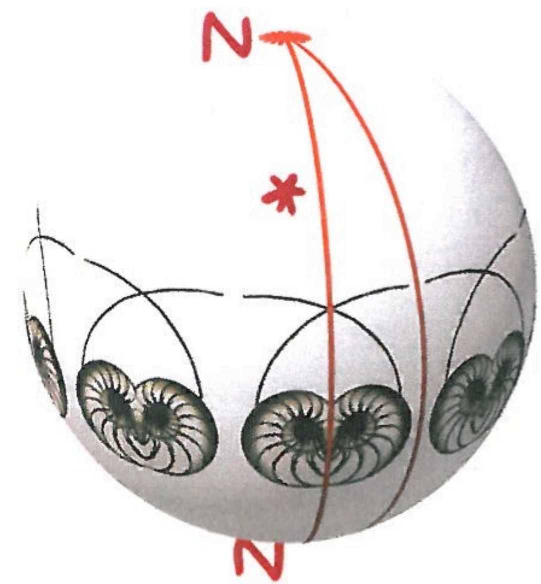
ch(*AB…N)=2-1-((A-1)/2A+…+(N-1)/2N),

ch(AB…N)=2-((A-1)/A+…+(N-1)/N),

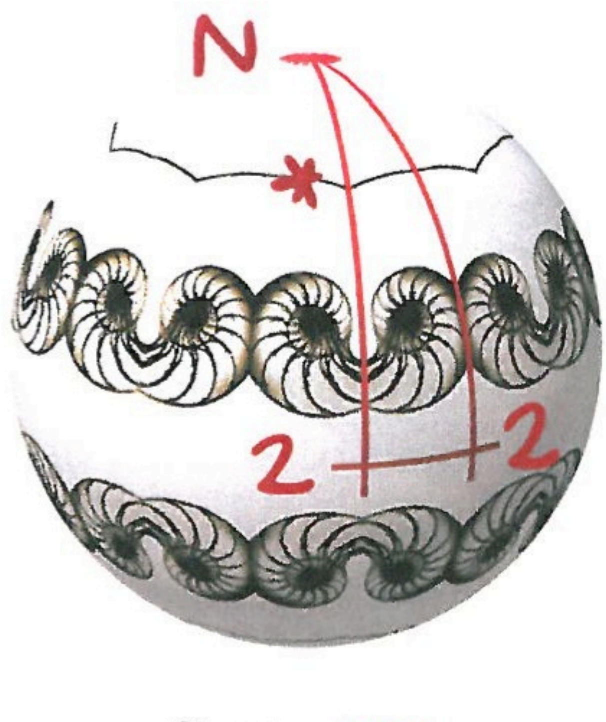
=>

ch(*AB…N)=½ch(AB…N)

Note: only *NN is possible with two digits !



*22N



*MN2

*432, *532, *332

ch(*332) = $2-(1+\frac{1}{4}+\frac{1}{3}+\frac{1}{3}) = 1/12$

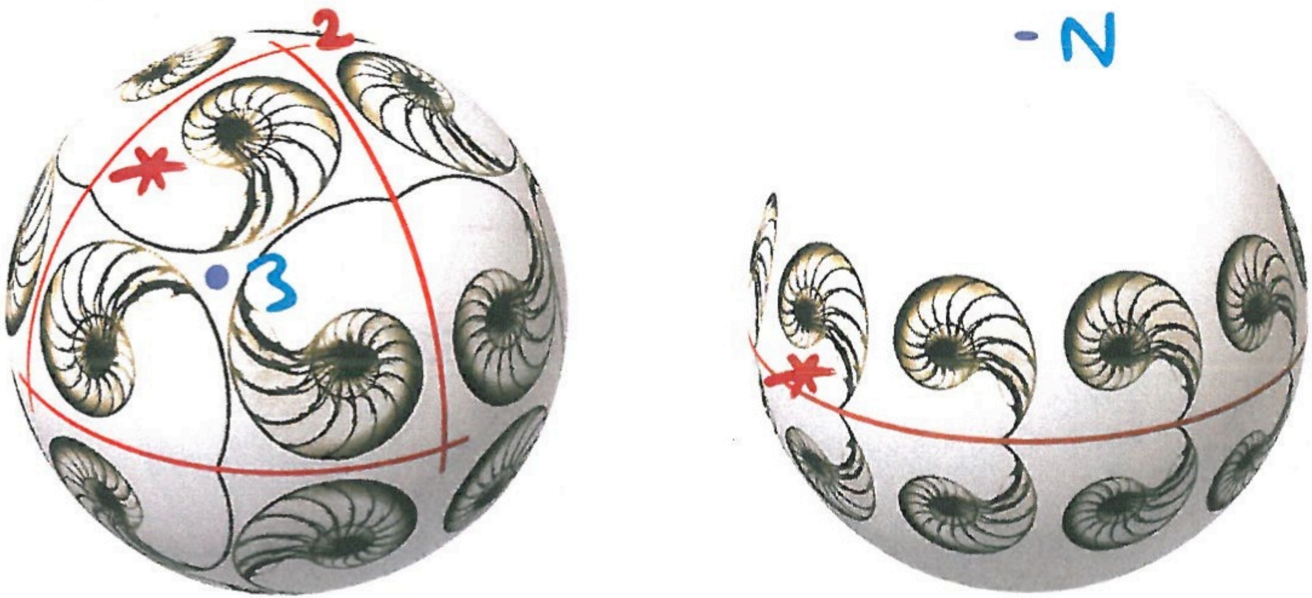
Compare with orientation reversing symmetries of five platonic solids.

The four Hybrid types

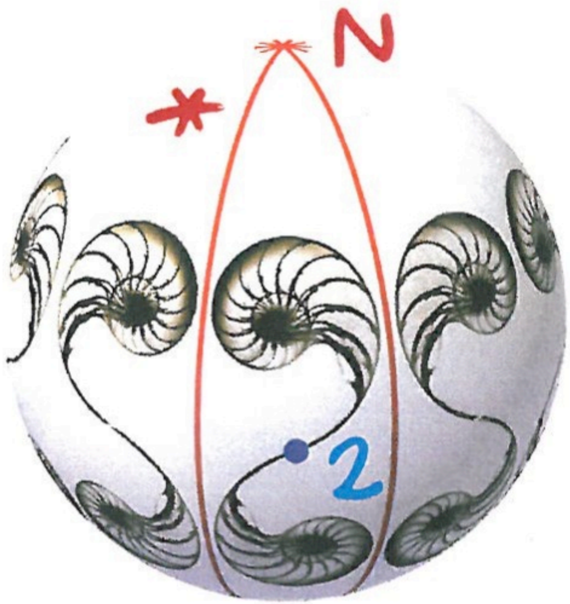
All possible variants (as in the plane case)

- *532
- *432
- *332 -> 3*2
- *22N -> 2*N
- *NN -> N* -> Nx

3*2 and N*



2*N and Nx

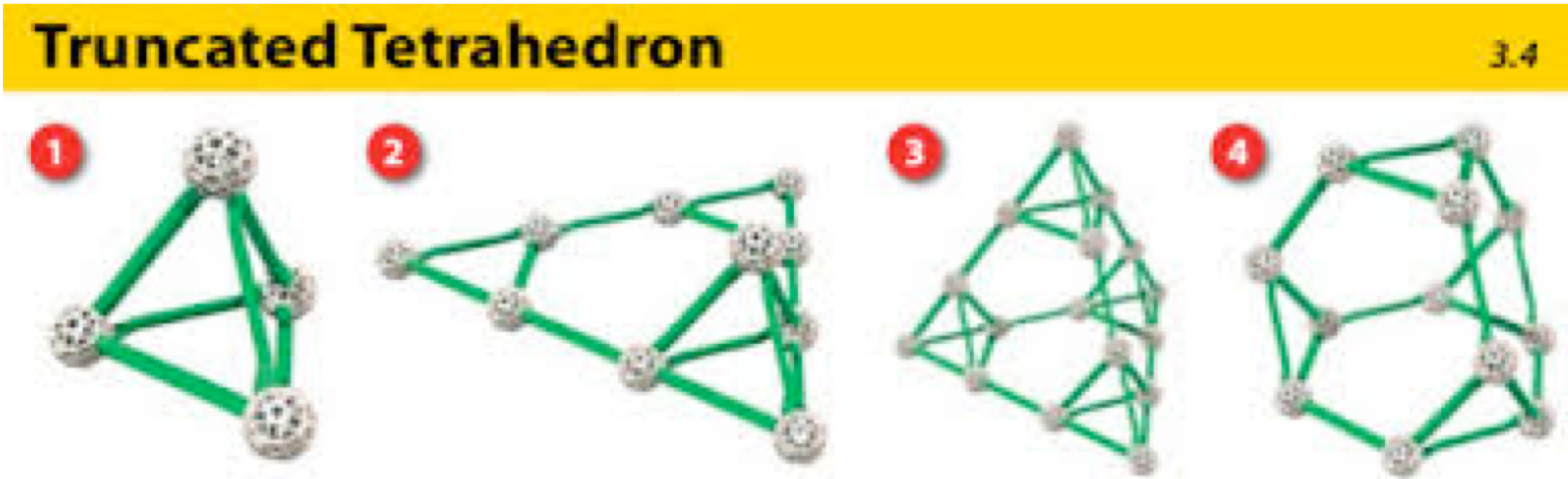


Build an Archimedean solid (or its dual)

- **Study its symmetries**
- **What is the number of vertices (V) ?**
- **What is the number of edges (E) ?**
- **What is the number of faces (F) ?**

- **Calculate V-E+F for each polyhedron**

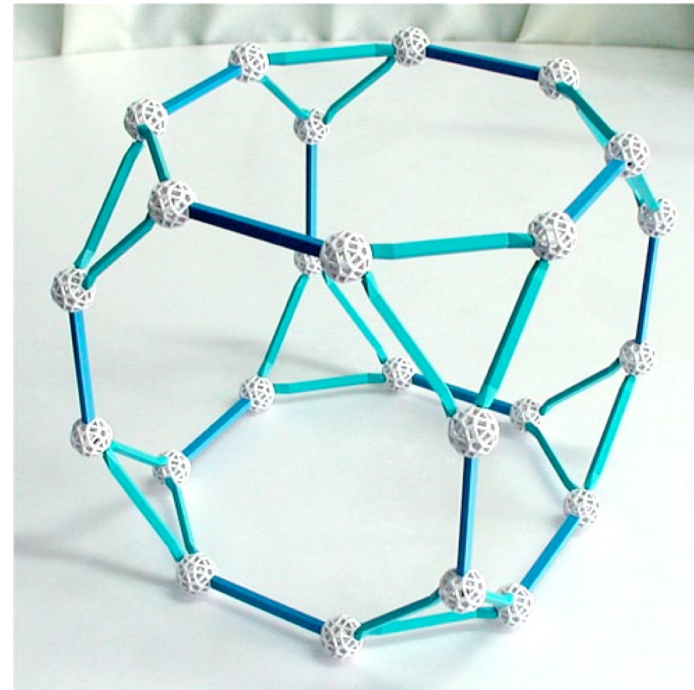
1) Truncated tetrahedron



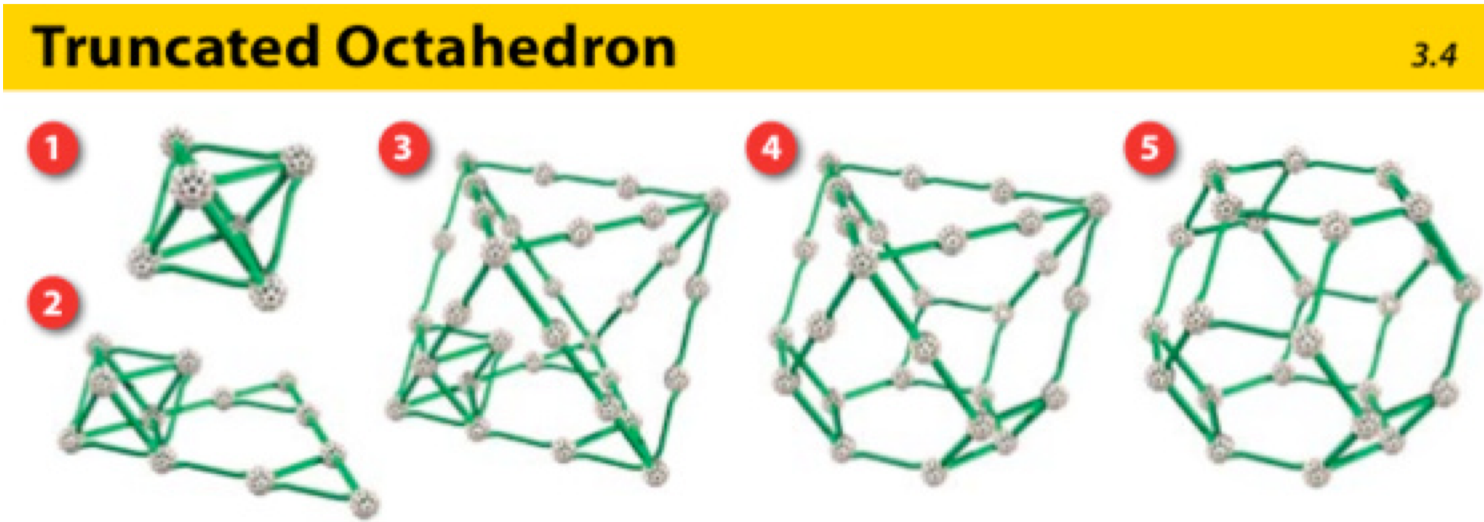
(**Dual Catalan solid:** Triakis truncated tetrahedron)

2) Truncated cube

Dual Catalan solid: triakis octahedron



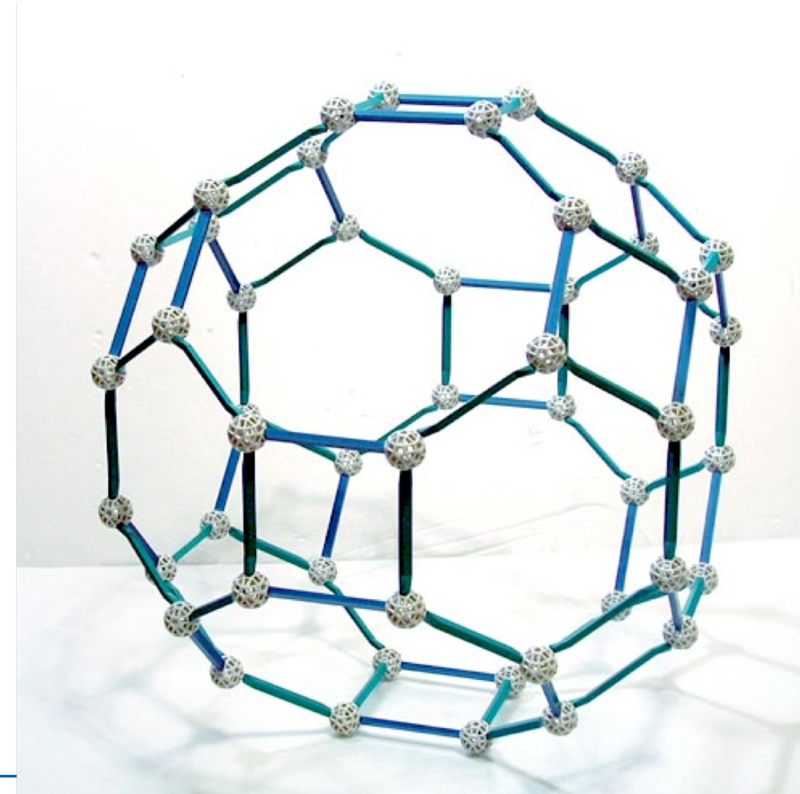
3) Truncated octahedron



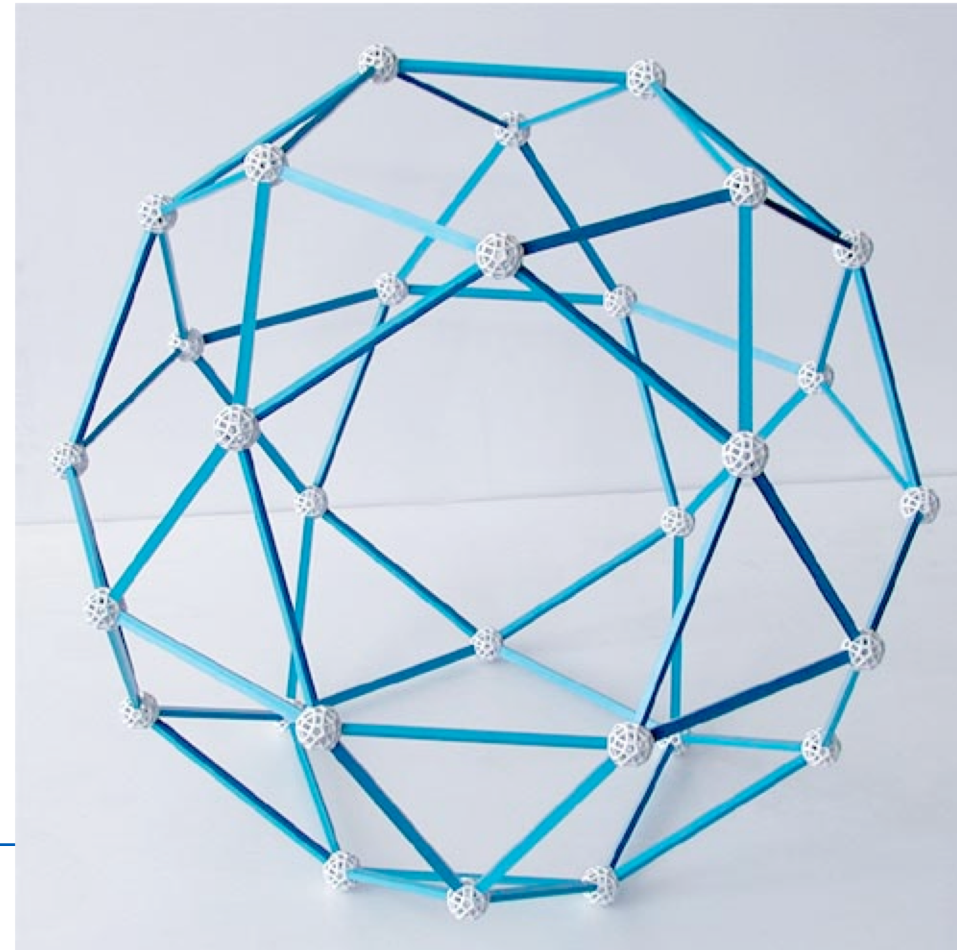
Dual Catalan solid: tetrakis hexahedron

4) Truncated cuboctahedron

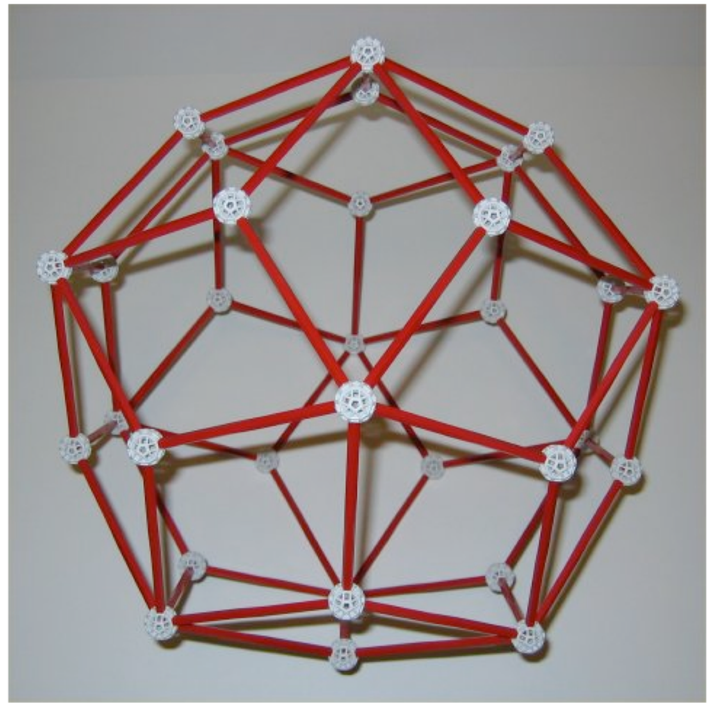
Dual Catalan solid:
disdyakis dodecahedron



5) Icosidodecahedron

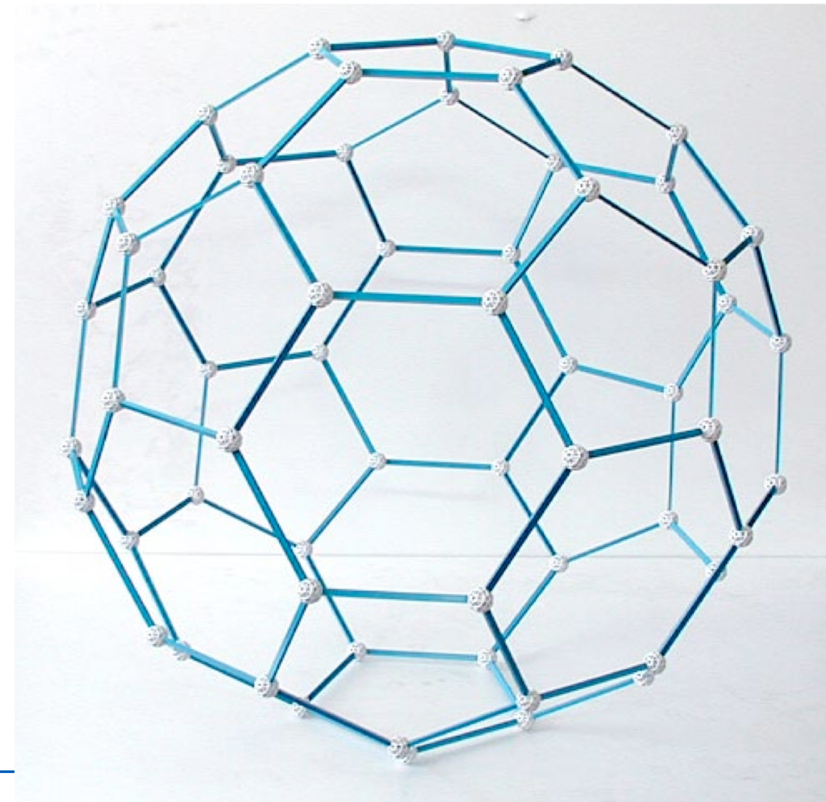


5b) Dual Catalan solid: Rhombic triacontahedron



6) Truncated icosahedron

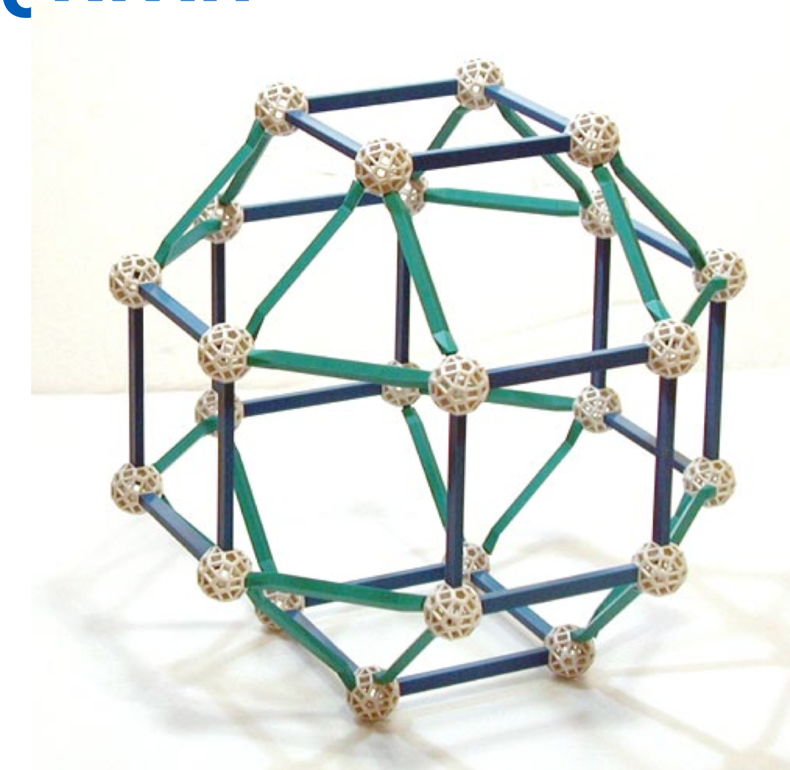
Dual Catalan solid: pentakis dodecahedron



7) Rhombicuboctahedron

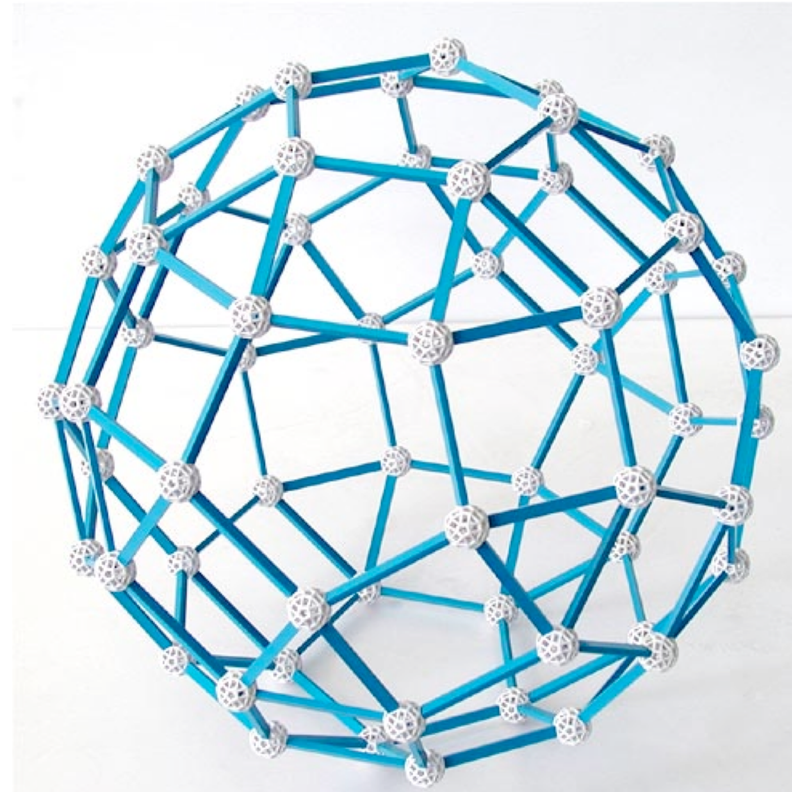
Dual Catalan solid:

Deltoidal icositetrahedron



8) Rhombicosidodecahedron

Dual Catalan solid:
Deltoidal hexacontahedron



9) Truncated dodecahedron

Dual Catalan solid:
Triakis icosahedron

10) Icosidodecahedron

Dual Catalan solid: Disdyakis triacontahedron

Exercise to be returned on 2^{nd} Oct

1) Find fundamental domain and signature of Platonic solids and check the validity of Miracle theorem:

Prize(symmetry)=2-2/d, d=number of symmetries (*Make use of the models you built)*

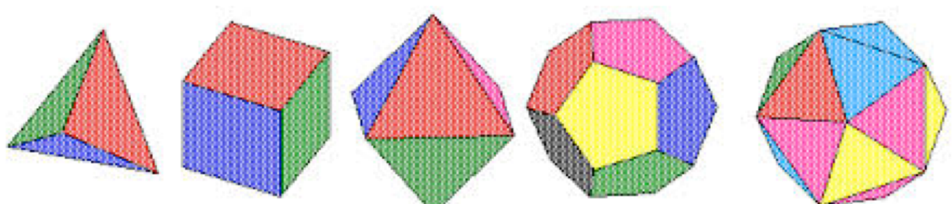
2) Find signature of at least four Archimedean/Catalan solids

3) What is the value of V-E+F in each case?

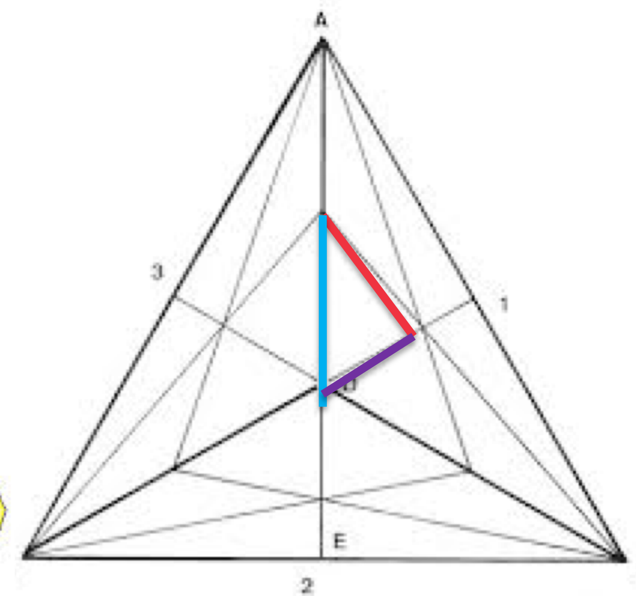
Ex: tetrahedron *332

1+2/6+2/6+1/4= 1+11/12 =2-2/24,

d=24=4x6



The Tetrahedron The Cube The Octahedron The Dodecahedron The Icosahedron



Some ideas for an essay

- Nanoscale symmetries (scanning electronic microscope available, own samples + support from Aalto Junior)
- 3D printing (or other method) to produce examples of spherical symmetries (Henry Segerman)
- Programs, other methods to produce planar tilings in practice
- Continue processing textiles
- Patterns from specific cultures, traditions etc.
- ….