SHAPES IN ACTION

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Symmetry in Projective Geometry

Taneli Luotoniemi

Axioms of geometry:

Two points define a line

Two lines define a point

unless their 'parallel' ☹

Point at Infinity

→ projective line

=the 'space' of lines through a point

Segments in the projective line

Line at Infinity

$\longrightarrow$ projective plane

=the 'space' of planes through a point

Polygons in the projective plane

Incidence structure

the *meet* of two lines is a point

the *join* of two points is a line

Projective duality:

Points & Lines $\longleftrightarrow$ *Lines & Points*

Point-wise world vs. Line-wise world

Exercise:

What is the dual of translation?

Pappus' Theorem

Pappus of Alexandria (circa 290-350 AD)

Exercises:

What if the two ranges of points are in perspective?

What if two lines are parallel?

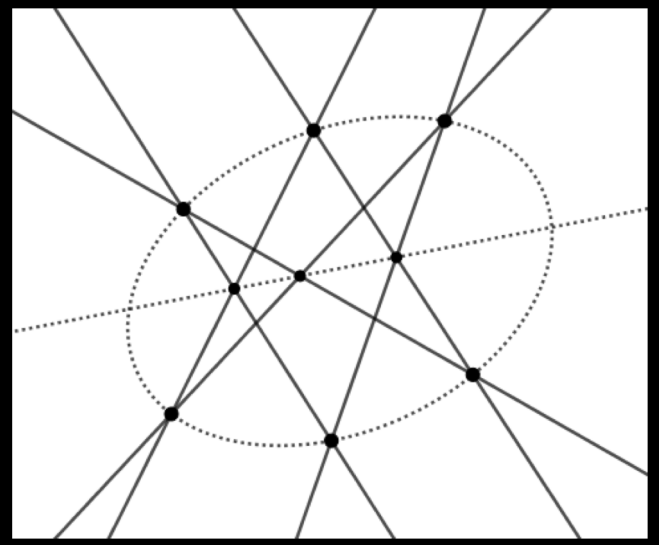
Exercise:

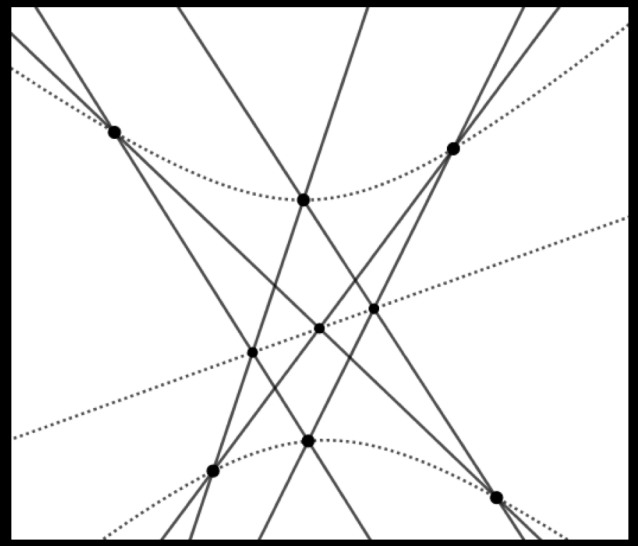
What is the dual of Pappus' theorem?

Projective Configurations

Blaise Pascal (1623-1662)

Pascal's theorem:





Exercise:

What is the dual of Pascal's theorem?

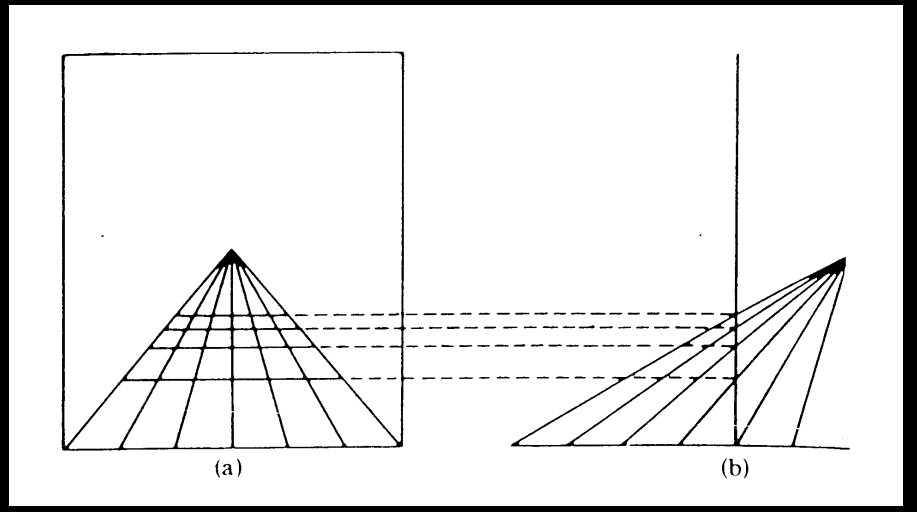
Brianchon's theorem

Charles Julien Brianchon (1783–1864)

Perspective

Filippo Brunelleschi (1377–1446)

Leon Battista Alberti (1404–1472)



Alberti's distance point method
from "Della Pittura", 1435

Exercise:

How to tile the plane with a given rectangle in perspective?

René Magritte: ”The Promenades of Euclid”, (1955)

Cross ratio

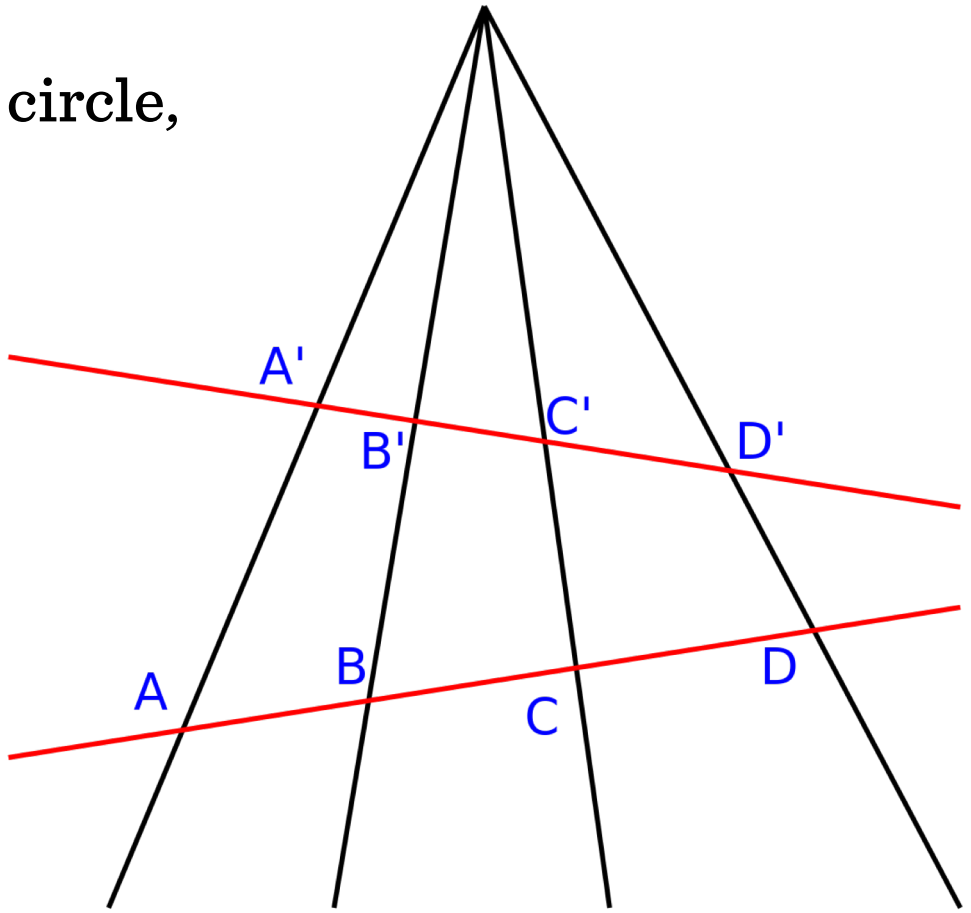
most important invariant in the subject

not just points, but of lines as well

Special case: Chasles' theorem

(related to inscribed angle theorem on the circle,

but works on any conics too)



Harmonic conjugates / harmonic range of points

dividing internally & externally

Exercise:

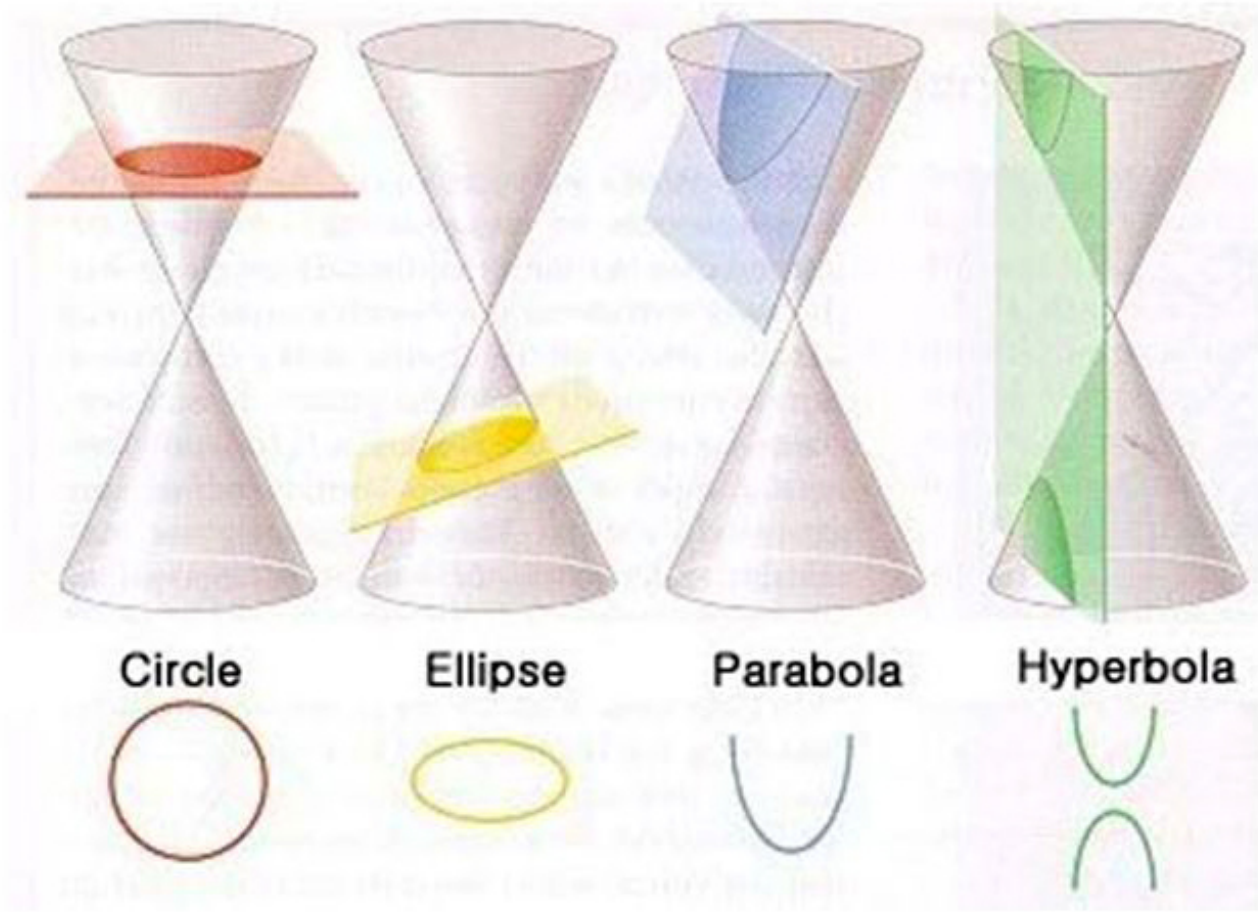
Construct a harmonic range

if one point is the midpoint of the two others, where is its harmonic conjugate?

Harmonic pencils of lines
e.g. angle bisectors, quadrilateral diagonals

Perspectivity & Projectivity

Projective view on conic sections



Euclidean geometry
in the context of projective geometry?

Euclidean plane = projective plane with ’one line removed’

Hierarcy of geometries:

Euclidean geometry (notion of 'perpendicular')

↑

affine geometry (notion of 'parallel')

↑

projective geometry

Exercise:

Desargues' Theorem

Girard Desargues (1591 – 1661)

THE DESARGUES CONFIGURATION

10 points
10 lines
5 planes

(3 lines and 3 planes per point
3 points and 2 planes per line
6 points and 4 lines per plane)

DESARGUES' THEOREM

if two triangles are in perspective from a point (point of perspectivity), they are also in perspective from a line (axis of perspectivity)