Aalto School of Electrical Engineering

ELEC-A7200 Signals and Systems

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Department of Communications and Networking

ELEC-A7200 Signals and systems

What is covered in the course?

- basic concepts of signals and systems
- basic methods of signal and system analysis
- basics of signal transmission
- basics of signal measurement

Where is this information needed?

- when something is measured
- when a signal is transmitted
- when the signals are filtered
- when the signals are generated
- when any system is controlled

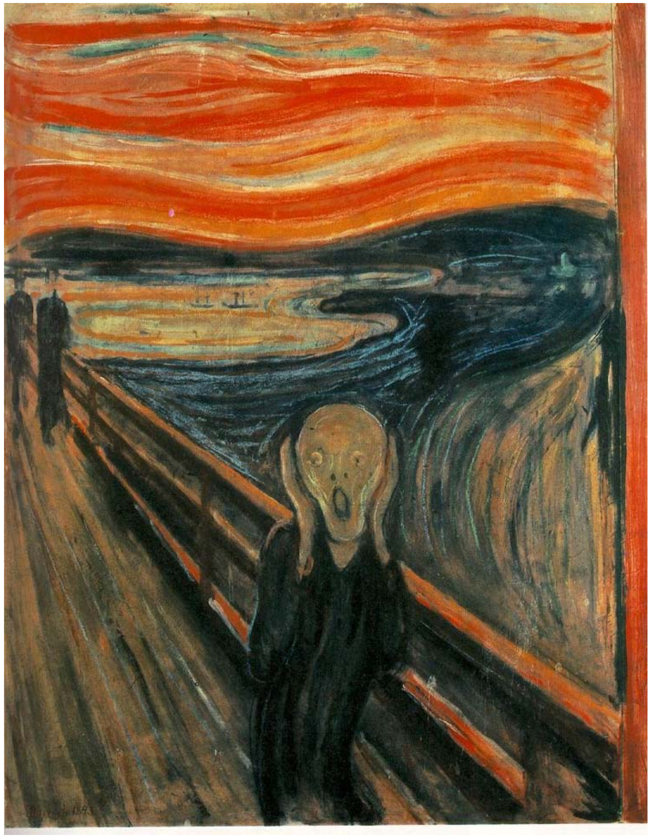
TIM-material

Chapter 01: Introduction - Signal Power and Energy
Chapter 02: Special Signals and Convolution
Chapter 03: Signal Space
Chapter 04: The Fourier Series
Chapter 05: Fourier Transform I
Chapter 06: Fourier Transformation II
Chapter 07: Sampling and Discrete Fourier Transform
Chapter 08: LTI Systems in the time domain (and Laplace Transform)
Chapter 09: LTI Systems in the frequency domain.
Chapter 10: Linear Filtering of Signals.
Chapter 11: Modulation and Memoryless Nonlinear Systems
Chapter 12: Random Signals

https://tim.aalto.fi/view/elec-a7200/syksy19/luku-00/en

How to study?

- **The course is demanding!**
- **Allow enough time (approx. 10 h / week) for reading, understanding and completing the assignments.**
- **As you read the material, try to understand everything!**
- **If you don't understand, just ask**
  - friends
  - assistants
  - professor
  - in slack
  - during exercises
- **Give feedback also during the course**
- **We will try to improve the material based on your feedback.**



Excercise sessions

MON	TUES	WED	THURS	FRI
	8.15-10.00 U5			
			14.15-16.00 U3	
	16.15-18.00 U351		**16.15-18.00** U351	

Grading

- **Grading rule**
  - Weekly exercises in TIM system (30%)
  - Two homework: (20%)
  - Two midterm exams or a final exam (50%) [Traditional paper exams]
  - Laboratory works: pass/fail

- **Each course section must be passed individually in order to pass the course.**

Objectives for the lecture

You will learn the basic concepts of signals and systems

- What is a signal?
- What is a system?
- What is a spectrum?
- What does signal filtering mean?

More in TIM.

NOTE! After today's lecture you are not supposed to know how to calculate anything! Solving problems is practiced with TIM & homework.

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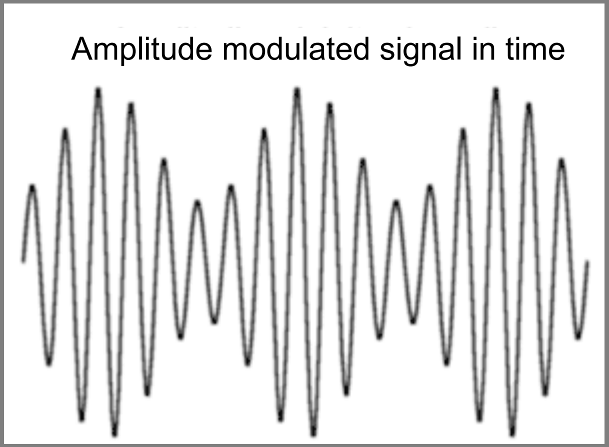
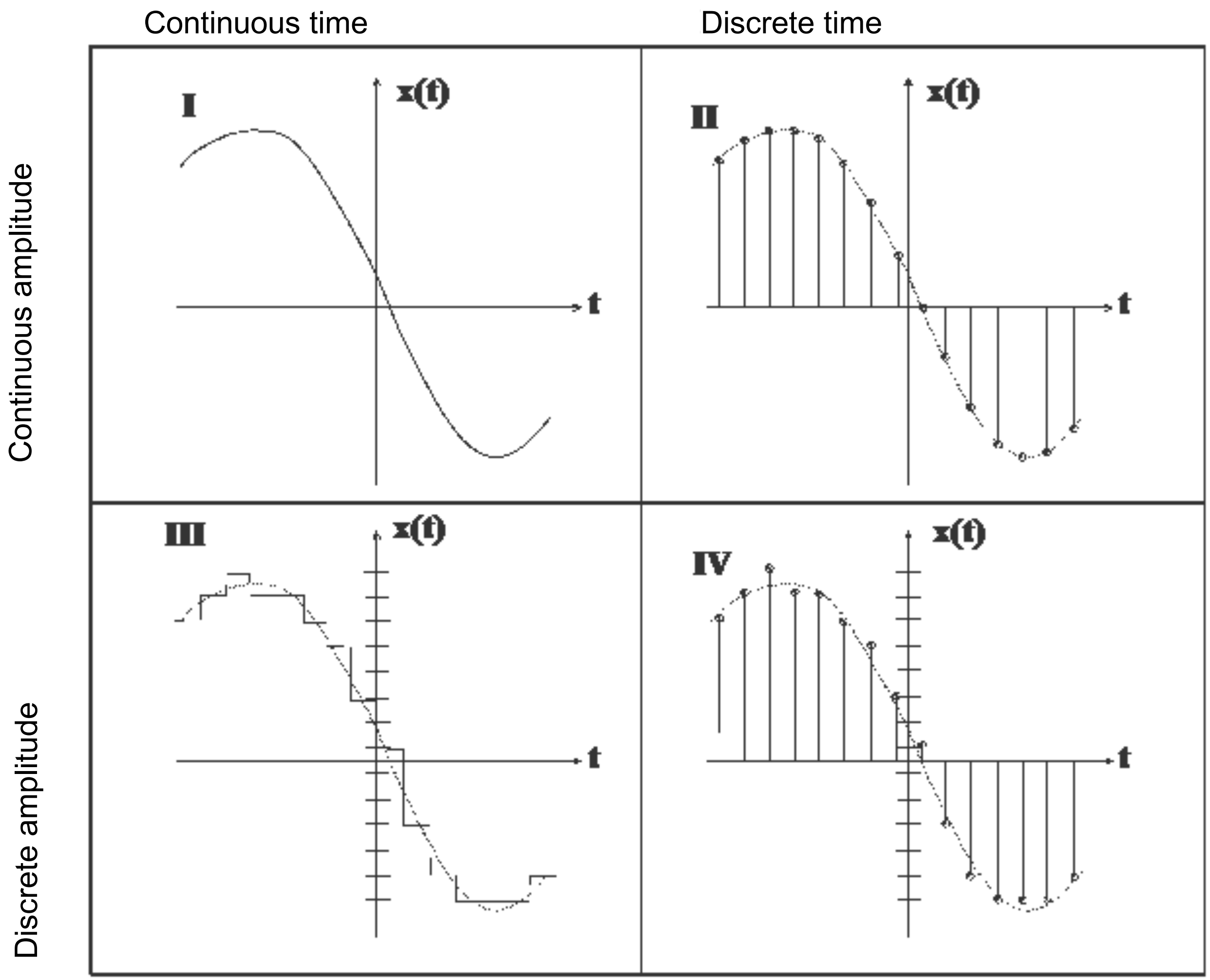
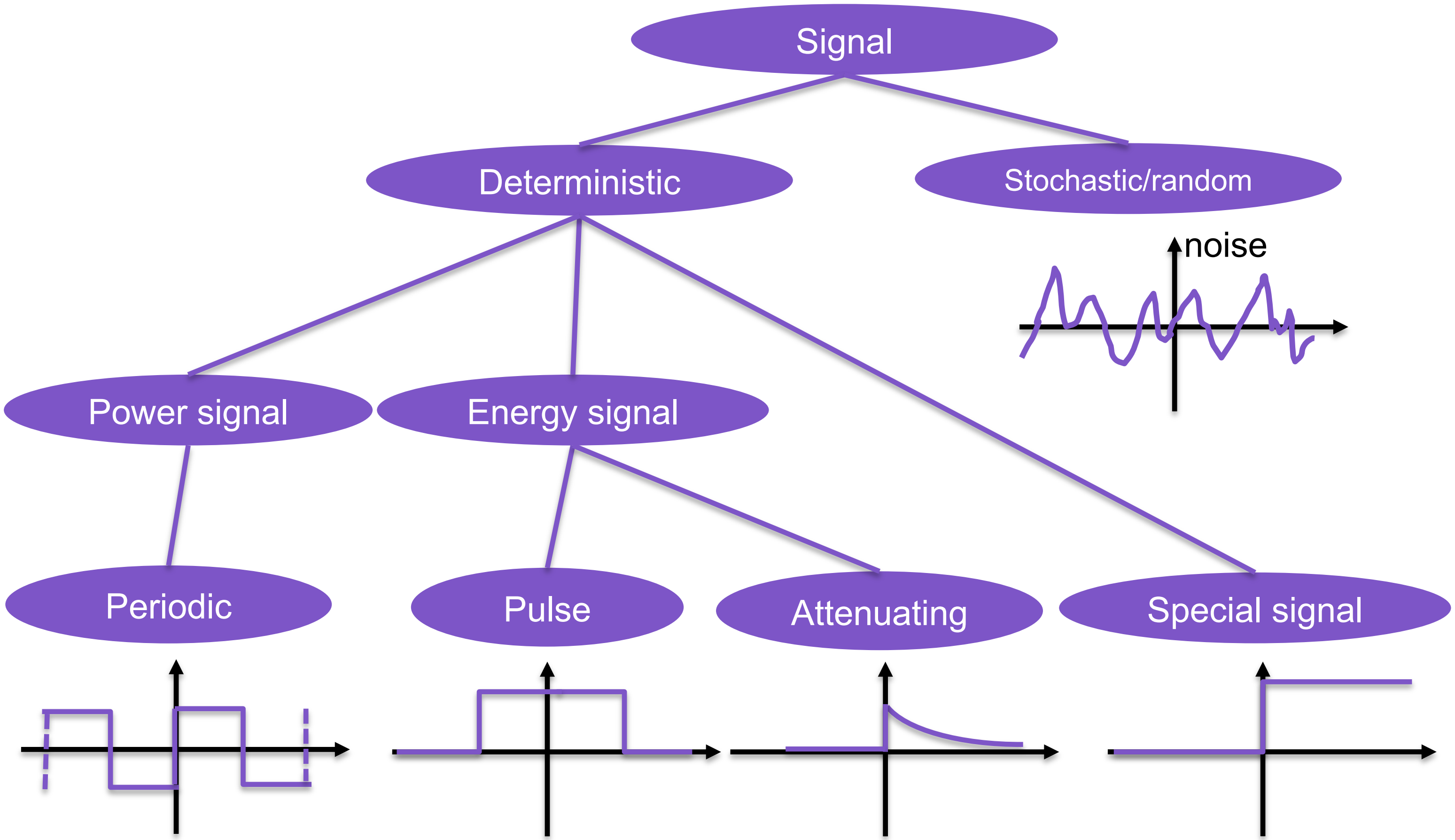


What is a signal?

A signal $x(t)$ is a function of time



Classification of signals



Signal amplitude can be real or complex

- All natural (measurable) signals are real
- Complex signal is a practical model for modulated signals

$$s(t)=x_I(t)\cos(\omega t)+x_Q(t)\sin(\omega t)$$

$$s(t)=\mathrm{Re}\{(x_I(t)-jx_Q(t))\exp(j\omega t)\}$$

$$e^{jx}=\cos x+j\sin x$$

$$j=\sqrt{-1}$$

$$x(t)=x_I(t)-jx_Q(t)$$

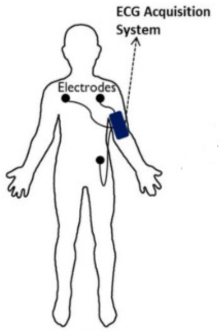
Baseband signal [kantataajuinen signaali]
Complex signal

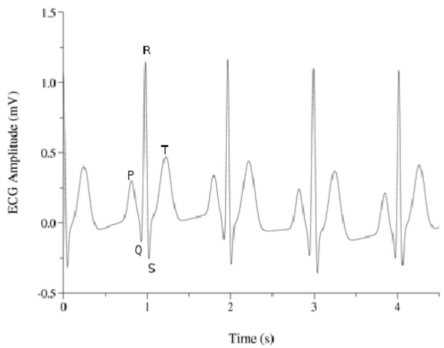
Problem

Consider examples of signals in a group.

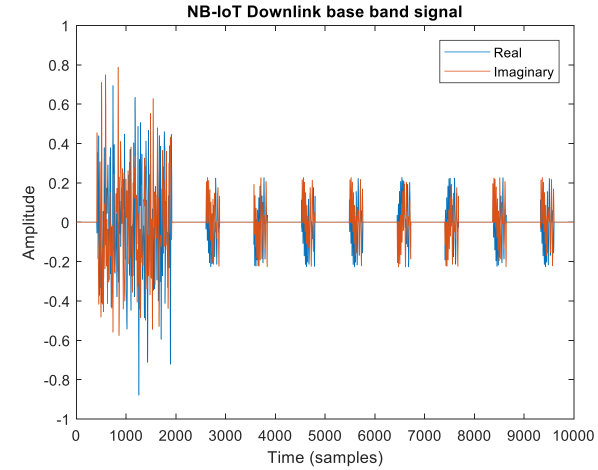
Examples of time domain signals

ECG

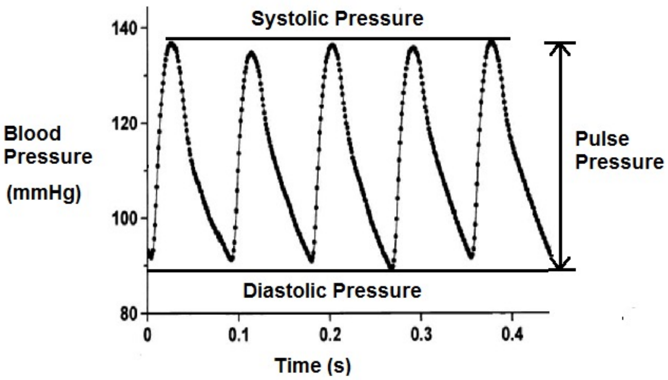




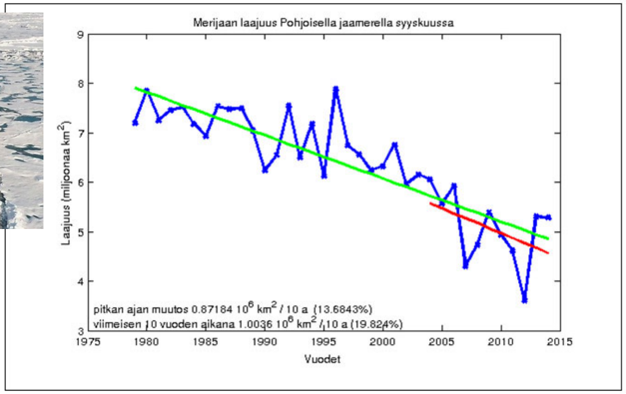
NB-IoT base station downlink base band signal (complex signal)



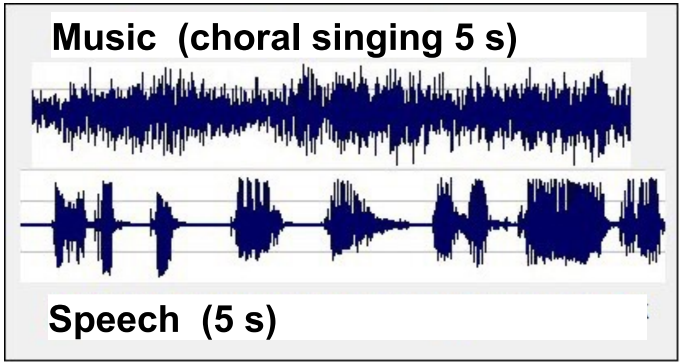
Blood pressure



Ice thickness (time series)



Sound



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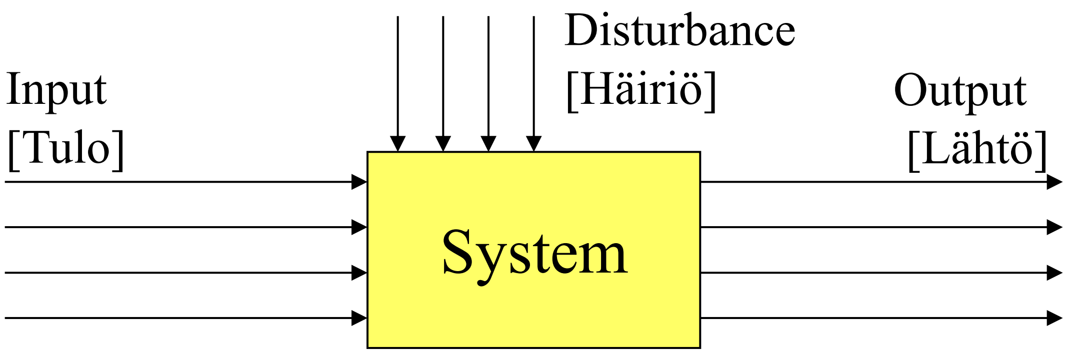
What is a system

What's not?

System

- System / Process [Järjestelmä/Prosessi] is an object that defines the relationships between a set of signals.
- System signals are often divided into input and output quantities.
- The input signals are system independent.
- The output signals contain information provided by the system.
- Typically there is a causal relationship between the input and output signals.



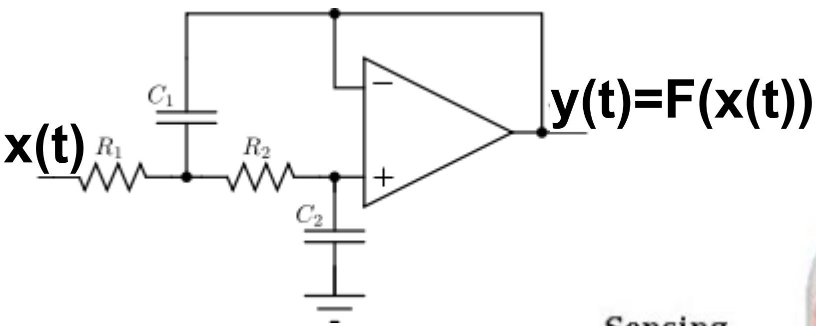
SISO	Single Input-Single Output
MISO	Multiple Input – Single Output
SIMO	Single Input – Multiple Output
MIMO	Multiple Input – Multiple Output

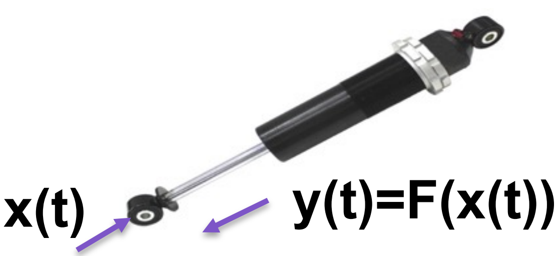
Problem

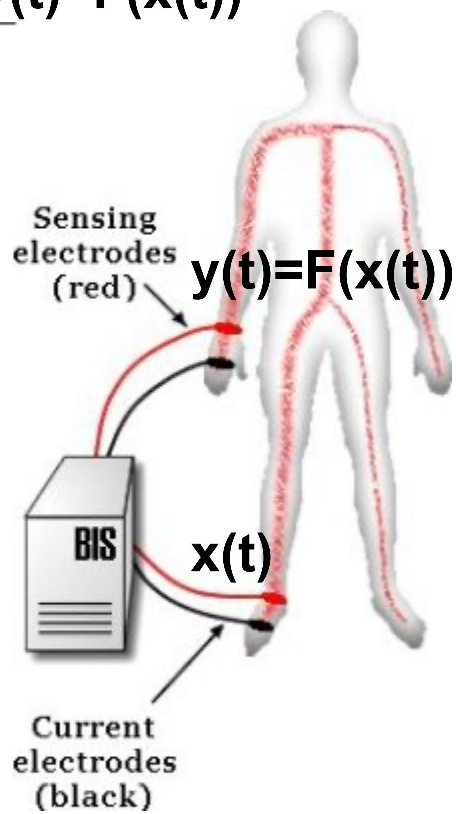
Consider examples of systems in a group.

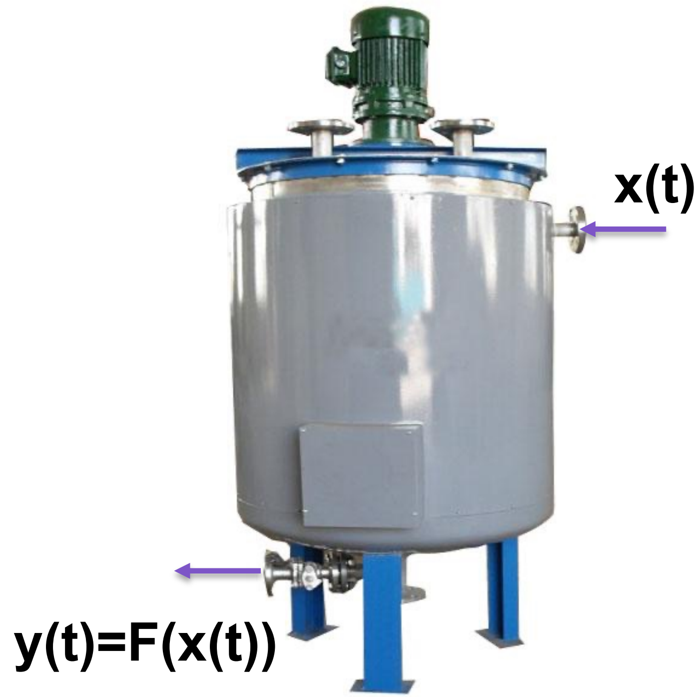
What is not a system?

Examples of systems









F is an operator that describes the operation of the plant

A?

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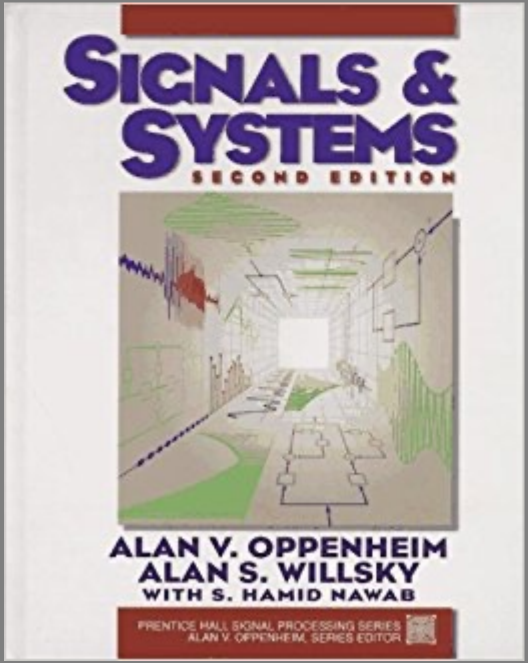
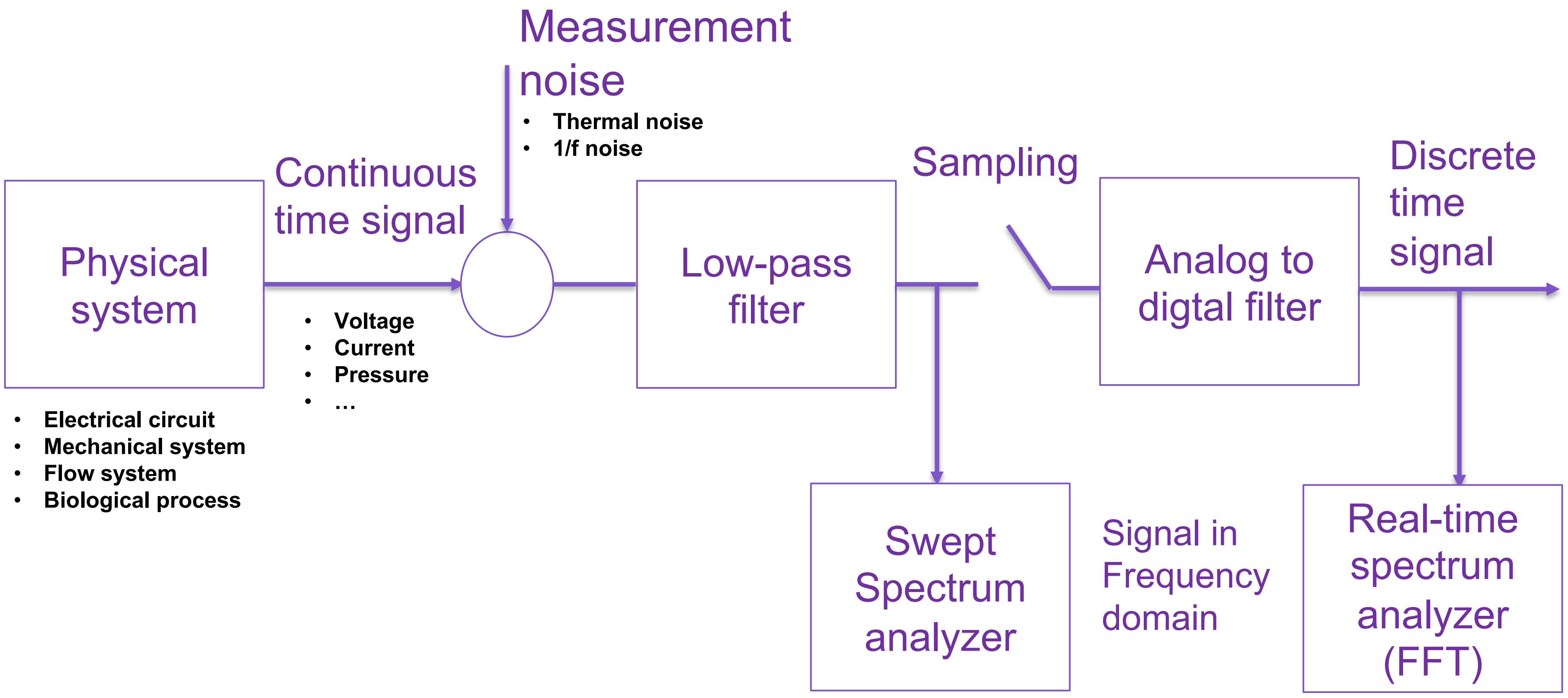
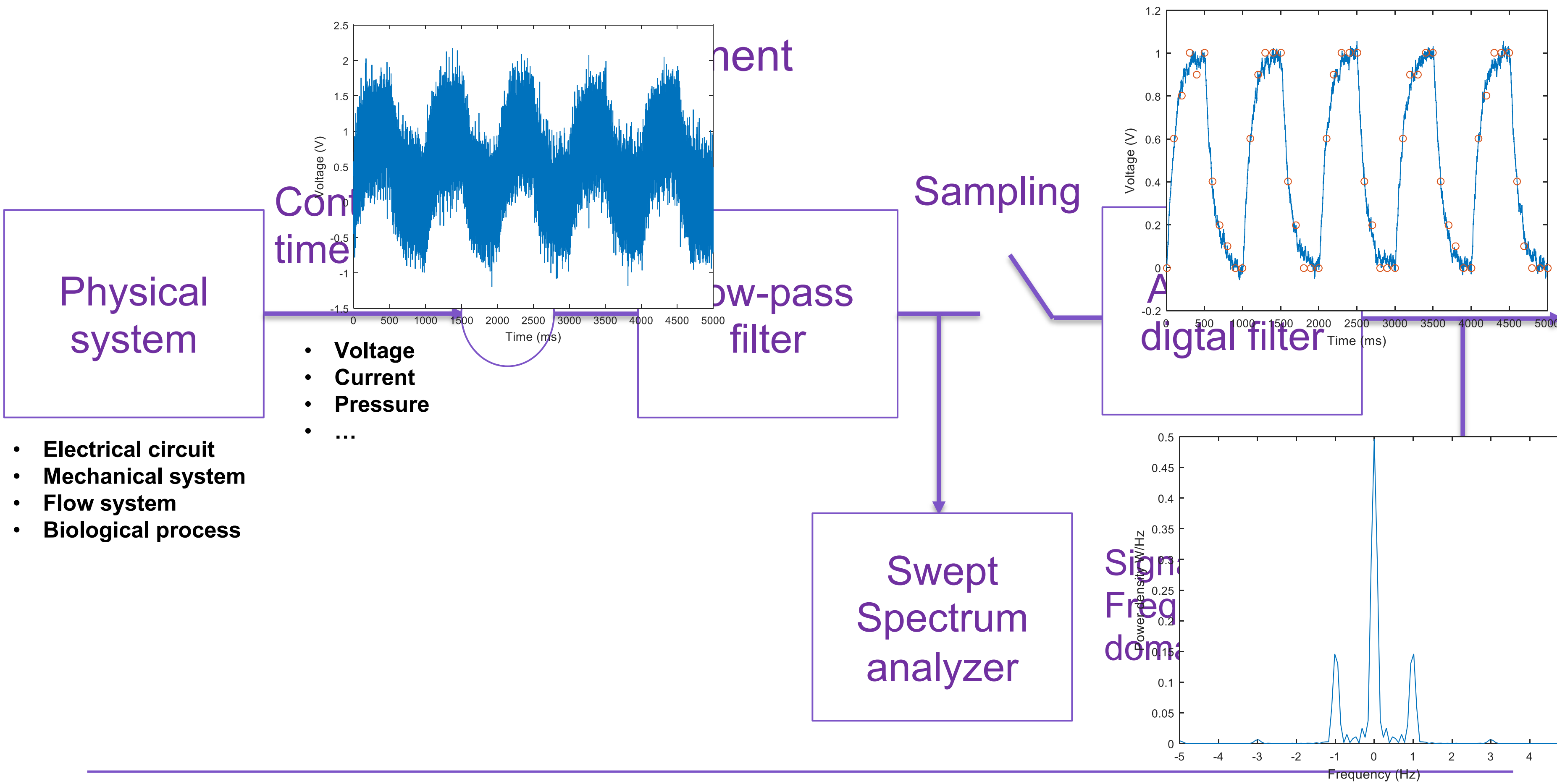


Signals and systems

Signals and systems - overall picture



Signals and systems - overall picture



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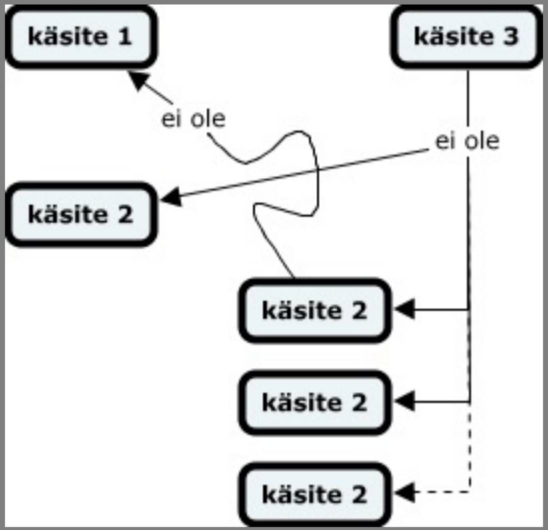

Basic concepts

Signal power and energy
Power and energy spectrum of the signal
Filtering the signal

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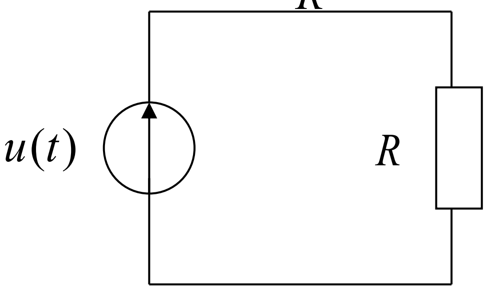
Signal power and energy

Chapter 1 in TIM

Signal power and energy

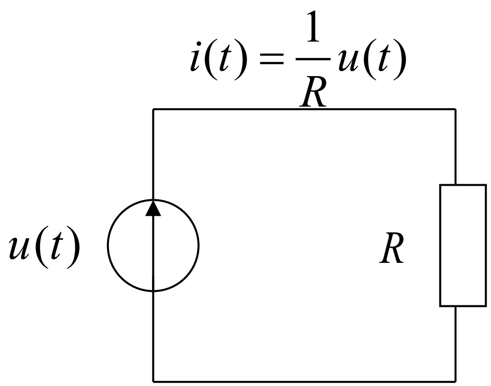
Basic circuit analysis: P=u·i, u=r · i

$$i(t) = \frac{1}{R} u(t)$$



Signal power and energy

Circuit with 1 Ohm resistive load (R=1 Ω)



Instantaneous (apparent) power

$$P(t) = u(t) i^*(t) = \frac{1}{R} |u(t)|^2 = |u(t)|^2$$

Energy consumed at the resistor during the time interval $[t_0, t_1]$

$$E = \int_{-t_0}^{t_1} P(t) dt = \int_{-t_0}^{t_1} |u(t)|^2 \, dt$$

Average power consumption of the resistor during the time interval $[t_0, t_1]$

$$P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} P(t) dt = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} |u(t)|^2 \, dt$$

Generalized energy and power

Arbitrary signal *s*(*t*) (not necessarily current or voltage)

Energy signal case

$$E = \lim_{T\to\infty} \int_{-T}^{T} |s(t)|^2 \, dt$$

A signal is called energy signal if $0 < E < \infty$

Power signal case

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |s(t)|^2 \, dt$$

A signal is called power signal if $0 < P < \infty$

Periodic signals

are power signals

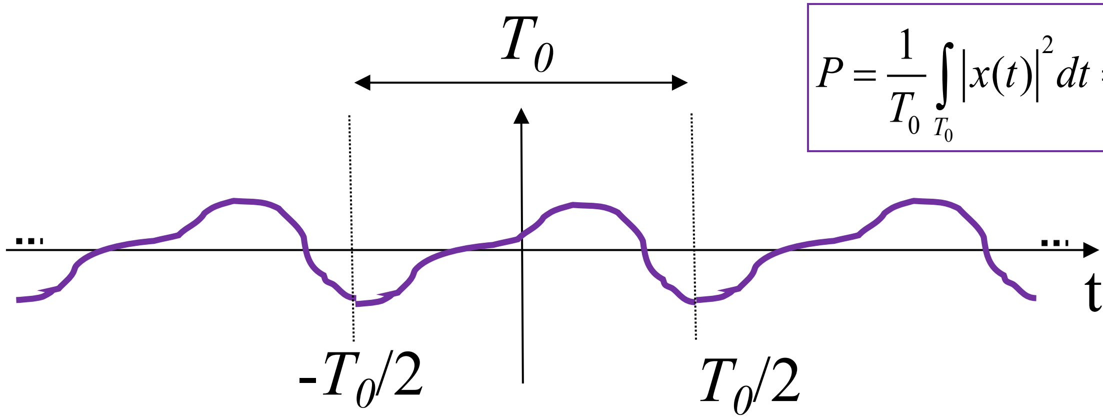
- **A periodic signal has the property**

$$x(t) = x(t + T_0), \quad t \in \mathbb{R}$$

T_0 denotes the duration of a period and $1/T_0$ is the nominal frequency

To calculate average power
it is sufficient to look at one period of time.
The location of the period can be chosen arbitrarily

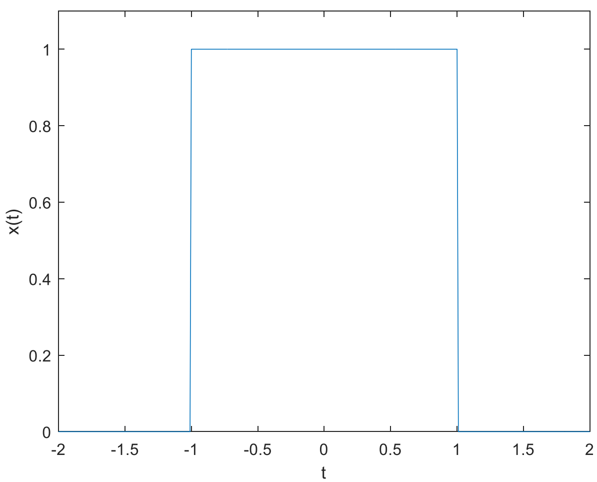
$$P = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \frac{1}{T_0} \int_{t_0}^{t_0 + T_0} |x(t)|^2 dt \quad \forall t_0 \in \mathbb{R}$$



Pulses ja attenuating signals

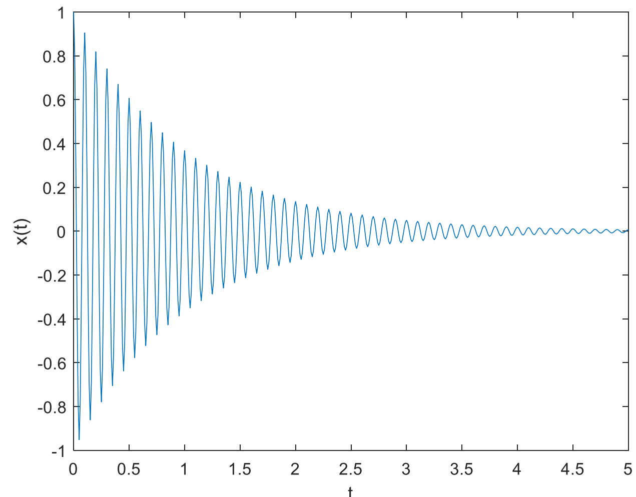
are energy signals

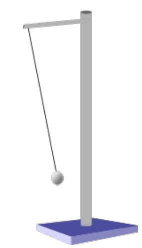
Pulse



E.g. the unit pulse

Attenuating signal





E.g. the damped harmonic oscillator

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$

Problem

Think of examples in the group of the energy signals and the power signals

Are there signals that are not power nor energy signals?

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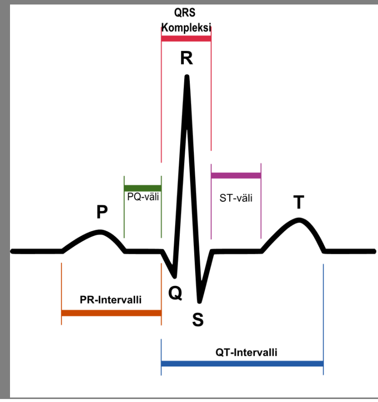
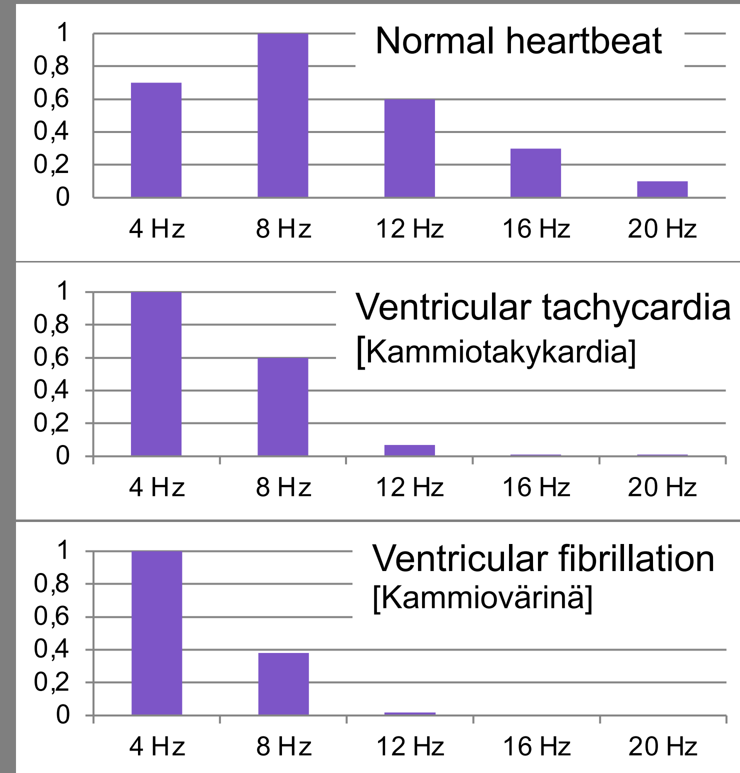


ECG PQRST Complex



Signals in frequency domain

What's a spectrum?

Chapters 4-6 in TIM

A?

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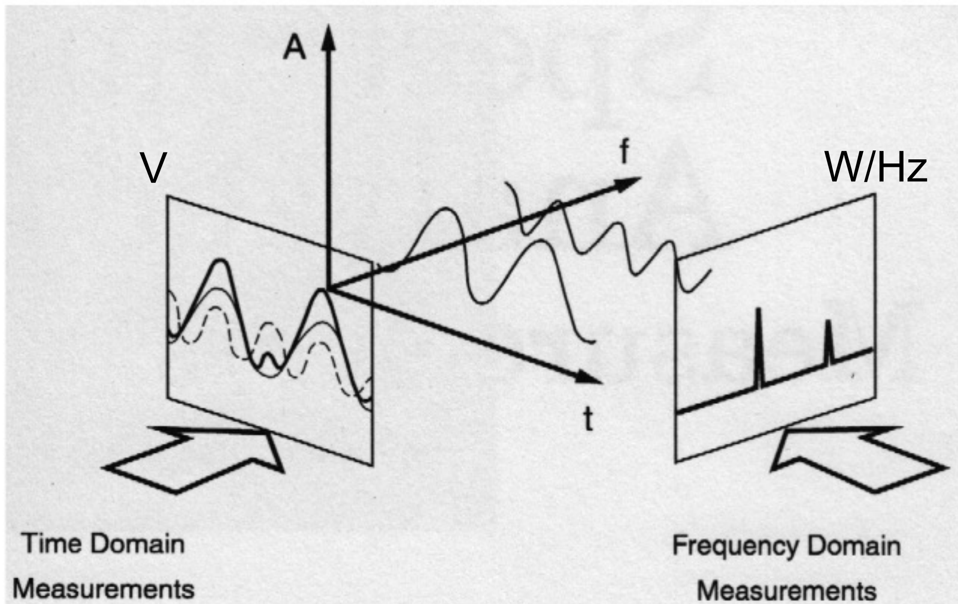
Periodic signals and their Fourier-series representations

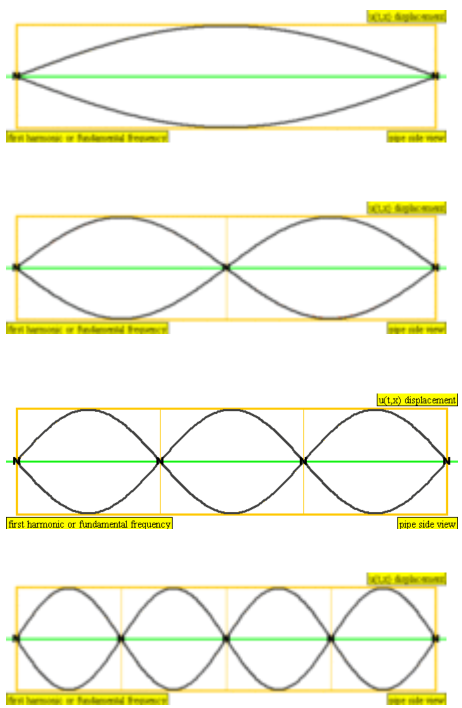
Chapter 4

Time and frequency domains



Kuva 3: Aika- ja taajuustasojen suhde (Lähde: Hewlett-Packard Company, Application Note 1286-1. 1997).

Periodic signals are composed of harmonic frequency components



Nominal frequency $1/T_0$ Hz

2. Harmonic frequency $2/T_0$ Hz

3. Harmonic frequency $3/T_0$ Hz

4. Harminic frequency $4/T_0$ Hz

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.
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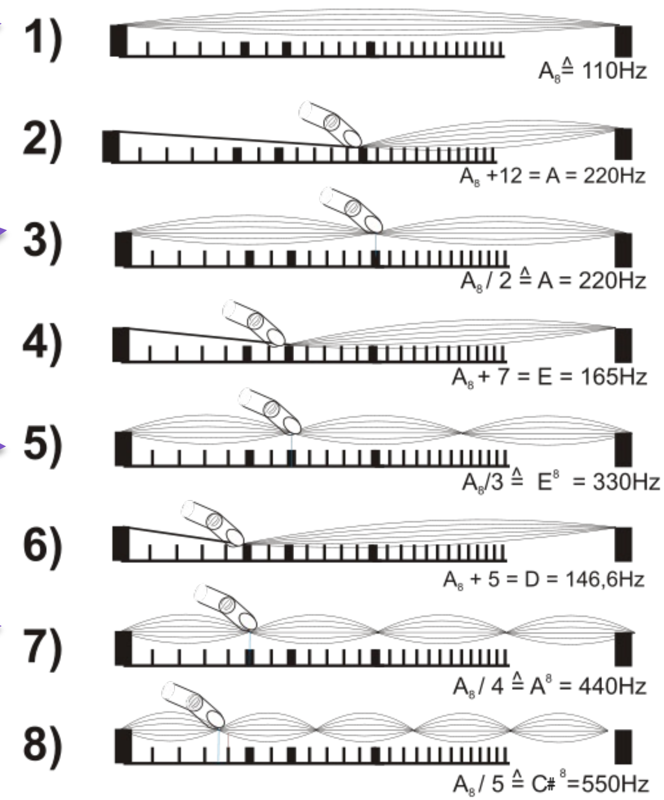


Playing harmonic frequencies with a guitar

https://en.wikipedia.org/wiki/Harmonic

Periodic signals are composed of harmonic frequency components

Real periodic signal can be written as

Fourier-series representation:

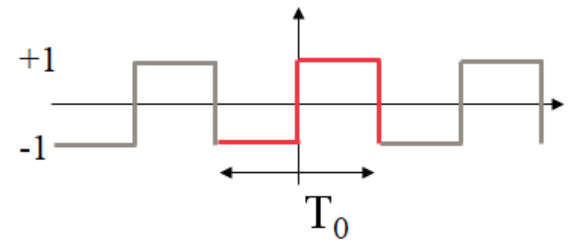
$$\boldsymbol{x}(\boldsymbol{t}) = x_0 + \sum_{\boldsymbol{k=1}}^{\infty} 2|x_k|\cos\left(\frac{2\pi k}{T_0}t + \arg\{x_k\}\right)$$

DC component — Amplitude — Harmonic frequency k/T_0 Hz — Phase

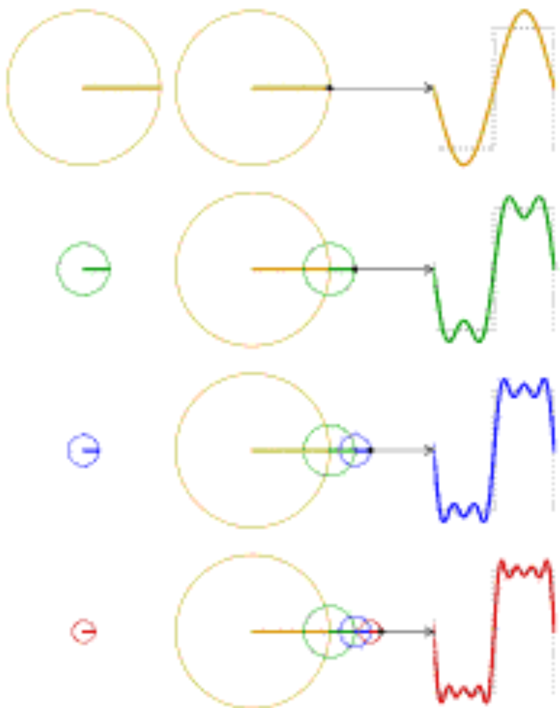
$$x_k = \frac{1}{T_0}\int_{\boldsymbol{T_0}} x(t)e^{-j\frac{2\pi}{\boldsymbol{T_0}}kt}dt$$

Fourier-series coefficient

Periodic signals are composed of harmonic frequency components



Square wave



Nominal frequency only

Nominal
+ 3rd harmonic

Nominal
+ 3rd harmonic
+ 5th harmonic

Nominal
+ 3rd harmonic
+ 5th harmonic
+ 7th harmonic

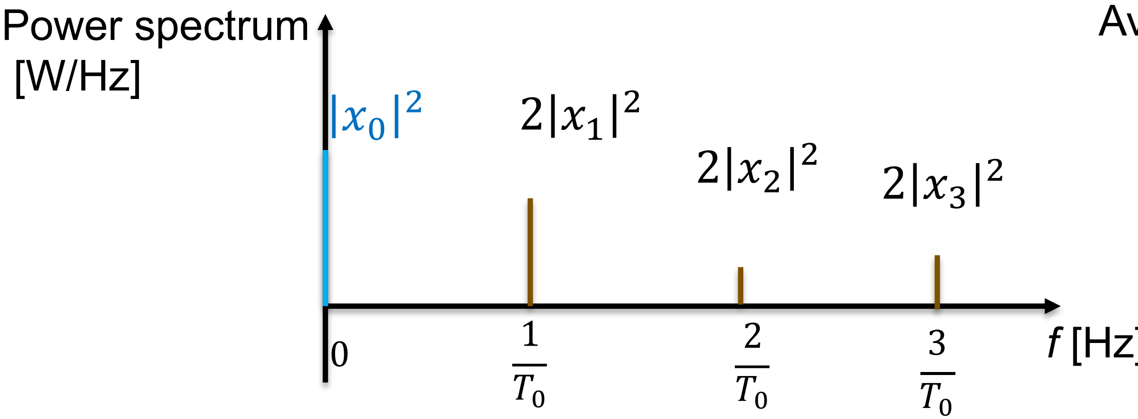
https://en.wikipedia.org/wiki/Fourier_series

One-sided power spectrum

If x (t) is real, then its Fourier series can be written as:

$$\boldsymbol{x}(\boldsymbol{t}) = x_0 + \sum_{\boldsymbol{k=1}}^{\infty} 2|x_k|\cos\left(\frac{2\pi k}{T_0}t + \arg\{x_k\}\right)$$

One-sided power spectrum



Average power

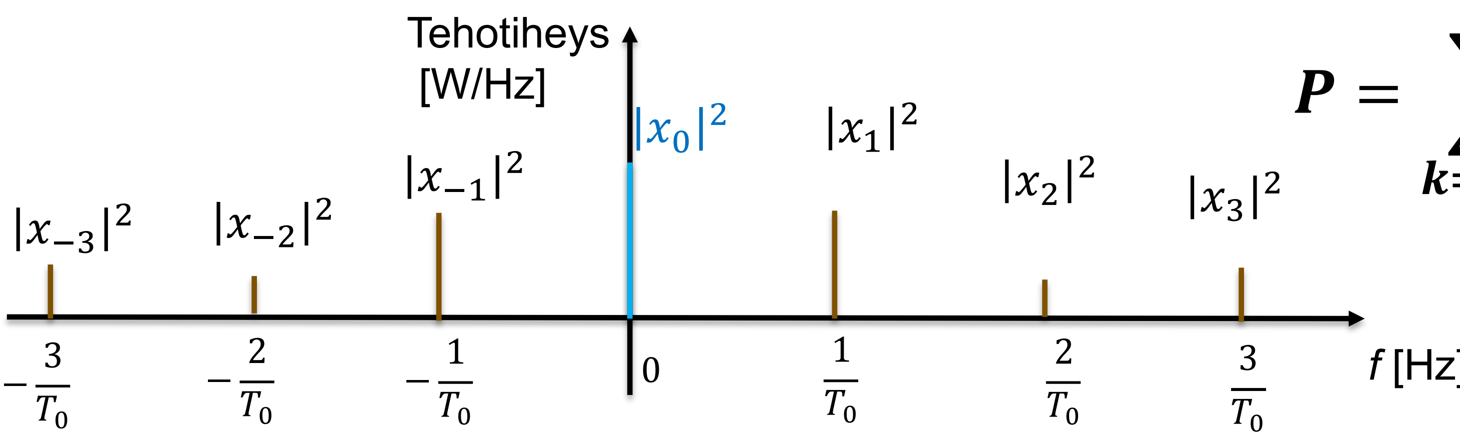
$$\boldsymbol{P} = |x_0|^2 + \sum_{\boldsymbol{k=1}}^{\infty} 2|x_k|^2$$

Two-sided power spectrum

In general case, the Fourier-series can be written as

$$\boldsymbol{x(t)} = \sum_{\boldsymbol{k}=-\infty}^{\infty} x_k e^{j\frac{2\pi k}{T_0}t}$$

Two-sided power spectrum

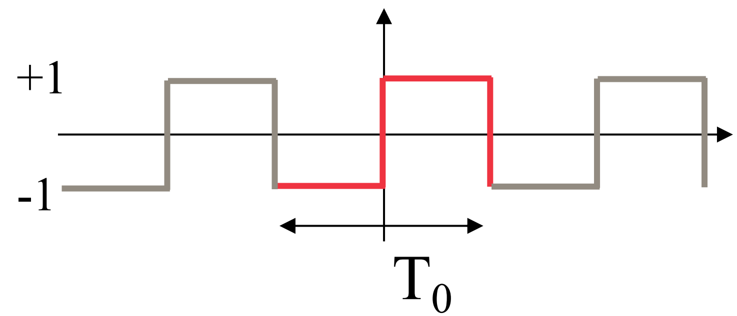


Average power

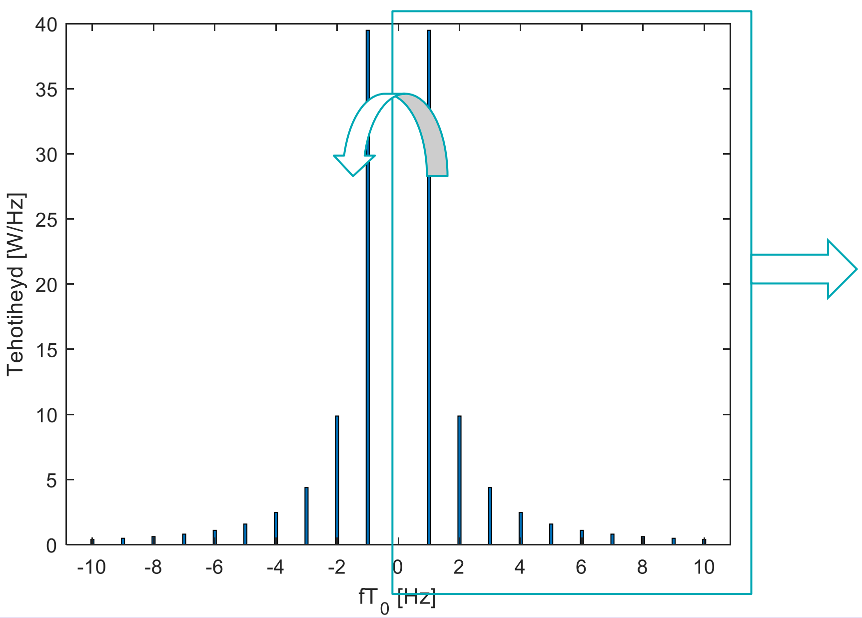
$$\boldsymbol{P} = \sum_{\boldsymbol{k}=-\infty}^{\infty} |x_k|^2$$

One and two sided power spectrum

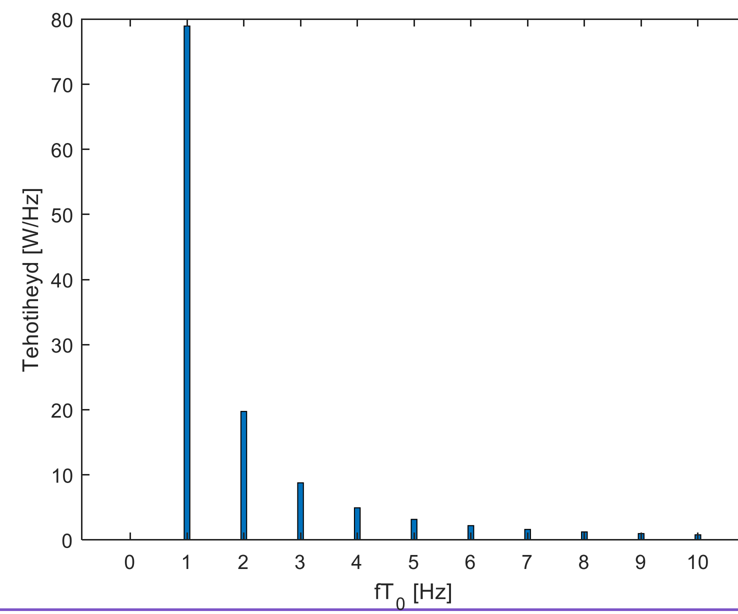
Example: Square wave



Two-sided power spectrum

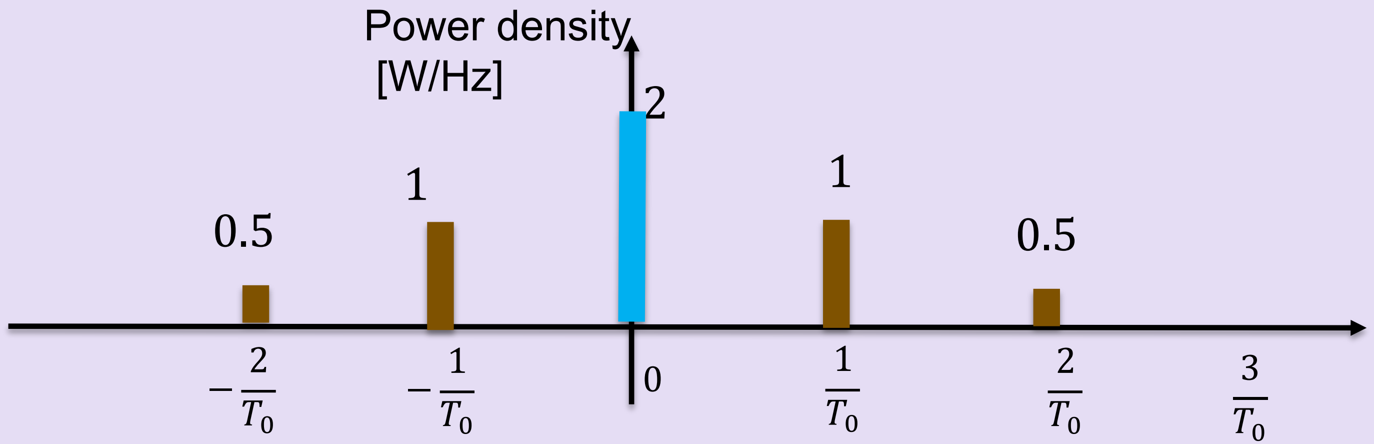


One-sided power spectrum



Problem

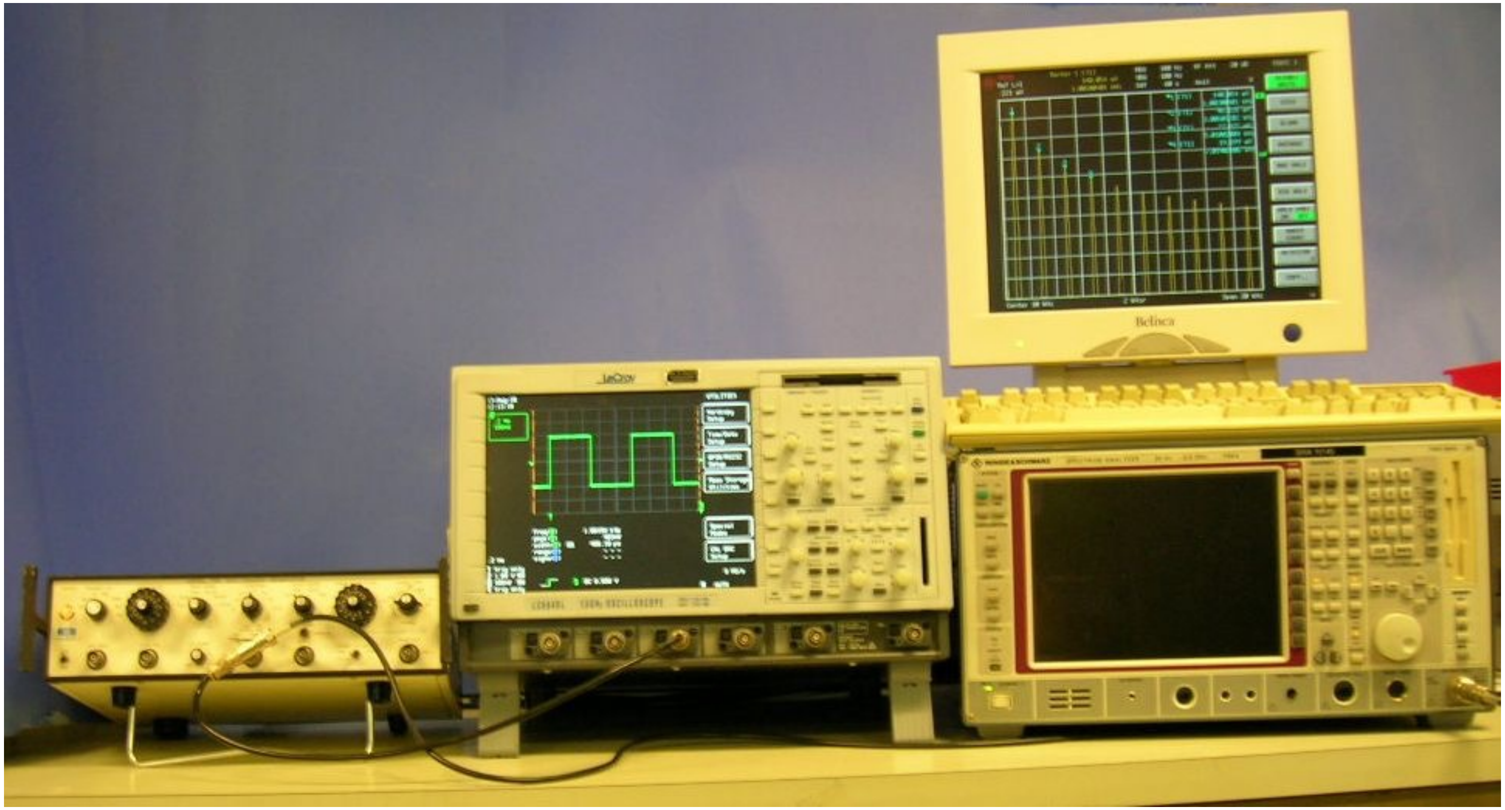
1. The two-sided power spectrum of a periodic signal is



a) Determine its average power

b) Draw its single-sided power spectrum

Signals in time and frequency domain



Signal generator

***Oscilloscope* showing the signal in time domain**

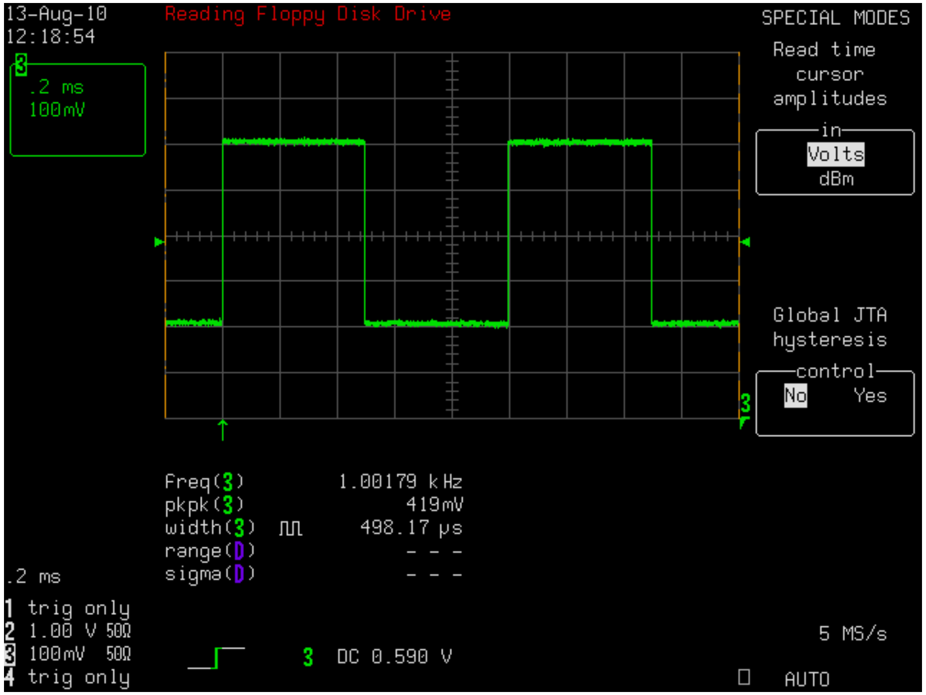
***Spectrum analyzer* showing the signal in frequency domain**

Signals in time and frequency domain

A square wave generated by a signal generator

E.g.

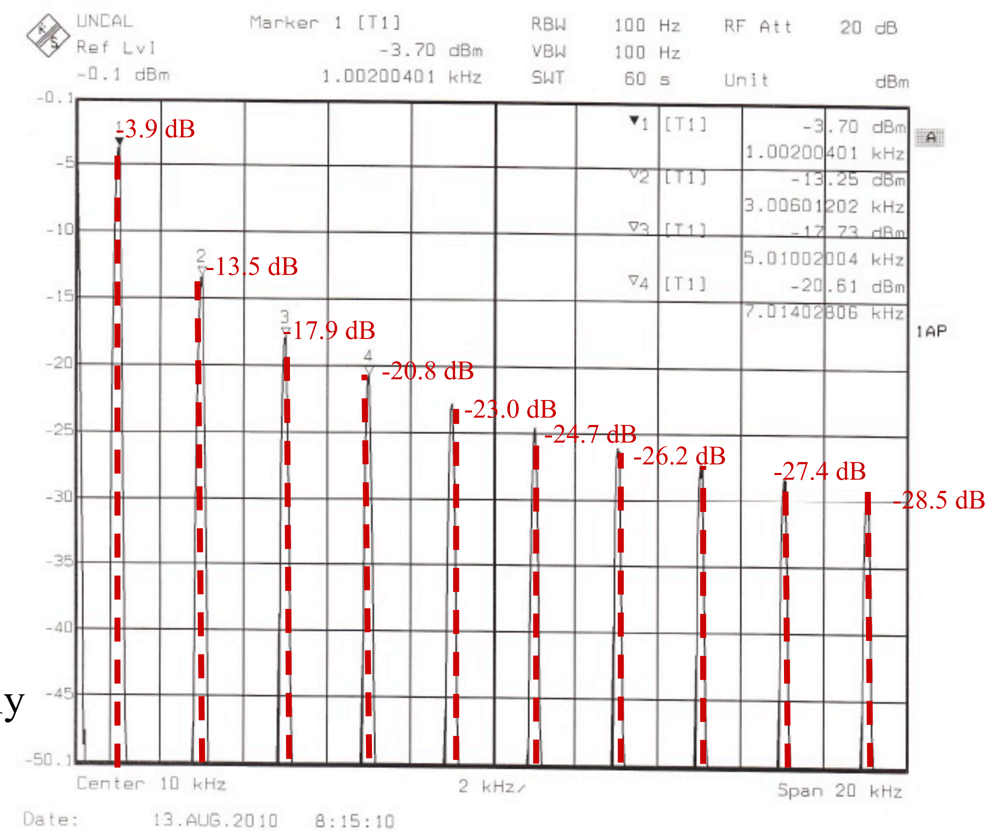
- Digital clock signal
- Alternating voltage generated by the switching power supply
- Test signal



Signals in time and frequency domain

The result of the spectrum analyzer.

Line spectrum calculated from a Fourier series presentation



The values predicted by theory are very close to those measured values!

The theory can be used to confirm that the measuring equipment is correctly calibrated!

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Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

Inverse Fourier Transform

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

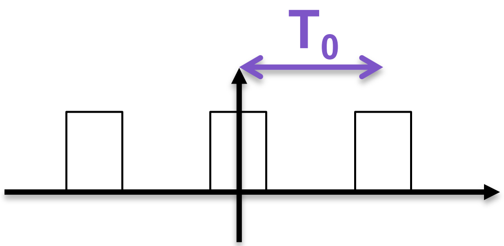
Fourier Transform

Chapters 5-6

Spectrum

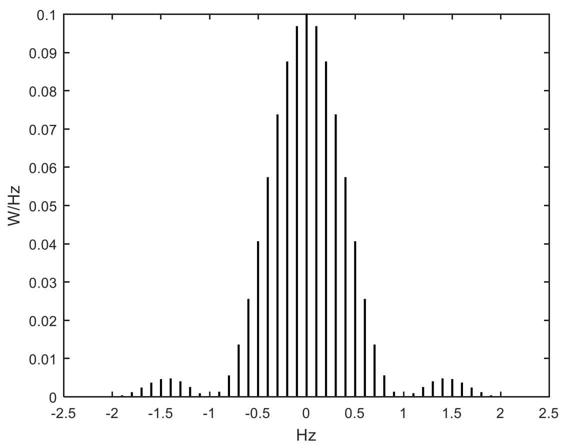
Power signal

=> **Power spectrum [W/Hz]**



$|x_k|^2$

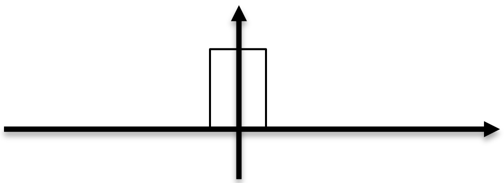
W/Hz



Distribution of the signal power to the frequency components

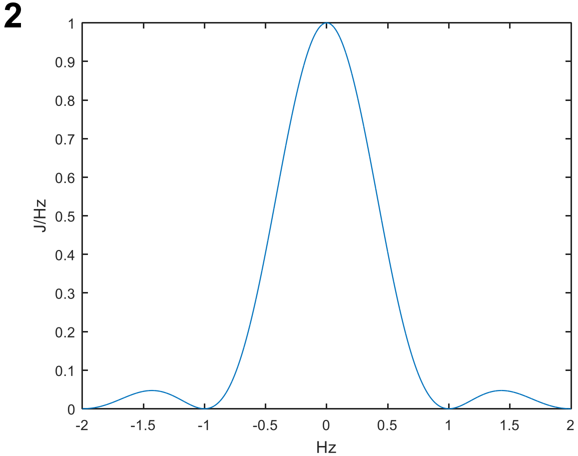
Energy signal

=> **Energy spectrum [J/Hz]**



$|X(f)|^2$

J/Hz



Distribution of the signal energy to the frequency components

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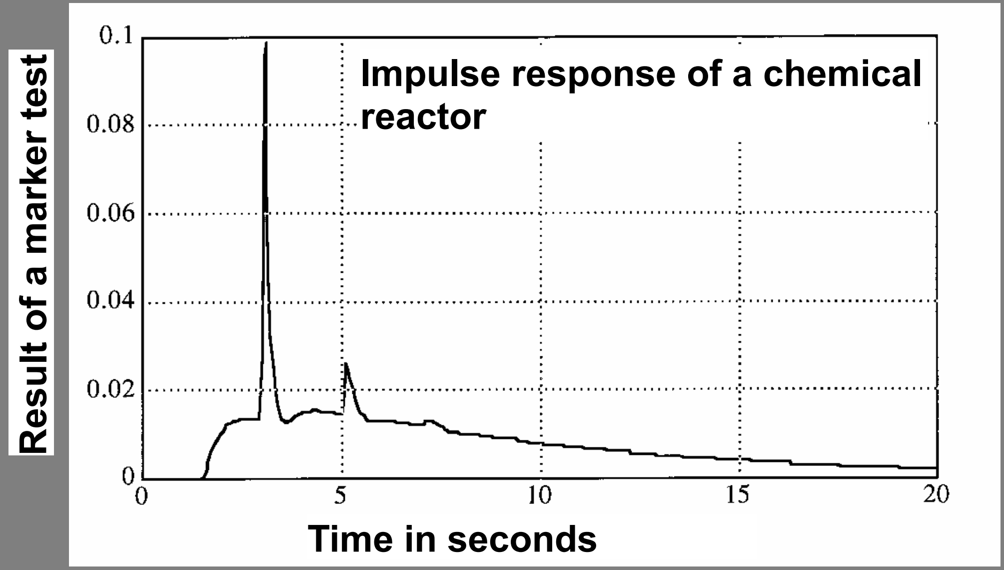


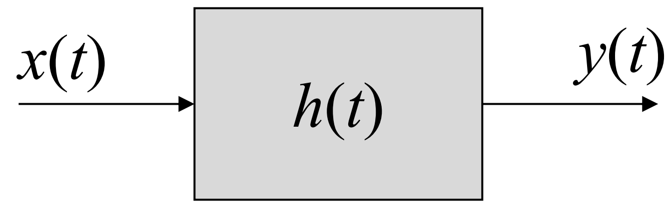
Systems in time domain

Impulse response and Convolution integral

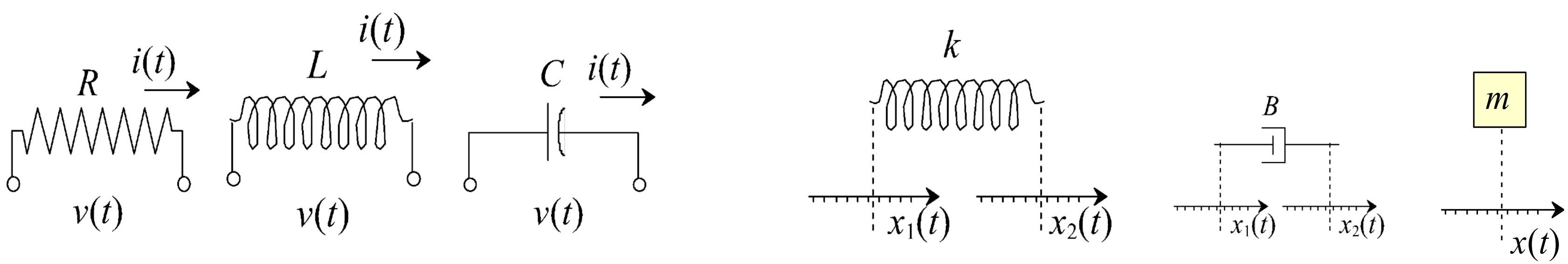
Luku 8

Linear Time Invariant (LTI) Systems
[Lineaariset aikainvariantit järjestelmät]

Differential equation describing the time dynamics of a linear time invariant system



$$\frac{d^n}{dt^n}y(t) = -a_1\frac{d^{n-1}}{dt^{n-1}}y(t) - \cdots - a_n y(t) + b_0\frac{d^m}{dt^m}x(t) + b_1\frac{d^{m-1}}{dt^{m-1}}x(t) + \cdots + b_m x(t)$$



$$v(t) = Ri(t) \qquad v(t) = L\frac{di(t)}{dt} \qquad i(t) = C\frac{dv(t)}{dt}$$

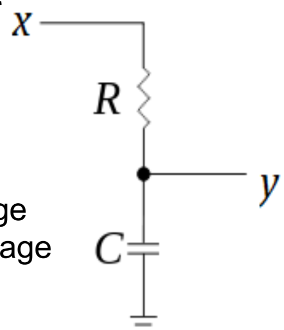
$$F_k(t) = k(x_1(t) - x_2(t)) = k\Delta x(t) \qquad F_b(t) = B\frac{d\Delta x(t)}{dt} \qquad F_m(t) = m\frac{d^2x(t)}{dt^2}$$

Linear Time Invariant (LTI) Systems

Examples of first order systems

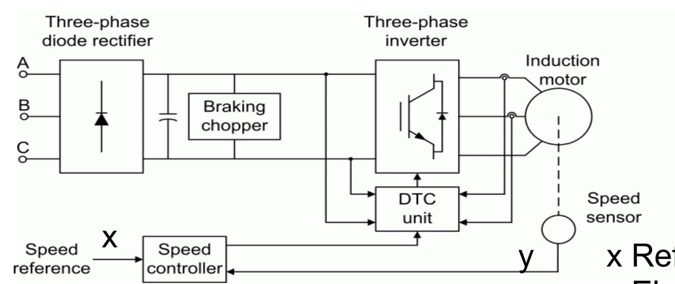
$$\frac{dy(t)}{dt} = -ay(t) + bx(t)$$

RC circuit



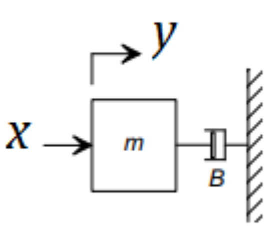
x Input voltage
y Output voltage

Direct torque controlled speed drive



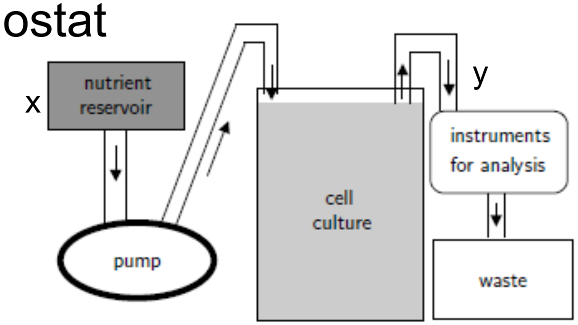
x Reference speed
y Electric motor speed

Shock absorber



x Force
y Speed

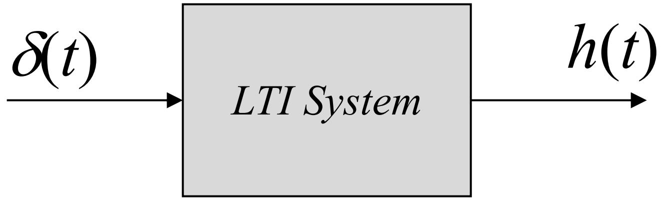
Chemostat



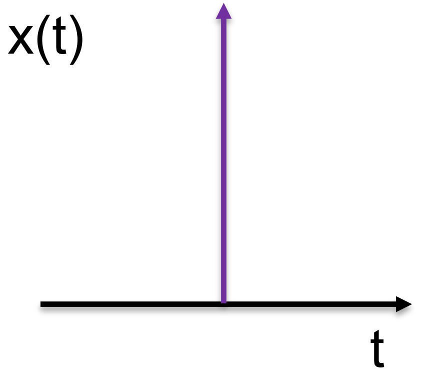
x Nutrient concentration
y Bio mass

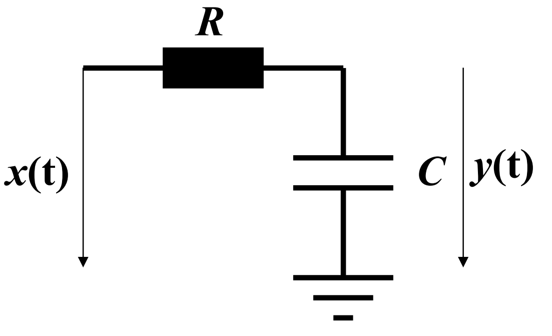
Impulse response of a LTI system

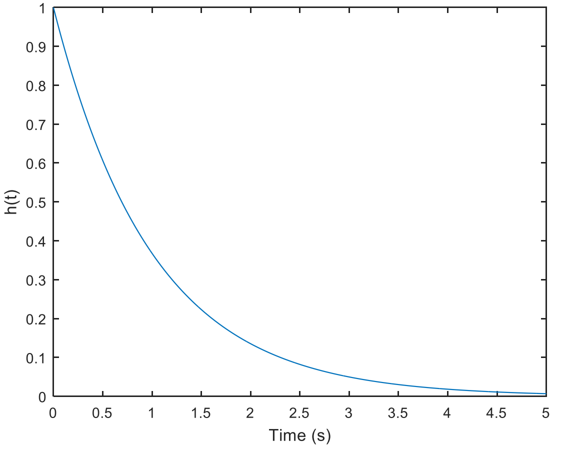
- **Impulse** h(t)



- E.g RC circuit

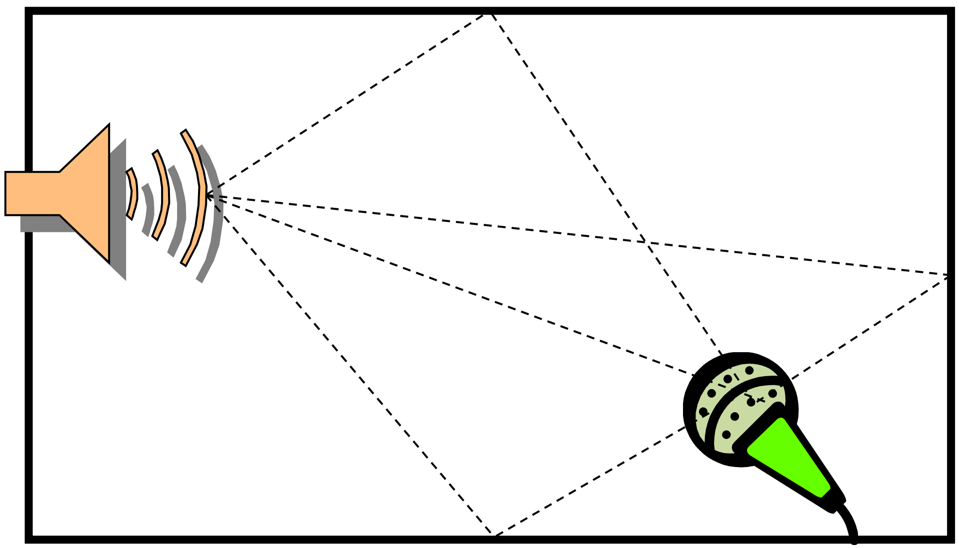






Impulse reponse

Modeling of acoustics in a concert hall



Singing in anechoic studio

$$x(t)$$

http://www.openairlib.net/anechoicdb/content/operatic-voice

Impulse response of a church hall

$$h(t)=\sum_k h_k \delta\left(t-\tau_k\right)$$

http://www.openairlib.net/auralizationdb/content/st-patricks-church-patrington-model

Singing in the church hall

$$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Convolution in chapter 2
+ FFT in chapter 7

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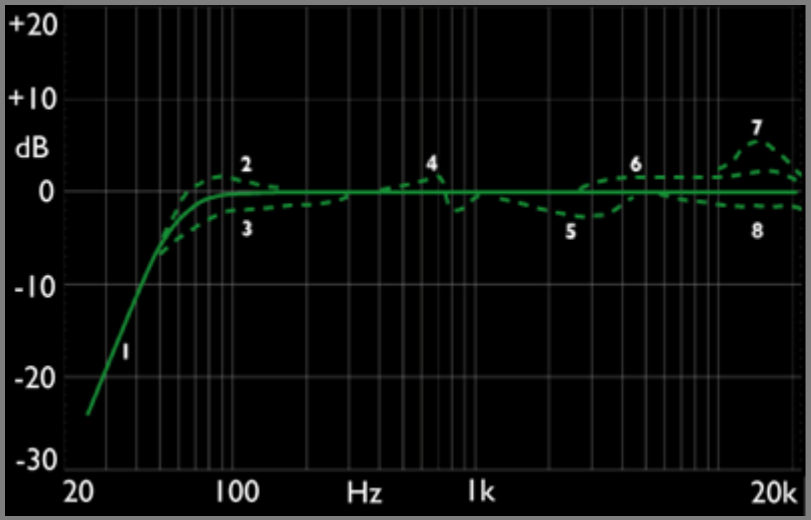
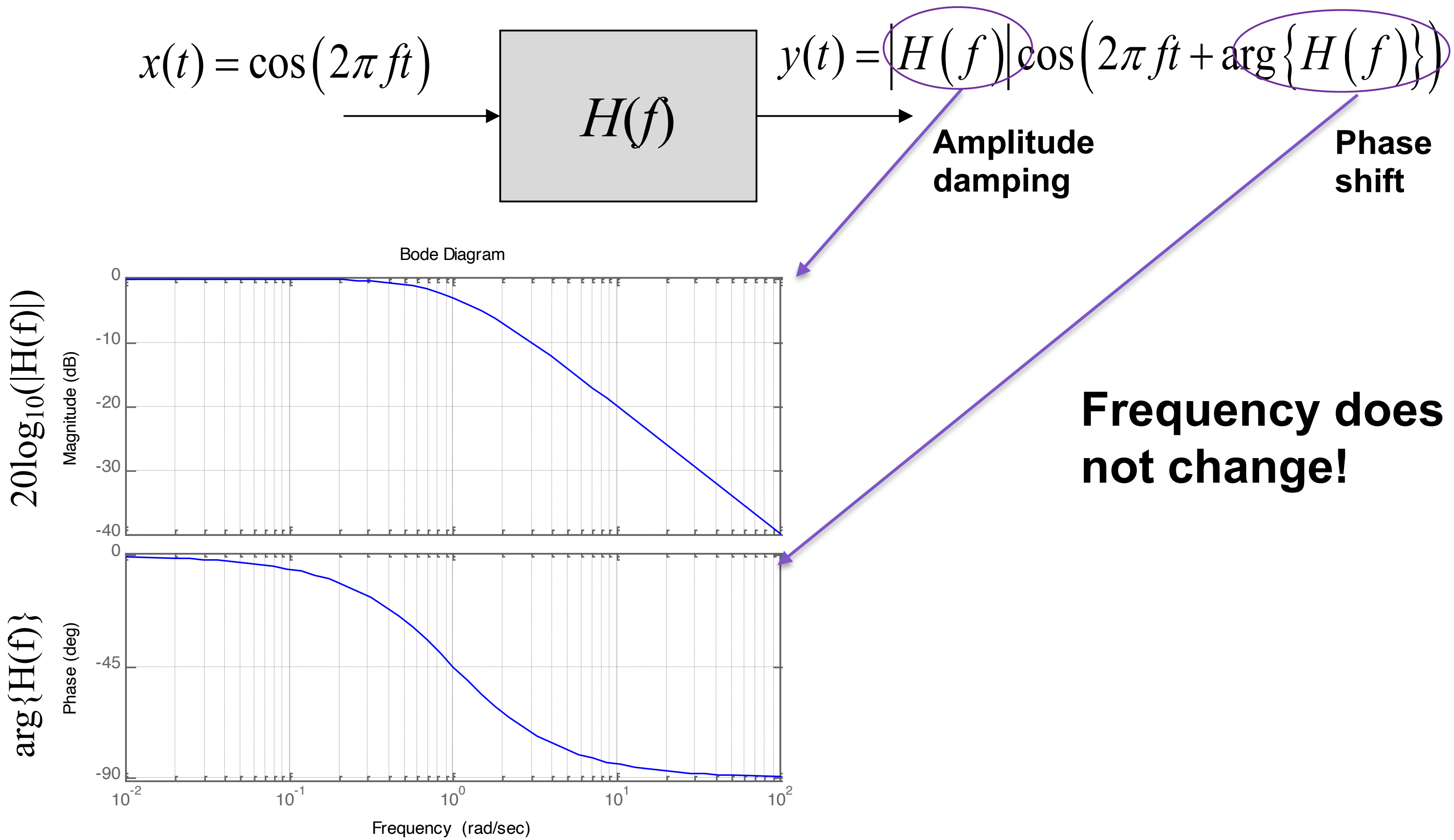


Systems in frequency domain

Chapters 9-10

Frequency response of a LTI system

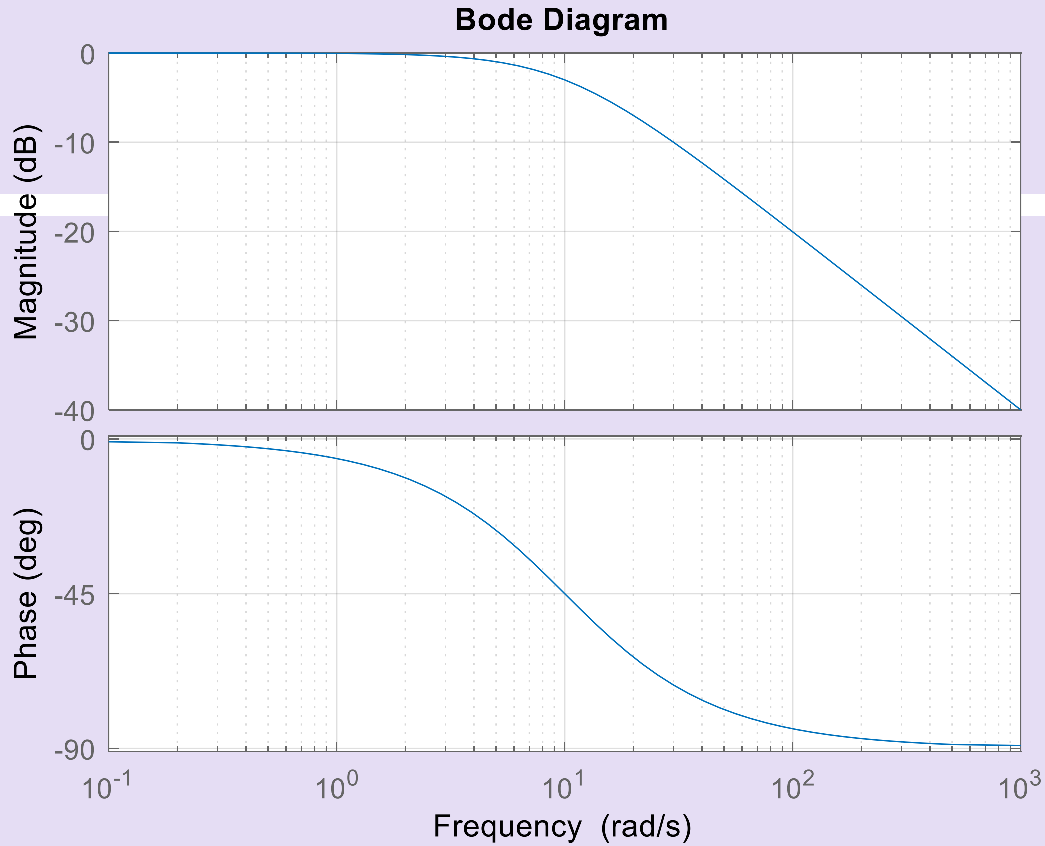


$\omega = 2\pi f$

Frequency does not change!

Problem

The frequency response of the system is



How much the system attenuates sinusoidal signal having angular frequency 1 rad/s? How about 100 rad/s?

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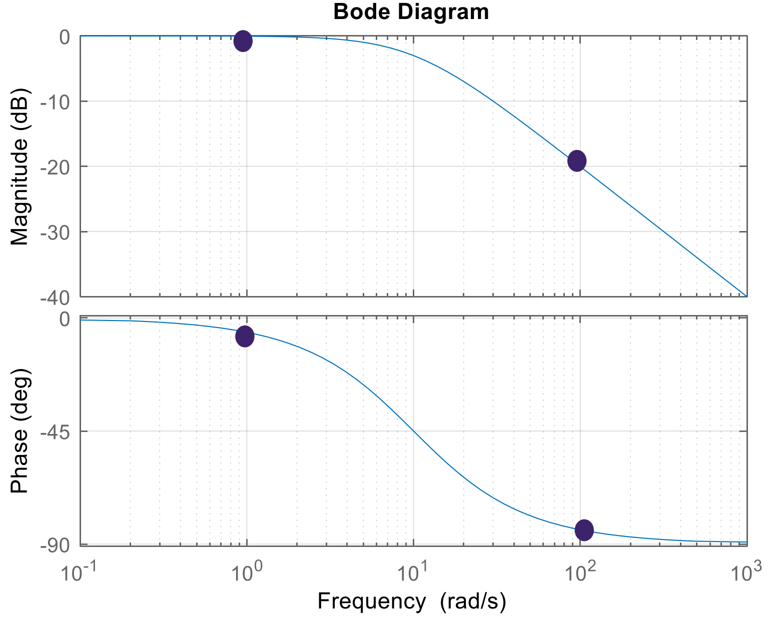
Filters

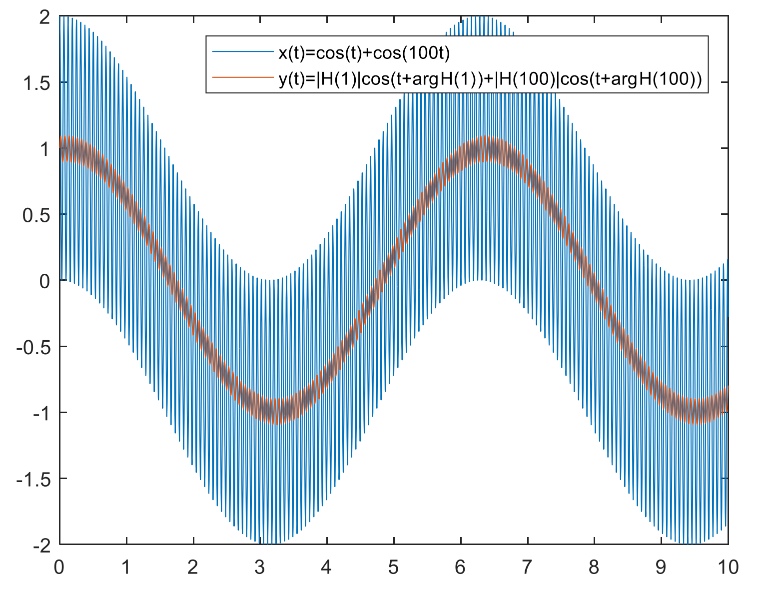
Filters

In signal processing, a filter is a device or process that removes some unwanted components or features from a signal.

Filters

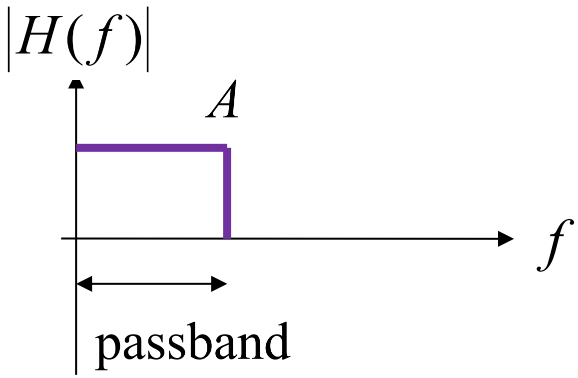
The low-pass filter attenuates the high frequencies but lets the low pass through.



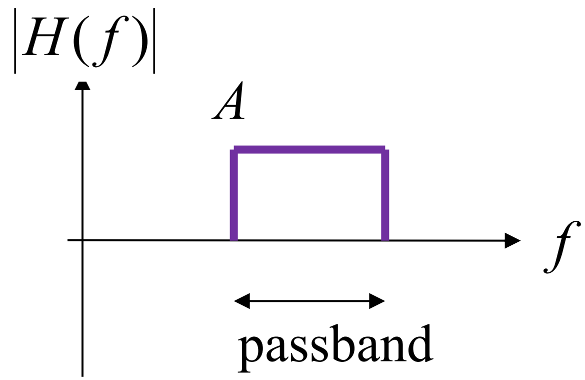


Ideal low-pass, band-pass, high-pass and band-stop filters

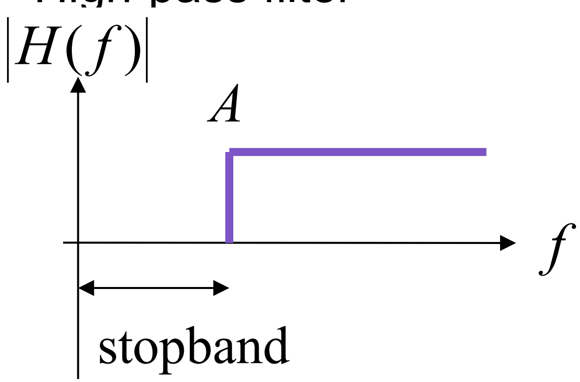
- Low-pass filter



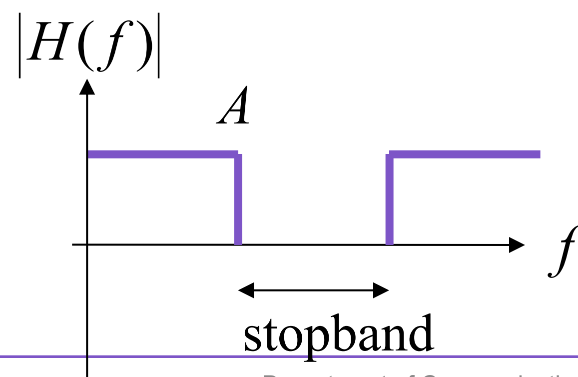
- Band-pass filter



- High-pass filter

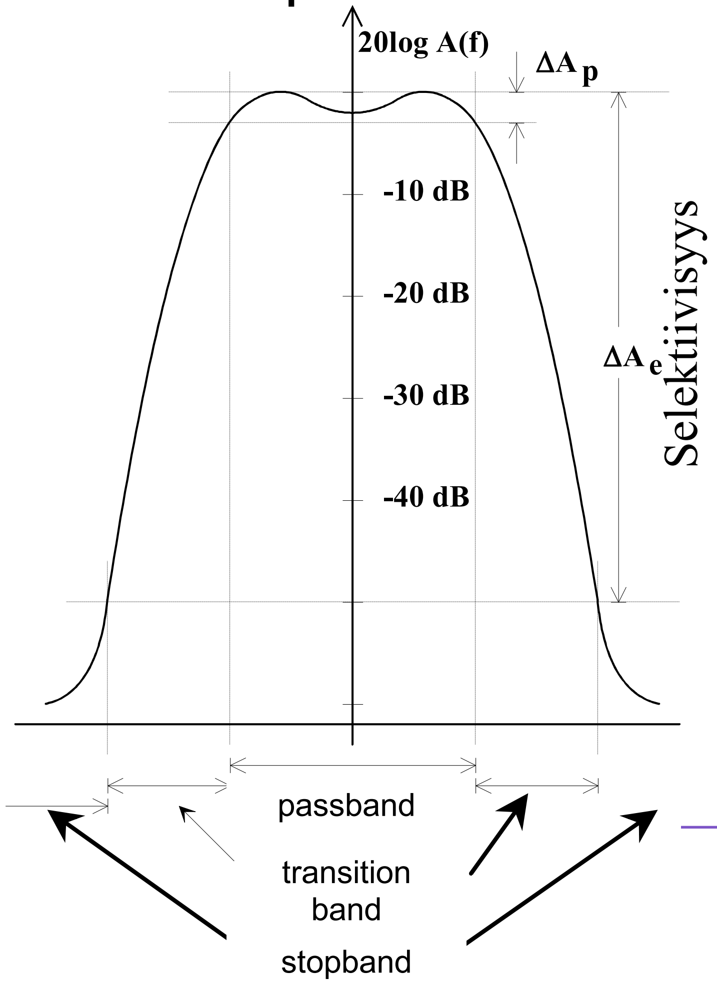


- Band-stop filter

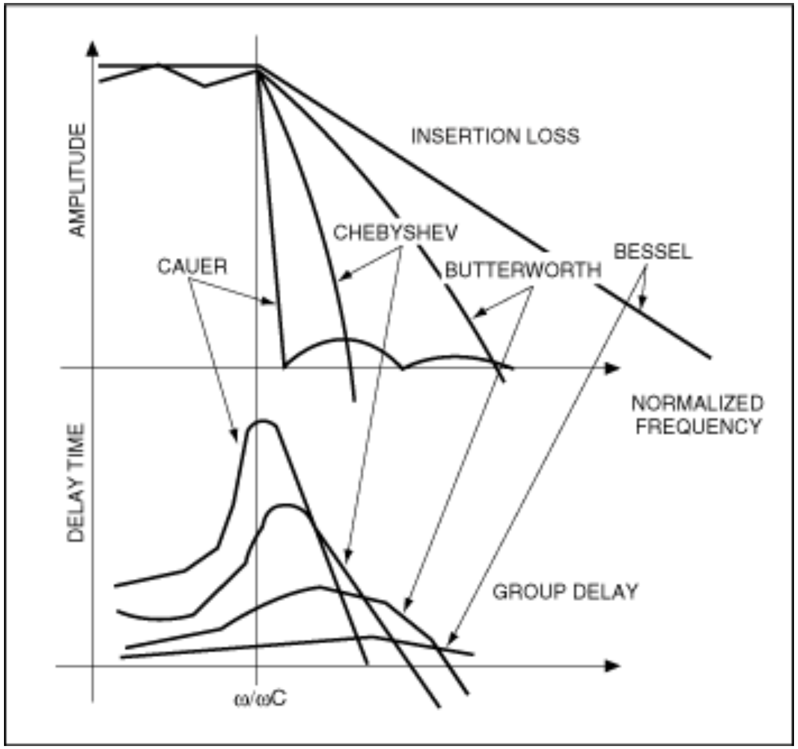


Practical filters

Bandpass filter



Filter families



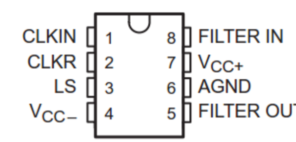
Practical filters

Example: Commerical Butterworth filter chip

SLAS021A – NOVEMBER 1986 – REVISED MARCH 1995

- **Low Clock-to-Cutoff-Frequency Ratio Error**
  TLC04/MF4A-50 . . . ±0.8%
  TLC14/MF4A-100 . . . ±1%
- **Filter Cutoff Frequency Dependent Only on External-Clock Frequency Stability**
- **Minimum Filter Response Deviation Due to External Component Variations Over Time and Temperature**
- **Cutoff Frequency Range From 0.1 Hz to 30 kHz, $V_{CC\pm} = \pm 2.5$ V**
- **5-V to 12-V Operation**
- **Self Clocking or TTL-Compatible and CMOS-Compatible Clock Inputs**
- **Low Supply-Voltage Sensitivity**
- **Designed to be Interchangeable With National MF4-50 and MF4-100**

D OR P PACKAGE
(TOP VIEW)



http://fi.mouser.com/images/texasinstruments/lrg/TI_SOIC_8.jpg

Problem

Give an example of an application where you need

- **a low-pass filter**
- **a bandpass filter**
- **a high-pass filter**

Filters

Filters are needed

- To remove high frequency components before sampling (anti-aliasing filter)
- To reconstruct original continuous time signal from samples
- To generate desired pulse shape
- To maximize signal-to-noise ratio by using matched filter
- To separate desired signal from unwanted signals e.g. at the radio receiver
- To reduce out-of-band interference
- To separate uplink and downlink signals using Duplex filter
- …

Aalto-yliopisto
Sähkötekniikan korkeakoulu

https://en.wikipedia.org/wiki/Audio_power_amplifier#/media/File:Unitra_ws-503_arch1_%281%29.jpg

Memoryless non-linear systems

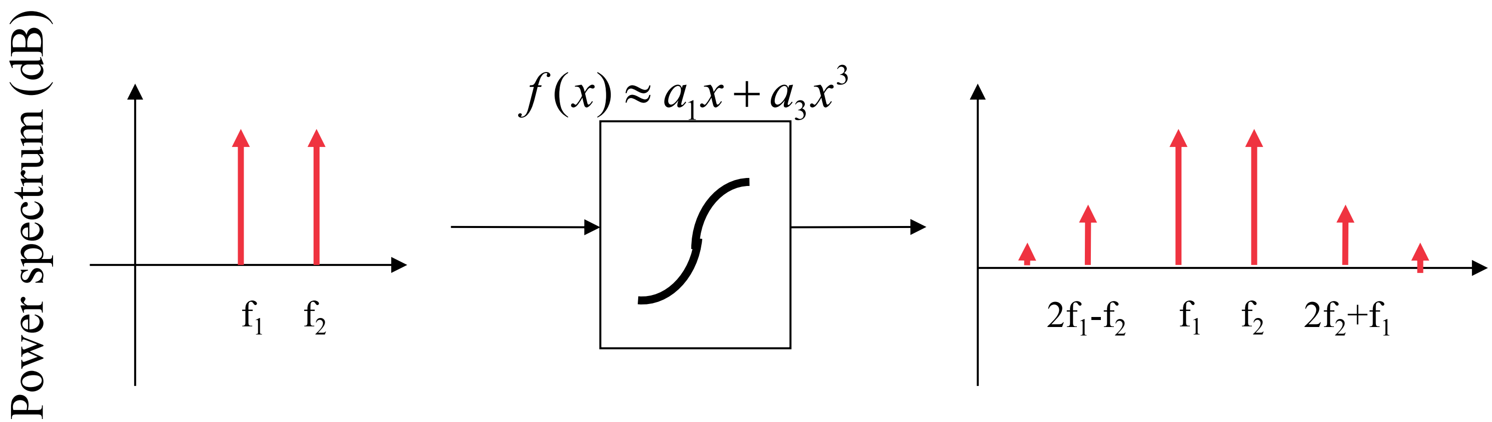
Such as amplifiers

Chapter 11

Distortion and intermodulation

- Memoryless non-linear components cause
  - Distortion (give rise to harmonic components)
  - Intermodulation (cause mixed frequencies)

Distortion_effect.ogg

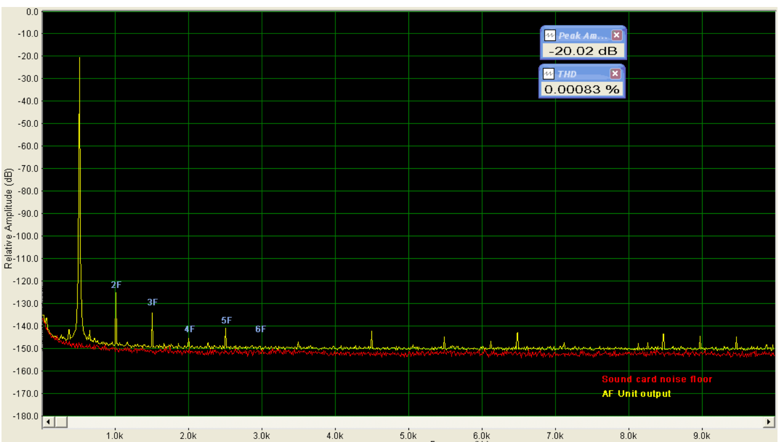


https://en.wikipedia.org/wiki/Distortion
http://en.wikipedia.org/wiki/File:Distortion_effect.ogg

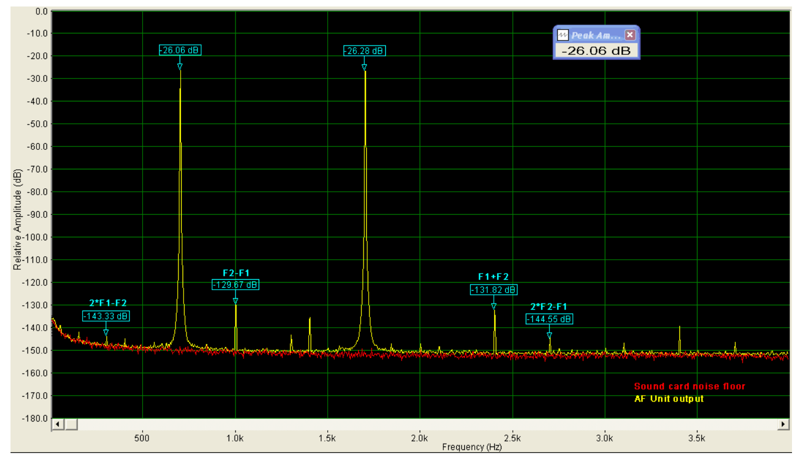
Distortion and intermodulation

Example: microphone preamplifier

Singe tone test



Two tone test



http://neon.skydan.in.ua/AF.php

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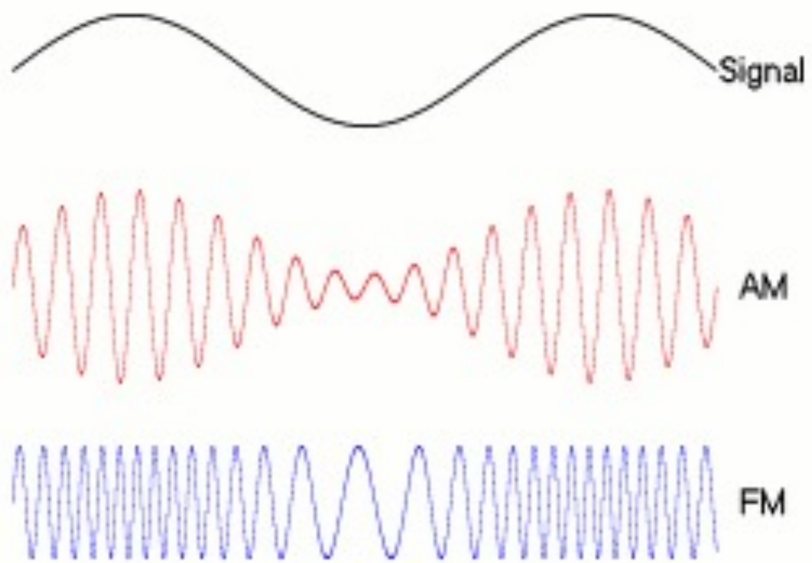


https://en.wikipedia.org/wiki/Modulation#/media/File:Amfm3-en-de.gif

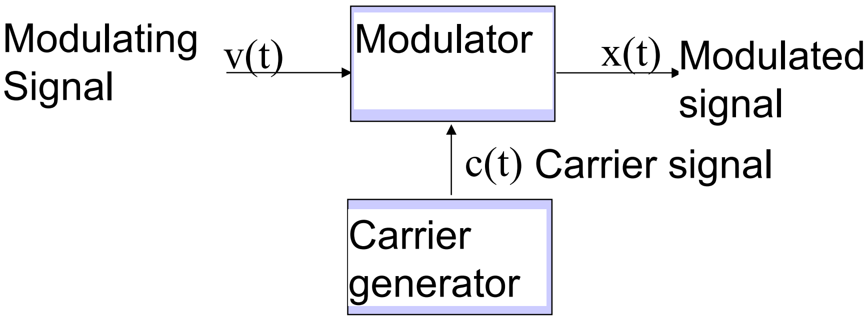
https://www.google.fi/search?q=FM+Radio&source=lnms&tbm=isch&sa=X&ved=0ahUKEwi9i9Kopc_NAhXGO5oKHbapC5cQ_AUICCgB&biw=1280&bih=953#imgrc=RmLLd5lFdjpjaM%3A

Modulation

Luku 11

Modulation

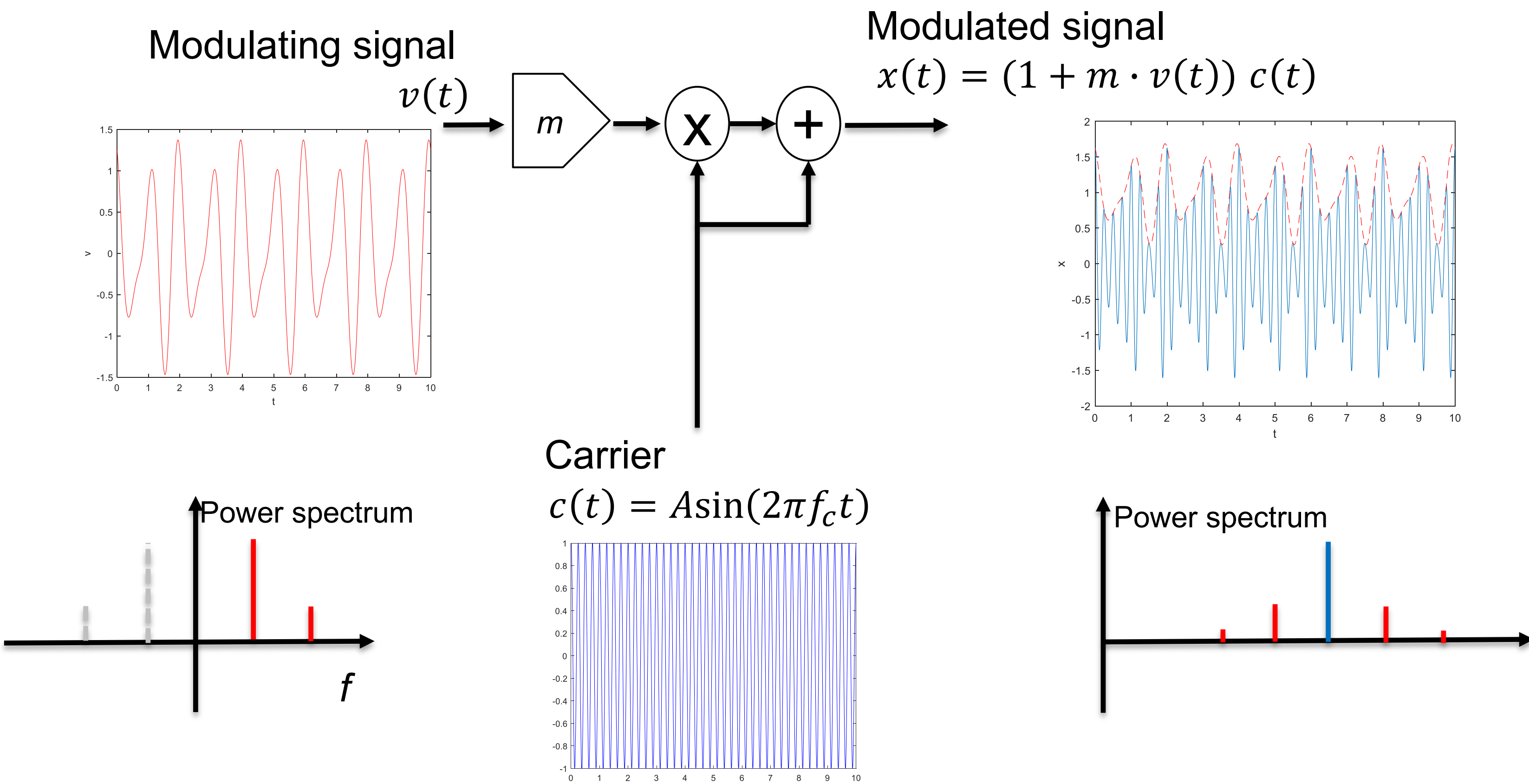
- The aim of a *modulation* is to transfer an analog baseband (or lowpass) signal at a different frequency.
- The reverse process is called *demodulation*.
- Modulation is done by varying one or more properties of a periodic waveform, called the carrier signal, with a modulating signal that typically contains information to be transmitted.

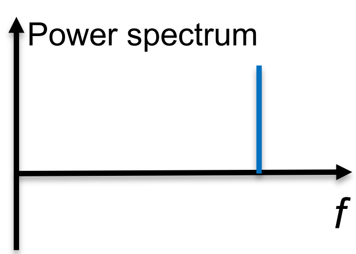


Linear modulation:
- v(t) control carrier amplitude

Nonlinear modulation:
- v(t) control carrier phase or frequency

Example: Amplitude modulation (linear modulation)





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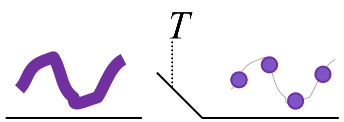
Sampling

Sampling theorem
Discrete Fourier Transform

Chapter 7

Sampling

- Let us take samples from a continuous signal with fixed time interval



$x(t)$ $\qquad \{x(kT), \quad k \in 0,1,2,\ldots\}$

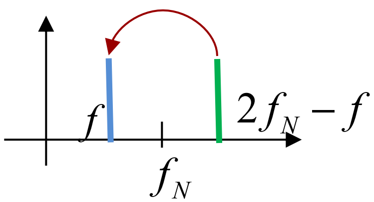
T_s sampling interval

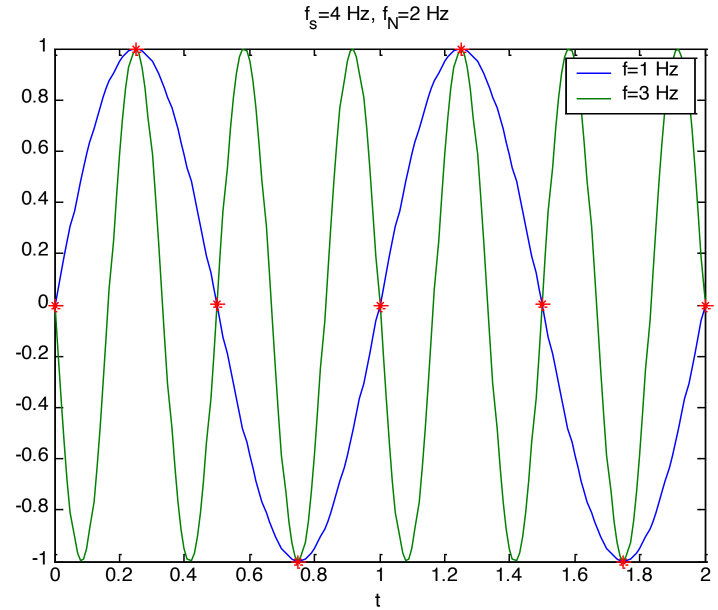
$f_s = 1/T_s$ sampling frequency

Nyquist theorem: Let x(t) be band limited signal with bandwidth B then the signal can be reconstructed from the samples without error if $f_s \geq 2B \triangleq f_N$ (Nyquist frequency).

Aliasing

Frequency components that exceeds the Nyquist frequency appear as a lower frequency component after sampling.





Because of this phenomenon called aliasing, all signals must be low-pass filtered before sampling (analog-to-digital conversion)

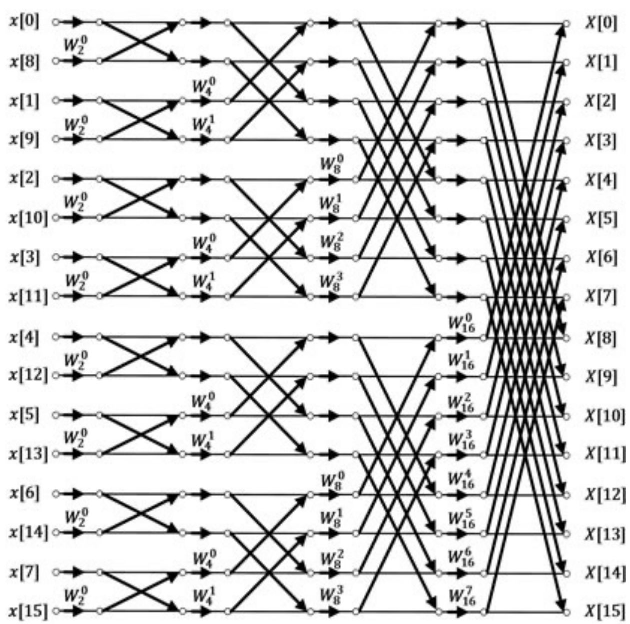
Problem

The sinusoidal signal has a frequency of 2 GHz.

What should be the sampling rate be to prevent aliasing?

Discrete and Fast Fourier Transform

- **In practice, the energy spectrum is measured by sampling the signal and applying a discrete Fourier transform (FFT) algorithm to the samples.**



Fast Fourier Transform (FFT) is an efficient algorithm to calculate discrete Fourier Transform.

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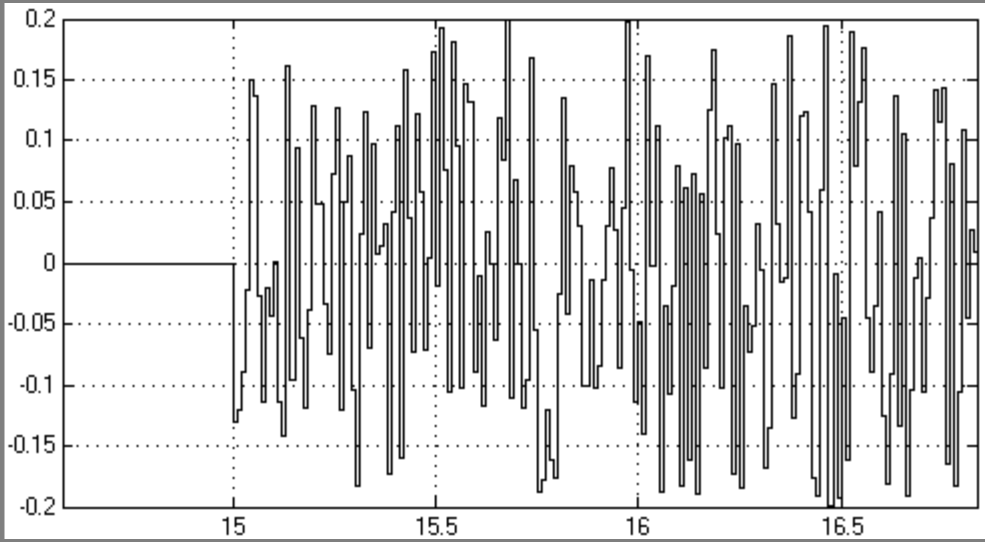


Random signals and noise

Noise

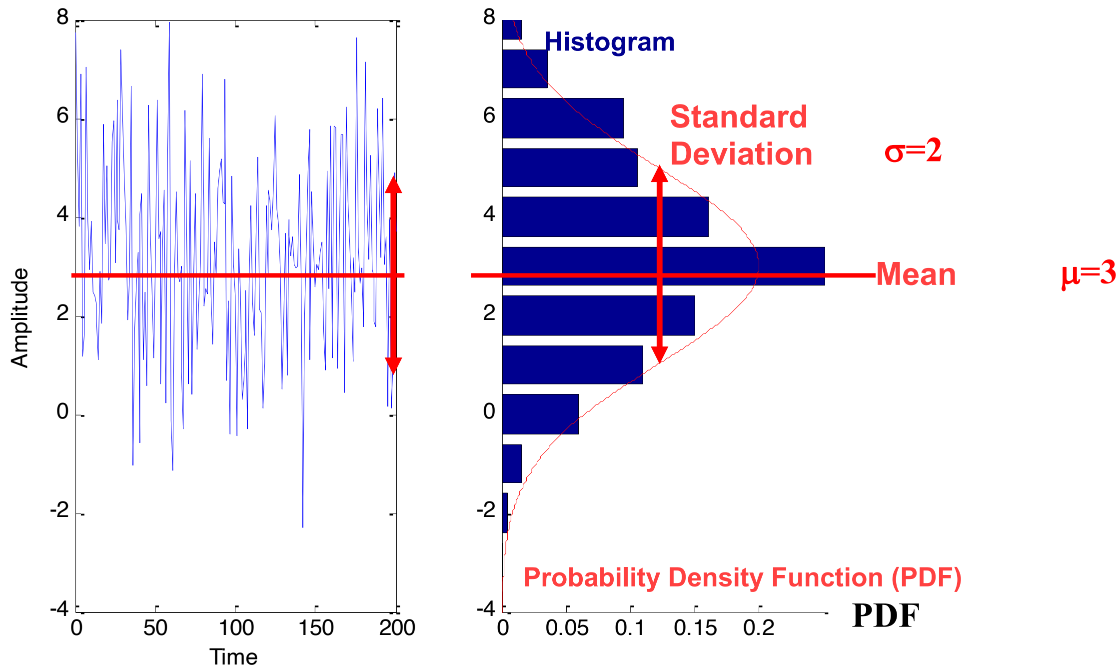
Chapter 12

Stochastic processes and noise in time domain

The value of a random signal cannot be accurately predicted. We can only give the probability for its amplitude to be on certain interval.

$$\Pr\left(x(t) \le x\right) = F_x(x;t)$$

Random signal is called stationary if its statistical properties do not change in time.



Thermal noise a.k.a. White noise

Thermal noise (Johnson-Nyquist noise) is the electronics noise generated by the thermal agitation of the charge carriers (usually electons) inside electrical conductor.

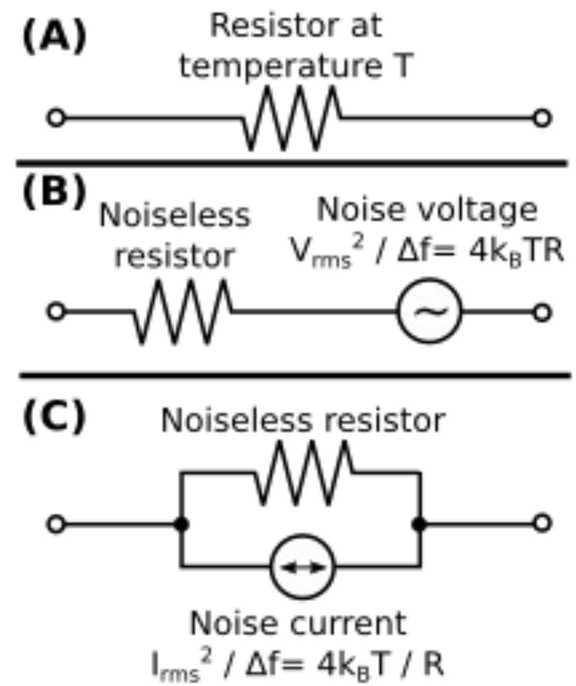
Thermal noise power

$$P = 4k_B T \Delta f$$

- k_B Bolzmann's constant
- T Temperature in Kelvins
- Δf Bandwidth in Hz

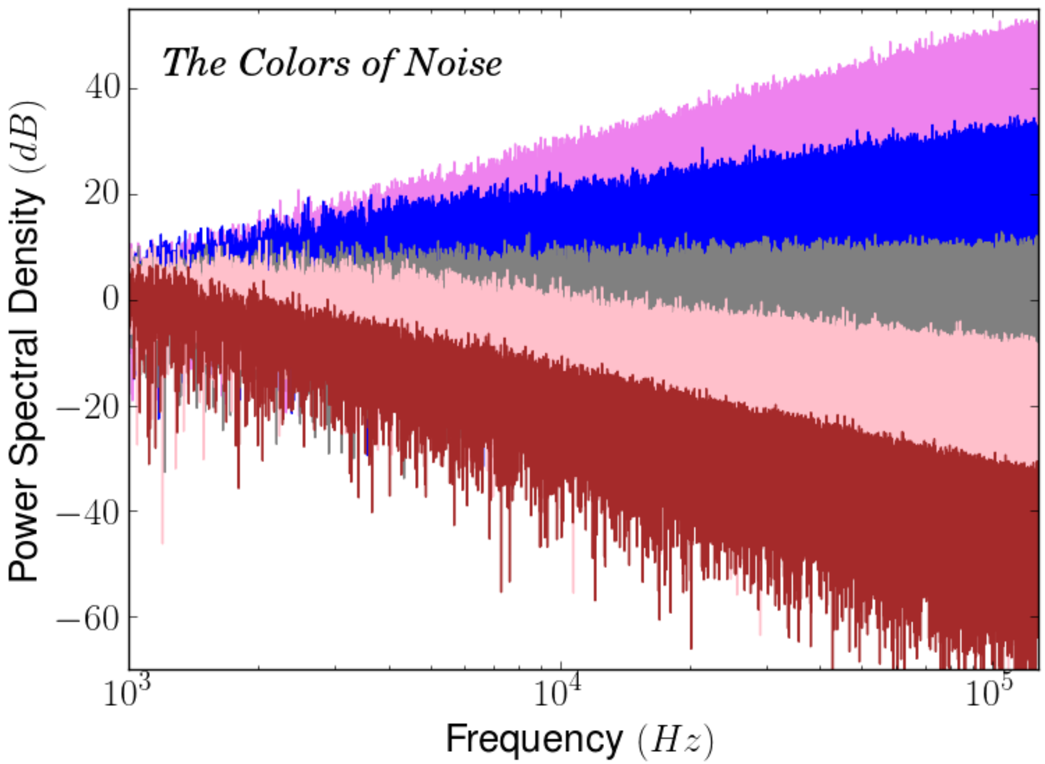
Thermal noise power at 300K for 1 Hz band ins -174 dBm

Thermal noise voltage follows Gaussian distribution with zero mean and *P* variance



Noise power density

All measured signals contain noise.



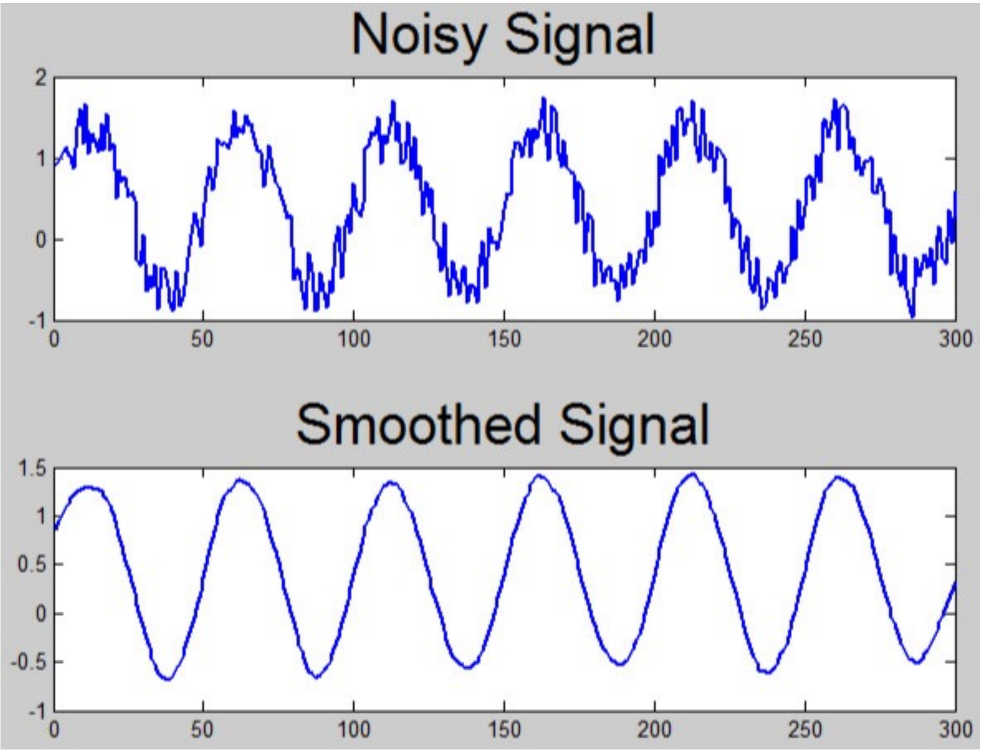
Blue_noise.ogg

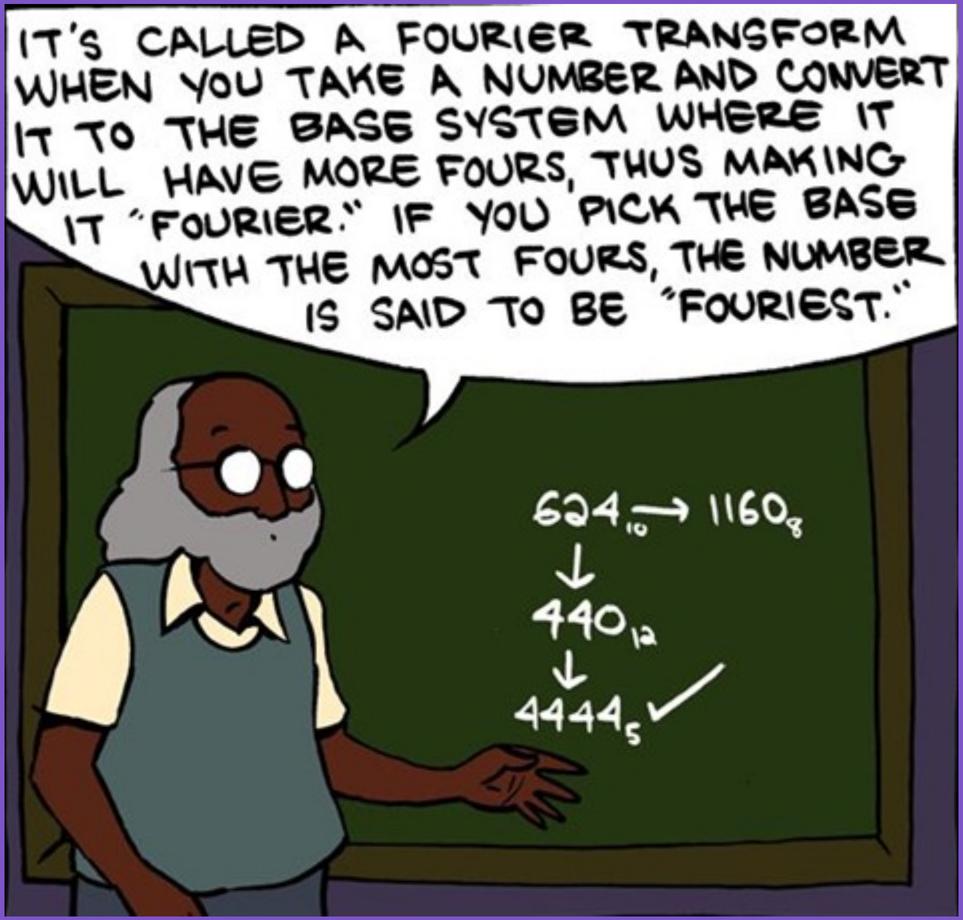
White-noise-sound-20sec-mono-44100Hz.ogg

Pink_noise.ogg

https://en.wikipedia.org/wiki/Colors_of_noise

Noise can be reduced by filtering




IT'S CALLED A FOURIER TRANSFORM WHEN YOU TAKE A NUMBER AND CONVERT IT TO THE BASE SYSTEM WHERE IT WILL HAVE MORE FOURS, THUS MAKING IT "FOURIER." IF YOU PICK THE BASE WITH THE MOST FOURS, THE NUMBER IS SAID TO BE "FOURIEST."
$624_{10} \rightarrow 1160_8$
440_{12}
4444_5 ✓