ELEC-E8101 Digital and Optimal Control

Exercise 1

The problems marked with an asterisk (*) are not discussed during the exercise. The solutions are given in MyCourses, and these problems belong to the course material.

1.

Take the Z-transform of the following sequences by using the definition

$y(k) = 1$, $k = 0,1,2,3,\ldots$

$y(k) = e^{-ak}$, a = constant and $k = 0,1,2,\ldots$

2.

a. Let $f(k) = a^k$, a constant. Show that $F(z) = \frac{z}{z-a}$.

b. In the Z-transform tables it is stated that $Z\left[e^{-kh/T}\right] = \frac{z}{z - e^{-h/T}}$. Is this the same as in part a. of the problem?

3.

Define the value of $y(kh)$, while $k \to \infty$ (use the final-value theorem).

$$Y(z) = \frac{0{,}792z^2}{(z-1)(z^2 - 0{,}416z + 0{,}208)}$$

Verify your result by using Matlab.

4.

Take the inverse-transform of the following expression

$$Y(z) = \frac{\left(1 - e^{-ah}\right)z}{(z-1)\left(z - e^{-ah}\right)}, \quad a \text{ constant.}$$

***5.**

Prove that the following holds

$$Z\left\{\frac{1}{2}(kh)^2\right\} = \frac{h^2 z(z+1)}{2(z-1)^3}$$

Hint: begin by transforming $Z\{kh\}$.

6.

By using the Z-transform solve for $y(k)$ from the following difference equation

$$y(k+2)-1{,}5y(k+1)+0{,}5y(k)=u(k+1)$$

where $u(k)$ is the unit step at k = 0, $y(0)=0{,}5$ and $y(-1)=1$. Verify your solution.

Final-value theorem: If $\lim_{k\to\infty} y(kh)$ exists, then it holds $\lim_{k\to\infty} y(kh)=\lim_{z\to 1}(1-z^{-1})Y(z)$, A sufficient (but not necessary) condition for the existence of $\lim_{k\to\infty} y(kh)$ is that $(1-z^{-1})Y(z)$ has no poles on or outside the unit circle.

Theorems of Z-transformation

Definition: $F(z)=Z\{f(kh)\}=\sum_{k=0}^{\infty} f(kh)z^{-k}$	
z-transformation	Sequence in time domain
$F(z)$	$f(k)$
$C_1F_2(z) + C_2F_2(z)$	$C_1f_2(k) + C_2f_2(k)$
$z^{-n}F(z)$	$q^{-n} f(k)$
$z^n\left(F(z)-\sum_{j=0}^{n-1} f(jh)z^{-j}\right)$	$q^n f(k)$
$F_1(z)F_2(z)$	$\sum_{n=0}^{k} f_1(n) f_2(k-n)$
If the limits of $f(kh)$ and $F(z)$ exist, they satisfy	
$\lim_{k\to\infty}\{f(kh)\}=\lim_{z\to 1}\{(1-z^{-1})F(z)\}$	$f(0)=\lim_{z\to\infty} F(z)$

Z-transformations and corresponding sequences

z-transformation	Sequence in time domain
1	$\delta(k)$
$\frac{z}{z-1}$	$1,\ k \geq 0.$
$\frac{hz}{(z-1)^2}$	kh
$\frac{h^2 z(z+1)}{(z-1)^3}$	$(kh)^2$
$\frac{z}{z-e^{-h/T}}$	$e^{-kh/T}$
$\frac{z\sin(\omega h)}{z^2-2z\cos(\omega h)+1}$	$\sin(\omega kh)$