



**Aalto University**  
School of Electrical  
Engineering

# **ELEC-E8740 — Course Overview and Introduction to Sensor Fusion**

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**Aalto University**

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# The Team

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**M.Sc. Zaeed Khan**

Zulip support, grading



# Intended Learning Outcomes of The Course

After successfully completing this course, you are able to:

- explain the principles and components of sensor fusion systems,
- identify and explain the differences between linear and nonlinear models and their implications on sensor fusion,
- construct models of multi-sensor systems and use least-squares algorithms for sensor fusion,
- construct continuous and discrete time state-space models based on ordinary differential equations, difference equations, and physical sensor models,
- develop and compare state-space models and Kalman as well as particle filtering algorithms for solving sensor fusion problems.

# Schedule

- 10+2 lectures
  - Tuesday, 12:15 - 14:00 in TU2.
  - Detailed schedule (incl. topics) on MyCourses
  - Previous year's recorded (Zoom) lectures will be available as backup
- 10+2 exercise sessions
  - Exercise sessions are on Fridays, 12:15 - 14:00 in AS2.
  - Exercises start on Friday, September 9, 2022.
- First exam is on Monday, October 17, 2022 at 9-12.
- Second exam is on Friday, December 9, 2022 at 13-16.
- The tentative project work deadlines are Sunday, November 20, 2022, and Sunday, December 18, 2022.

Check MyCourses regularly for updates!

# Course Material and Zulip

- Lecture notes and slides are the main course literature
  - Lecture notes (~ course book) are already available on the course homepage in MyCourses.
  - Slides will be made available in MyCourses just before each lecture.
- There is also Zulip chat space for the course (see MyCourses).

# Exercises and Homeworks

- Exercise sessions are held on Fridays, 12:15 - 14:00 in AS2, starting on Friday, September 9, 2022.
- In the exercise sessions, the teacher shows you hands on how to solve the exercises.
- Pen & paper and computer exercises (mainly Python)
- Exercise sessions are not mandatory but highly recommended – the exam questions are likely to be related to the exercises
- In the end of each exercise paper there is a homework. Homeworks affect grading.
  - The homeworks need to be generally returned on MyCourses before the next exercise session day at 12:00.

# Assessment and Grading

- The high level formula for the grade is  
$$\text{final grade} = \max(\text{exams+homework grade}, \text{project grade})$$
- You still must pass both the exams+homework and project!
- The exams and homeworks give total of 100 points, which determine the grade via mapping  $\geq 50\text{pts} \leftrightarrow \text{grade } 1$ ,  
 $\geq 60\text{pts} \leftrightarrow \text{grade } 2$ ,  $\geq 70\text{pts} \leftrightarrow \text{grade } 3$ ,  
 $\geq 80\text{pts} \leftrightarrow \text{grade } 4$ ,  $\geq 90\text{pts} \leftrightarrow \text{grade } 5$ .
- Each of the 2 exams give maximum of 30 points and the homeworks (10) give 4 points each.
- The project work grading is clarified later.

# Project work

- Track an **autonomous robot** using multiple sensors
- **Details** of the project work will be provided later.



# Presemo Questionnaire

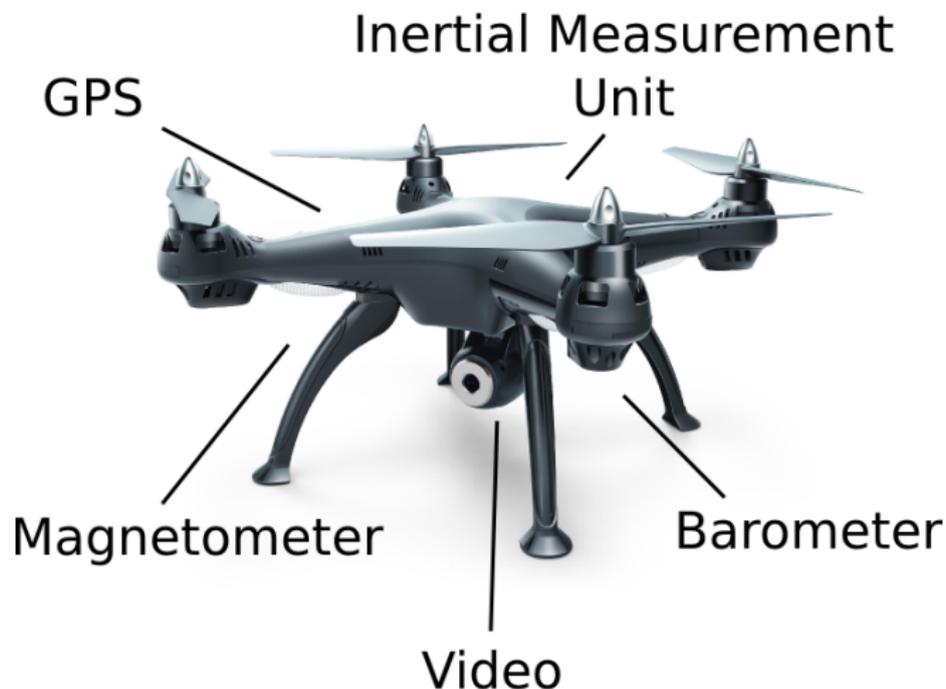
- We are using **presemo** on this course.
- Please use your computer or mobile phone and go to:

<http://presemo.aalto.fi/fusion>

# Definition of Sensor Fusion

- One possible definition of **sensor fusion**:  
*"computational methodology which aims at combining the measurements from multiple sensors such that they jointly give more information on the measured system than any of the sensors alone."*
- The important aspects are:
  - It is **computational methodology**.
  - Uses measurements from **multiple sensors**.
  - Attempts to use the **information** from all the sensors **jointly**.

# Sensor Fusion Applications: Drones



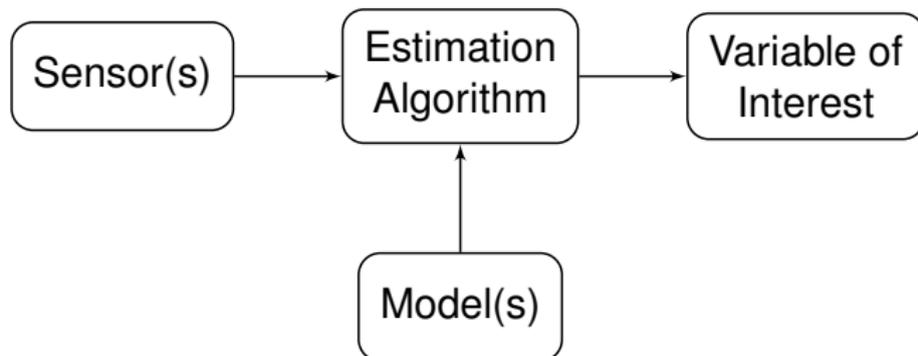
# Sensor Fusion Applications: Autonomous Cars



# Sensor Fusion Applications: Smartphones



# The Components of Sensor Fusion



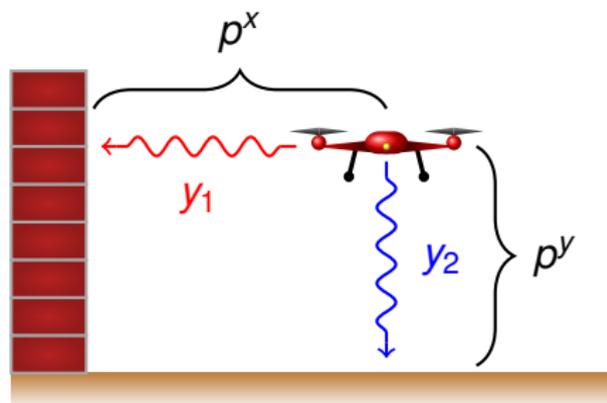
# Model of a Drone (1)

- We measure  $y_1$  with e.g. radar or ultrasound.
- We measure  $y_2$  with e.g. radar or barometer.
- We wish to "fuse" the sensor measurements to get the location  $(p^x, p^y)$ .
- The model in this case is

$$y_1 = p^x + r_1,$$

$$y_2 = p^y + r_2. \quad (r_1 \text{ and } r_2 \text{ here denote measurement noises})$$

- Sensor fusion amounts to just  $p^x \approx y_1$  and  $p^y \approx y_2$ .



## Model of a Drone (2)

- We could also measure the distance  $y_3$  to an additional **tilted wall**.
- The **model** now becomes

$$y_1 = p^x + r_1,$$

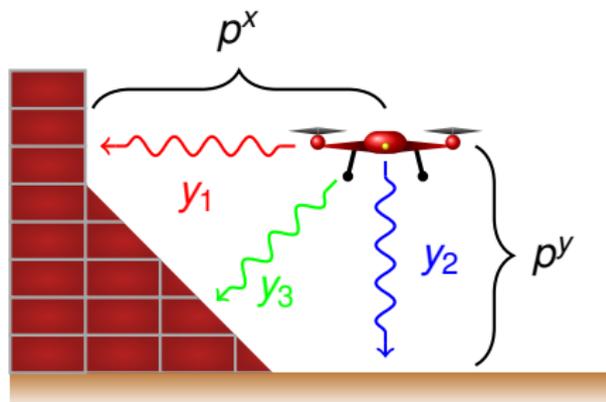
$$y_2 = p^y + r_2,$$

$$y_3 = \frac{1}{\sqrt{2}}(p^x - x_0) + \frac{1}{\sqrt{2}}p^y + r_3.$$

- In **vector form**:

$$\mathbf{y} = \mathbf{G} \mathbf{x} + \mathbf{b} + \mathbf{r}.$$

- **Linear least squares method** gives  $\mathbf{x} = (p^x, p^y)$ .



# Model of an Autonomous Car (1)

- We measure **relative positions** of  $M$  landmarks.
- We get  $2M$  measurements ( $M = 4$  here):

$$y_1 = s_1^x - p^x + r_1,$$

$$y_2 = s_1^y - p^y + r_2,$$

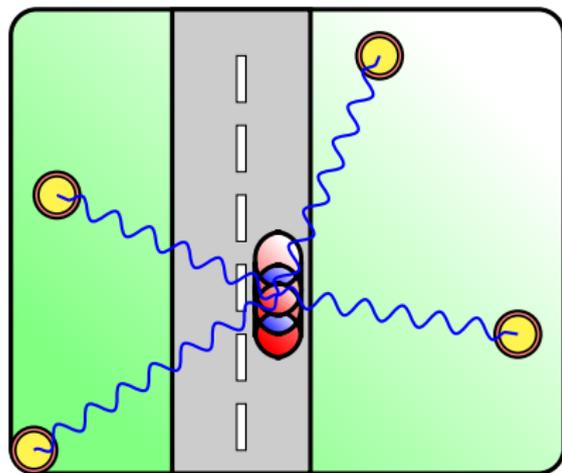
⋮

$$y_{2M-1} = s_M^x - p^x + r_{2M-1},$$

$$y_{2M} = s_M^y - p^y + r_{2M}.$$

- Again leads to form

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{b} + \mathbf{r}.$$



## Model of an Autonomous Car (2)

- We only measure the **range** to **each landmark**.
- In that case we have

$$y_1^R = \sqrt{(s_1^x - p^x)^2 + (s_1^y - p^y)^2} + r_1^R,$$

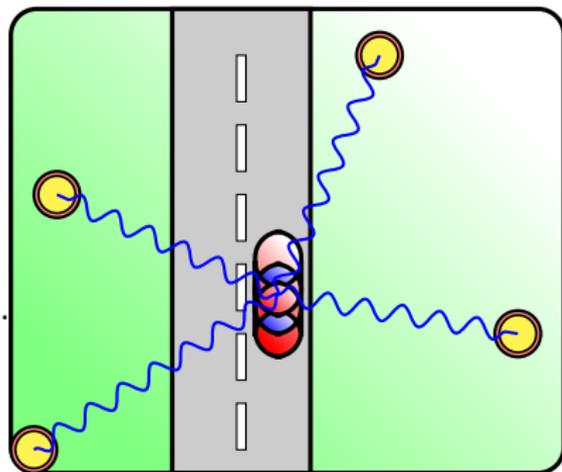
⋮

$$y_M^R = \sqrt{(s_M^x - p^x)^2 + (s_M^y - p^y)^2} + r_M^R.$$

- This is a **non-linear model**

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r}$$

- **Non-linear least squares method** is needed.



# Dynamic Models

- The object of interest might also be **moving**.
- We can model time-continuity with a **dynamic model**.
- For example, we might have

$$\mathbf{x}_n = \mathbf{x}_{n-1} + \mathbf{q}_n \quad (\text{here } \mathbf{q}_n \text{ is a noise process})$$

- More generally we get **state-space models** of the form

$$\mathbf{x}_n = \mathbf{f}(\mathbf{x}_{n-1}) + \mathbf{q}_n,$$

$$\mathbf{y}_n = \mathbf{g}(\mathbf{x}_n) + \mathbf{r}_n.$$

- Can be coped with **Kalman filters, and extended/unscented Kalman filters**.

# Technical Contents of the Course

- Formulation of sensor fusion as a least squares problem.
- Solution methods for linear least squares problems.
- Solution methods for non-linear least squares problems.
- Solution methods for dynamic least squares (state-estimation) problems.
- Implementation of the methodology to robot platform.

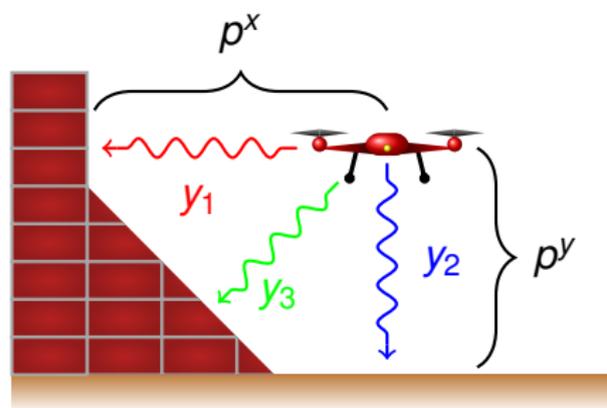


# Summary (1)

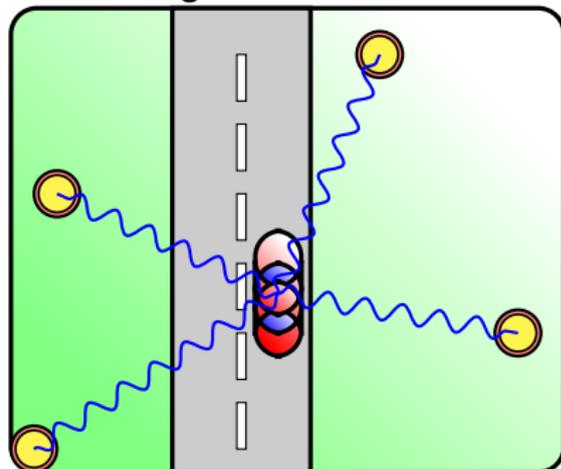
- Lectures are on Tuesdays in 12:15-14:00
- Exercises on Fridays in 12:15-14:00
- Teaching materials are lecture notes and slides on MyCourses.
- Project work starts later and it is about sensor fusion in a mobile robot.
- There are two mid-term Exams.
- The grade is determined by exams, homeworks, and project work.
- Sensor fusion is methodology for intelligent processing of measurements from multiple sensors.
- In practice, linear/non-linear least squares methods and Kalman filtering methods.

## Summary (2)

Typical models that we saw are the following:



$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{b} + \mathbf{r}$$



$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r}$$

# Presemo Questionnaire

<http://presemo.aalto.fi/fusion>