Lecture 0
Introduction and overview

- prerequisites
- some general features of convex optimization
- example
- duality example
- what we will/won't do
- how many problems are convex?

Prerequisites

- good knowledge of linear algebra
- elementary probability
- exposure to engineering (mechanical, electrical, civil, . . .)
- elementary analysis (norms, limits, . . .)
- knowledge of Matlab, or willingness to learn

Not required but helps

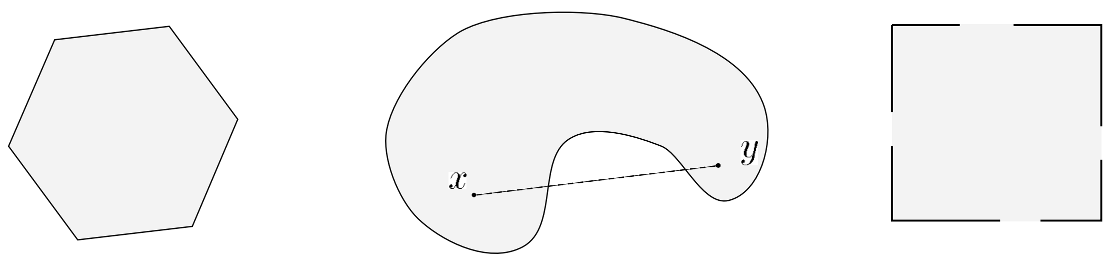
- exposure to optimization
- numerical linear algebra

Convex set

$C \subseteq \mathbf{R}^n$ is convex if

$$x, y \in C, \quad \lambda \in [0,1] \Rightarrow \lambda x + (1-\lambda)y \in C$$

convex not convex

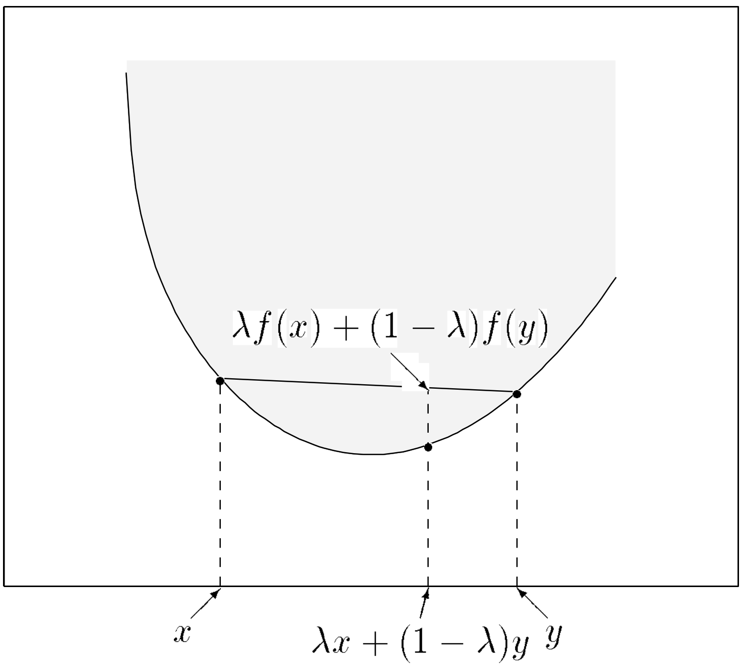


(more later!)

Convex function

$f : \mathbf{R}^n \to \mathbf{R}$ is convex if

$$x, y \in \mathbf{R}^n, \quad \lambda \in [0, 1]$$
$$\Downarrow$$
$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda) f(y)$$



(much more later!)

Convex optimization problem

$$\begin{array}{ll}\text{minimize} & f(x)\\ \text{subject to} & x \in C\end{array}$$

f convex, C convex

convex optimization problems

- can be solved numerically with great efficiency
- have extensive, useful theory
- occur often in engineering problems
- often go unrecognized

tractable in theory and practice:

there exist algorithms s.t.

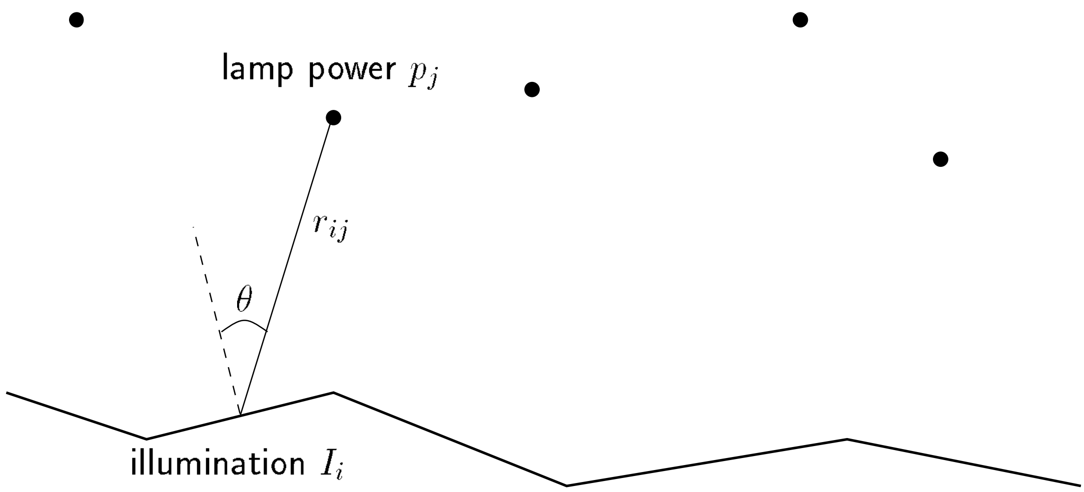
- computation time small, grows gracefully with problem size
- global solutions attained
- non heuristic stopping criteria; provable lower bounds
- handle nondifferentiable as well as smooth problems

duality theory:

- n.a.s.c. for global optimality
- certificates that **prove** infeasibility or lower bounds on objective
- sensitivity analysis w.r.t. changes in f, C

Example

m lamps illuminating n (small, flat) patches



$$I_i = \sum_{j=1}^{m} a_{ij} p_j, \qquad a_{ij} = r_{ij}^{-2} \max\{\cos\theta_{ij}, 0\}$$

lamp power limits: $0 \leq p_j \leq p_{\mathsf{max}}$

problem:

$$\underset{p_j}{\text{minimize}} \quad \max_{i=1,\ldots,n} \quad |\log I_i - \log I_{\mathsf{des}}|$$

How to solve?

1. uniform power: $p_i = p$, vary p
   could try heuristic adjustment of powers
2. least squares: minimize $\sum_j (I_j - I_{\mathrm{des}})^2$
   (closed form, widely available, reliable software, fast)
   what if $p_i \geq p_{\mathrm{max}}$ or $p_i \leq 0$?
   could 'saturate' or add weights:
   $$\text{minimize} \sum_j (I_j - I_{\mathrm{des}})^2 + \sum_i w_i (p_i - p_{\mathrm{max}}/2)^2$$
3. linear programming

. . . of course these are approximate 'solutions'

in fact this problem can be formulated as a convex optimization problem, hence is readily solved
exact solution obtained with effort ≈ modest factor times least squares effort

Two additional constraints

1. no more than half total power is in any 10 lamps
2. no more than half of the lamps are on

Does adding (1) or (2) complicate the problem?

With (1), still easy to solve
With (2), **extremely difficult** to solve

moral:
without the proper background (*i.e.*, this course)
very easy problems can appear quite similar to
very difficult problems.
(Untrained) intuition doesn't always work.

What we will cover

- recognizing & exploiting convexity in engineering context
- ideas of convex optimization
- a few algorithms (*less is more*, Le Corbusier) extremal on the run time/code time tradeoff curve

What we won't do

- details of convex **analysis**
- details of optimization **theory** (regularity conditions, constraint qualifications, ...)
- encyclopedia of algorithms

What fraction of ‘real’ problems are convex?

- by no means all
- many more than are recognized
- convex optimization plays important role in nonconvex optimization (more later)

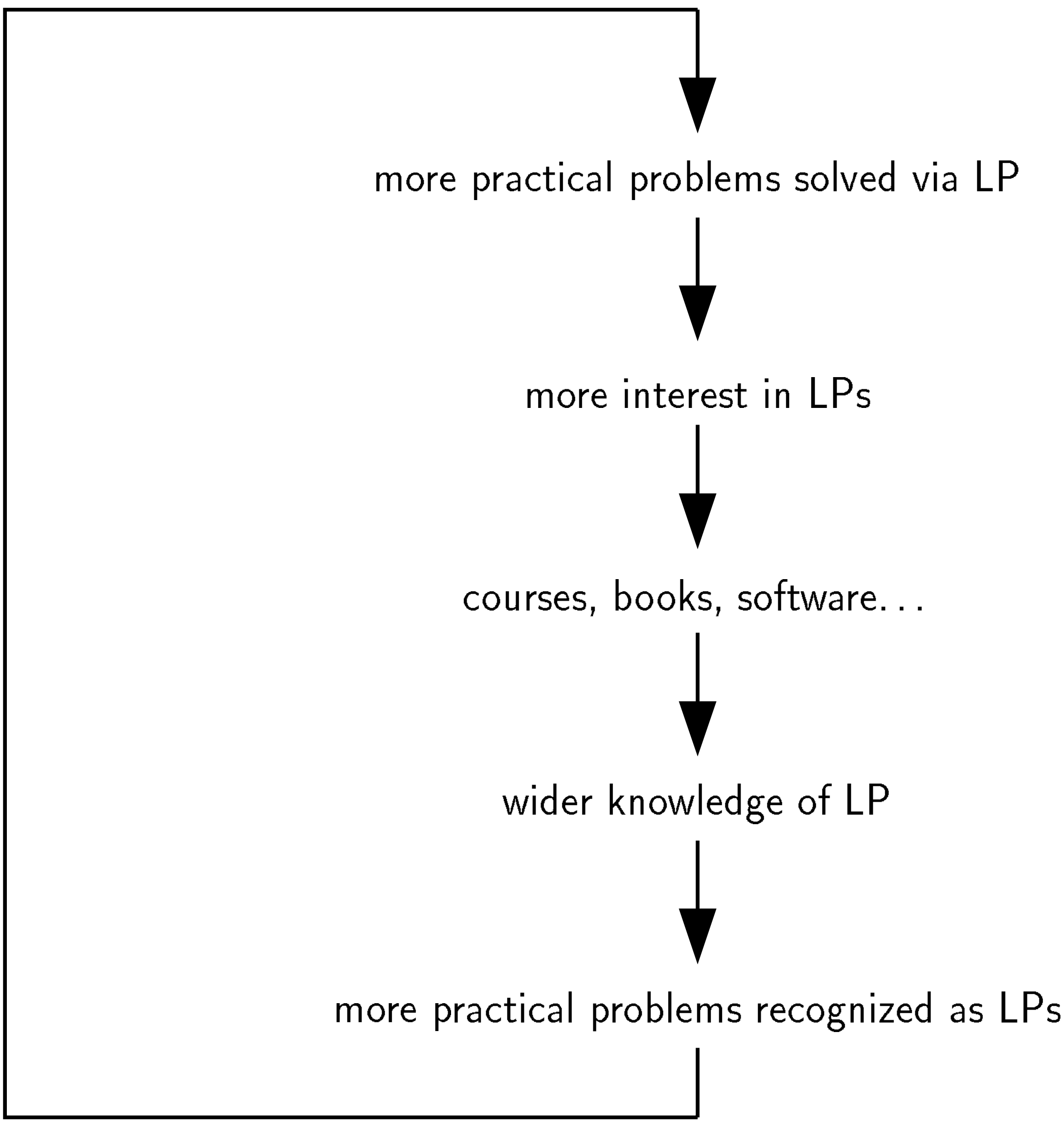
Analog: linear programs

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & a_i^T x \leq b_i, \; i = 1, \ldots, m \end{array}$$

- no “closed form” solution
- very large LPs solved very quickly in practice
- extensive, useful theory

How many problems are LPs?

1940s: “the real world is nonlinear, hence LP silly”
a few examples known (in planning)
many examples found **after** LP became widely known…



(we guess) same story for convex optimization convex optimization

- handles* some problems very well
- can say a lot about it

(simulated annealing, genetic algorithms, neural networks, . . .)

- handle† many problems
- can say very little about it

* means a lot — global solutions, always works, worst case computation time, etc.

† means much less — local solutions (sometimes), no complexity theory, etc.