

1. a) According to the first law, energy is conserved. The second law states that entropy increases when the system approaches thermodynamic equilibrium and  $dS = dQ/T$ . Use these to show that when two isolated bodies at different temperatures are thermally connected heat flows from the hotter to the colder body.
  - b) According to the third law,  $S = 0$  at  $T = 0$ . Using this and the second law show that the heat capacity of any system is zero at  $T = 0$ .
2. Consider two identical systems, one at temperature  $T = T_0$  and the other at negative temperature  $T = -T_0$ . The two systems are brought in thermal contact. What is the final temperature of the combined system?
3. Calculate how much liquid helium is needed to cool a 1 Kg copper object from room temperature to 4.2 K if the object is cooled by
  - (a) submerging it directly into a helium bath when only the latent heat is used
  - (b) cooling slowly in helium gas when the enthalpy of the gas is also used
  - (c) precooling the object first in liquid nitrogen ( $T = 77$  K) and then performing a) or b).
4. The filling line of an experimental cell (1 mK) containing  $^3\text{He}$  is thermally anchored to the mixing chamber of a dilution refrigerator (10 mK). The outer diameter of this CuNi tube is 0.3 mm, inner diameter 0.1 mm and the length 0.2 m. How large is the heat leak from 10 mK to the cell?  
 Thermal conductivities:  
 $^3\text{He}$  Fermi liquid:  $\kappa = \frac{0.36 \cdot 10^{-3}}{T}$  W/m, when  $T < 30$  mK  
 CuNi:  $\kappa = 0.07T$  W/(K<sup>2</sup>m)
5. Consider a cylindrical rod of length  $L$  with heat capacity  $C$  and thermal conductivity  $\kappa$ . Initially the rod is at temperature  $T_1$ . Now, at one end the temperature rises to  $T_2$  and stays there. How does the temperature of the other end behave?
6. Molecular hydrogen  $\text{H}_2$  has rotational states

$$E_R = \frac{\hbar^2}{2\Theta} J(J+1),$$

where  $\Theta = 4.59 \cdot 10^{-48} \text{ Kg m}^2$  and  $J = 0, 1, 2, \dots$ . Hydrogen nuclei have a nuclear spin  $I = 1/2$ . In a hydrogen molecule the spins can be either parallel (total spin  $I = 1$ ) or antiparallel ( $I = 0$ ). Due to symmetry reasons  $I = 1$  requires that  $J$  is odd (ortho-hydrogen). Calculate the ratio of para- and ortho-hydrogen at room temperature ( $T = 293 \text{ K}$ ) and at the boiling point of nitrogen ( $T = 77 \text{ K}$ ).