Aalto University
School of Science

MS-E2135
Decision Analysis Lecture 2

- *Biases in probability assessment*
- *Expected Utility Theory (EUT)*
- *Assessment of utility functions*

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Last week

- Decision trees provide a visual and structured way to modelling sequential decision-making problems which involve uncertainties
  - *Paths of decisions and random events*
- Probabilities are employed to model uncertainties
  - *Subjective probabilities can be employed even in the absence of data*
- The elicitation of probabilities may involve subjective judgements

Different kinds of uncertainties

- We frequently make statements about uncertainty
  - "We will have a white Christmas." ↔ subjective probability
  - "The 100 000th decimal of π is 6." ↔ a fact, the uncertainty lies in the available information
  - "I win in a lottery with probability 0,00005." ↔ frequentist or classical probability interpretation

- Uncertainties are associated with events with unknown outcome
- Probabilities provide a quantitative measure of this uncertainty

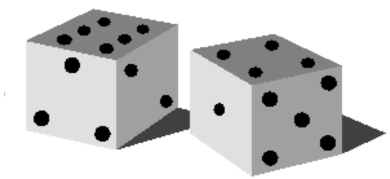
Classical interpretation

- Jacob Bernoulli (1685), Pierre-Simon Laplace (1814)
- Probability = The **ratio** between (i) the number of possible outcomes defining the event and (ii) the total number of possible outcomes which are assumed to be equally likely

$$\boxed{P(A) = \frac{\#(A)}{\#(S)}}$$

$\#(A)$ = Number of possible outcomes favourable to A

$\#(S)$ = Total number of possible outcomes



- Circular definition: Probability defined in terms of "equally likely"
- Principle of indifference:
  - *Each event is defined as a collection of outcomes*
  - *Events are "equally likely" if there is no known reason for predicting the occurrence of one event rather than another*
  - *The probability to get "6" when tossing a dice is 1/6*

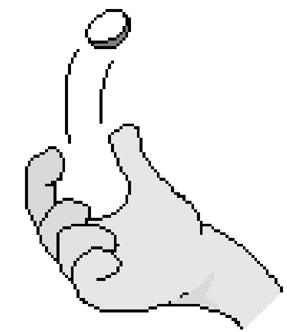
Frequentist interpretation

- Leslie Ellis, mid 19th century
- Probability = The **relative frequency** of trials in which the favourable event occurs as the number of trials approaches infinity

$$\boxed{P(A) = \lim_{n \to \infty} \frac{n(A)}{n}}$$

$n(A)$ = Number of times that A occurs

n = Total number of trials



- You may determine the probability of getting "heads" by tossing a coin (which may not be fair) a very large number of times
- Yet in many cases repeated trials cannot be carried out
  - *E.g., will there be a recession if the interest rates are raised by 1 %?*

Subjective (Bayesian) interpretation

- Bruno De Finetti (1937)

- Probability = An individual's **degree of belief** in the occurrence of a particular outcome
  - *The probability may change e.g. when additional information is received*
  - *The event may have already occurred*

- Examples
  - *"I believe there's a 40 % chance that we will have a white Christmas"*
  - *"I'm 15 % sure that Martin Luther King was 34 years when he died"*

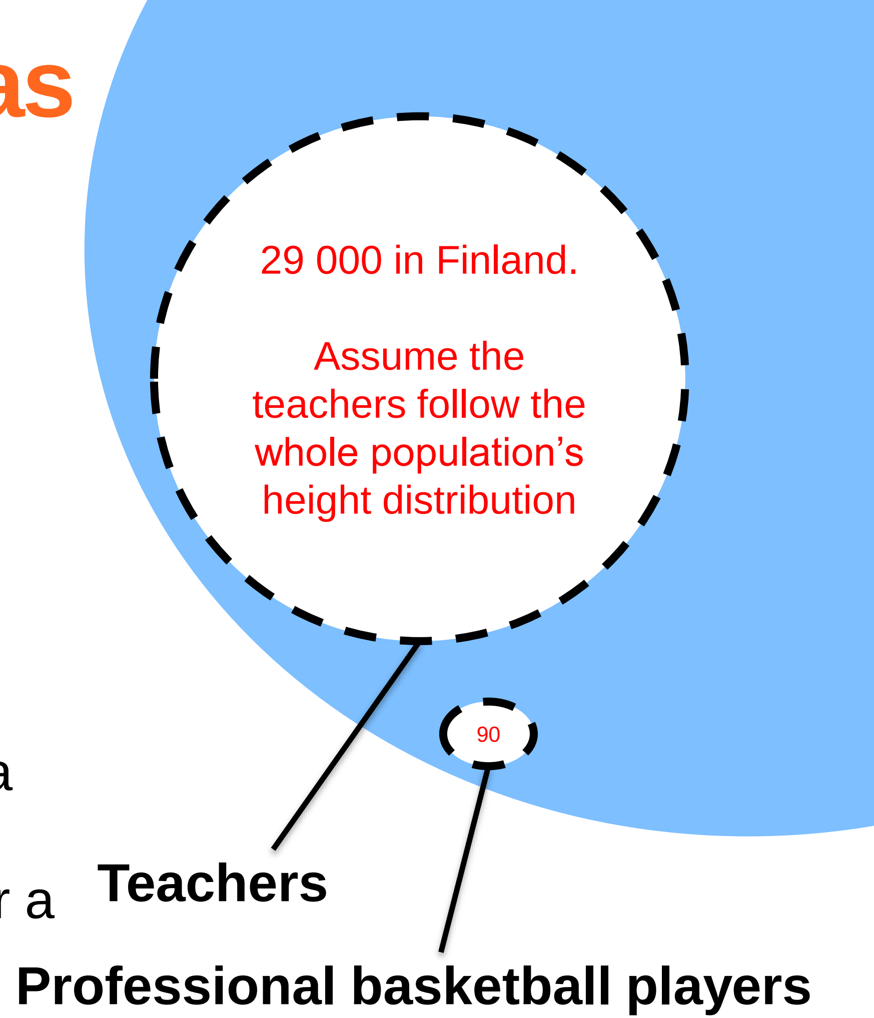
Biases in probability assessment

- Subjective judgements by “ordinary people” and “experts” alike are prone to different kinds of biases
  - *Cognitive bias: Systematic discrepancy between the ‘correct’ answer and the respondent’s actual answer*
    - E.g., assessments of conditional probabilities differ from the correct value given by the Bayes’ rule
  - *Motivational biases: judgements are influenced by the desirability or undesirability of events, e.g.*
    - Overoptimism about success probabilities
    - Strategic underestimation of failure probabilities

- Some biases can be difficult to correct

Representativeness bias (cognitive)

- If x fits the description of A well, then Prob(x∈A) is assumed to be large
- The ‘base rate’ of A in the population (i.e., the probability of A) is not taken into account
- Example: You see a very tall man in a bar. Is he more likely to be a professional basketball (BB) player or a teacher?

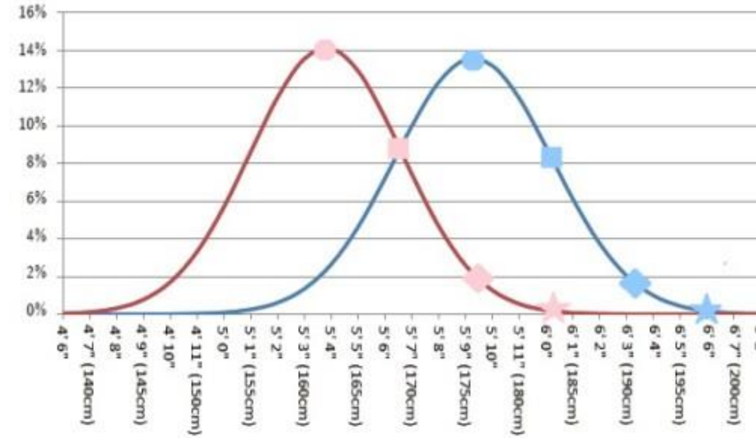


Representativeness bias

- What is meant by ‘very tall’?
  - 195 cm?
  - Assume all BB players are very tall

- The share of Finnish men taller than 195 cm is about 0.3%

- If BB players go to the bar as often as teachers, **it is more probable that the very tall man is a teacher, if the share of very tall men exceeds 0.31%**
  - Your **responses: 87,5% teacher, 12,5% basketball player**



Height	Males					
	20–29 years	30–39 years	40–49 years	50–59 years	60–69 years	70–79 years
Percent under—						
4'10"	–	–	–	(B)	–	–
4'11"	–	–	–	(B)	(B)	–
5'	(B)	–	–	(B)	(B)	–
5'1"	(B)	(B)	(B)	(B)	[1] 0.4	(B)
5'2"	(B)	(B)	(B)	(B)	(B)	(B)
5'3"	(B)	[1] 3.1	[1] 1.9	(B)	[1] 2.3	(B)
5'4"	3.7	[1] 4.4	3.8	[1] 4.3	4.4	5.8
5'5"	7.2	6.7	5.6	7.6	7.8	12.8
5'6"	11.6	13.1	9.8	12.2	14.7	23.0
5'7"	20.6	19.6	19.4	18.6	23.7	35.1
5'8"	33.1	32.2	30.3	30.3	37.7	47.7
5'9"	42.2	45.4	40.4	41.2	50.2	60.3
5'10"	58.6	58.1	54.4	54.3	65.2	75.2
5'11"	70.7	69.4	69.6	70.0	75.0	85.8
6'	79.9	78.5	79.1	81.2	84.3	91.0
6'1"	89.0	89.0	87.4	91.6	93.6	94.9
6'2"	94.1	94.0	92.5	93.7	97.8	98.6
6'3"	98.3	95.8	97.7	96.6	99.9	100.0
6'4"	100.0	97.6	99.0	99.5	100.0	100.0
6'5"	100.0	99.4	99.4	99.6	100.0	100.0
6'6"	100.0	99.5	99.9	100.0	100.0	100.0

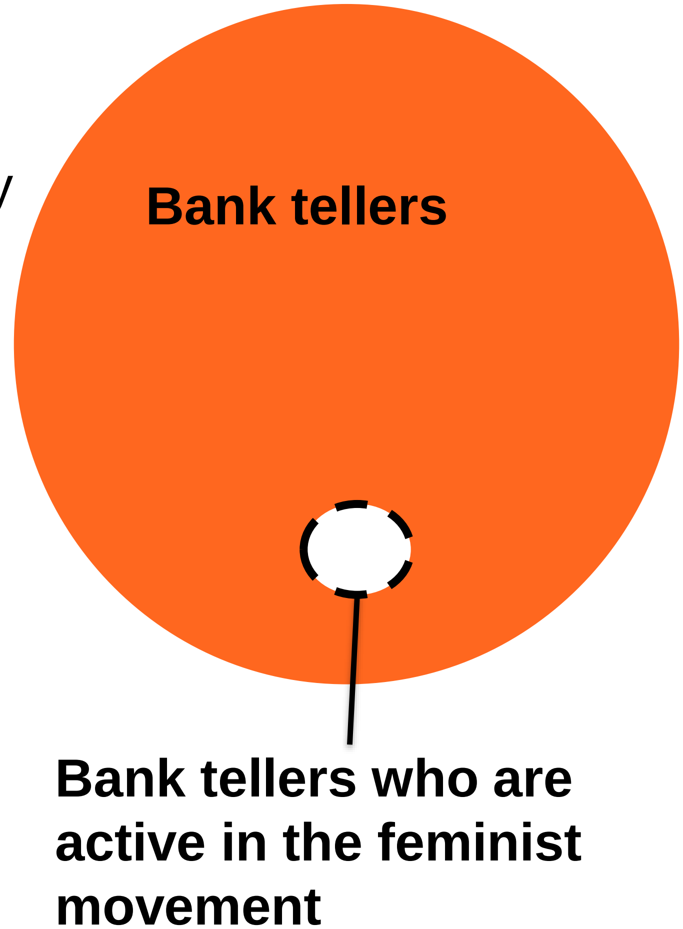
Representativeness bias

- Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

  Please check the most likely alternative:

  a. *Linda is a bank teller.*
  b. *Linda is a bank teller and active in the feminist movement.*

- Many choose b, although b⊂a whereby P(b)<P(a)
  - **Your responses:** 70% a, 30% b.



Conservativism bias (cognitive)

- After obtaining some information about an uncertain event, people typically do not adjust their initial probability estimate about this event as much as they should based on Bayes' theorem.

- Example: Consider two bags X and Y. Bag X contains 30 white balls and 10 black balls, whereas bag Y contains 30 black balls and 10 white balls. Suppose that you select one of these bags at random, and randomly draw five balls one-by-one by replacing them in the bag after each draw. Suppose you get four white balls and one black. What is the probability that you selected bag X with mainly white balls?

- Typically people answer something between 70-80%. Yet, the correct probability is 27/28 ≈ 96%.
- **Your responses**: mean response 55%. Many (20%) answered 50%.

Representativeness and conservativism bias - debiasing

- Pay attention to the logic of joint and conditional probabilities and Bayes' rule

- Split the task into an assessment of
  - The base rates for the event (i.e., prior probability)
    - *E.g., what are the <u>relative shares</u> of teachers and pro basketball players?*
  - The likelihood of the data, given the event (i.e., conditional probabilities)
    - *E.g., what is the relative share of people active in the feminist movement? Is this share roughly the same among bank tellers as it is among the general population or higher/lower?*
    - *What is the likelihood that a male teacher is taller than 195cm? How about a pro basketball player?*

Availability bias (cognitive)

- People assess the probability of an event by the ease with which instances or occurences of this event can be brought to mind.

- Example: In a typical sample of English text, is it more likely that a word starts with the letter K or that K is the third letter?
  - *Most (nowadays only many?) people think that words beginning with K are more likely, because it is easier to think of words that begin with "K” than words with "K" as the third letter*
  - *Yet, there are twice as many words with K as the third letter*
  - **Your responses:** 35% first letter, 65% third letter.

- Other examples:
  - *Due to media sensationalist reporting in the US, the number of violent crimes such as child murders seems to have increased*
  - *Yet, compared to 2000's, 18 times as many children were killed per capita in 1950's and twice as many in 1990's*
  - *Probabilities of flight accidents after the volcanic eruption in Iceland in 2011*

Availability bias - debiasing

- Conduct probability training
- Provide concrete counterexamples
- Provide statistics

- Still, based on empirical experimental studies, availability bias is **difficult to correct**

Anchoring bias (cognitive)

- When assessing probabilities, respondents may be guided by reference assessments

- Often, the respondent is *anchored* to the reference assessment

- Example: Is the percentage of African countries in the UN
  A. Greater or less than 65? What is the exact percentage?
     - *'Average' answer: Less, 45%.*
     - ***Your responses**: Less, median 39%, mean 39%.*
  B. Greater or less than 10? What is the exact percentage?
     - *'Average' answer: Greater, mean 25%.*

The correct share is 54/193 = **28%** (Wikipedia)

Anchoring bias - debiasing

- ❑ Avoid providing anchors
  - *But there are contexts where <u>deliberate</u> attempts to influence answers are made (e.g., marketing)*
- ❑ Provide multiple and counteranchors
  - *If you have to provide an anchor, provide several which differ significantly from each other*
- ❑ Use different experts who use different anchors
- ❑ Based on empirical evidence, anchoring bias is **difficult to correct**

Hindsight bias

- People falsely believe they could have predicted the outcome of an event
  - *Once the outcome has been observed, the DM may assume that they are the only ones that could have happened and underestimate the uncertainty*
- Undermines possibilities for learning from the past
- Alerting people to this bias has little effect

- **How to mitigate**:
  - *Argue against the inevitability of the reported outcome*
  - *Develop alternative descriptions of how the future might have unfolded <u>differently</u>*

Desirability / undesirability of events (motivational)

- People tend to believe that there is a less than 50 % probability that **negative outcomes** will occur compared with peers
  - *"I am less likely to develop long-term symptoms even if I catch COVID-19"*

- People tend to believe that there is a greater than 50 % probability that **positive outcomes** will occur compared with peers
  - *"I am more likely to become a homeowner / have a starting salary of more than 4,500€"*
  - **Earlier responses** on owning a home: **40%** (20%) more likely, **12%** (12%) less likely, **48%** (68%) equally likely
  - **Earlier responses** on salary: **23%** (20 %) more likely, **10%** (10%) less likely, **67%** (71%) equally likely

- People tend to underestimate the probability of negative outcomes and overestimate the probability of positive outcomes
  - The estimates are not conservative – the actual risks are higher than estimated

Desirability / undesirability of events - debiasing

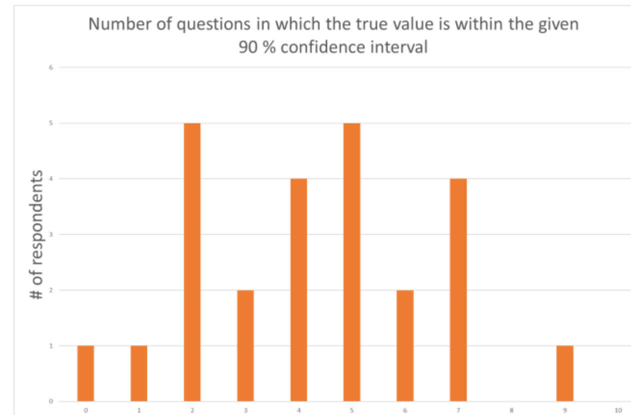
- Use multiple experts with alternative points of view
- Place hypothetical bets against the desired event
  - *“Make the respondent think about monetary consequences”*
- Use decomposition and realistic assessment of partial probabilities
  - *“Split the events”*
- Yet, empirical evidence suggests that **motivational biases are often difficult to correct**

Further reading: **Montibeller, G., and D. von Winterfeldt**, 2015. Cognitive and Motivational Biases in Decision and Risk Analysis, *Risk Analysis*

Overconfidence (cognitive)

- People tend to assign overly narrow confidence intervals to their probability estimates
  1. Martin Luther King's age at death **39 years**
  2. Length of the Nile River **6738 km**
  3. Number of Countries that are members of OPEC **13**
  4. Number of Books in the Old Testament **39**
  5. Diameter of the moon **3476 km**
  6. Weight of an empty Boeing 747 **176900 kg**
  7. Year of Wolfgang Amadeus Mozart's birth **1756**
  8. Gestation period of an Asian elephant **21.5 months**
  9. Air distance from London to Tokyo **9590 km**
  10. Depth of the deepest known point in the oceans **11033 m**

Responses by 25 subjects:



- There are 10 questions with 90% confidence intervals
  - *If the intervals are correct, each answer is within the confidence interval with probability 0.9*
  - *The probability that* n *estimates are within the intervals is* $\binom{10}{n}0.9^n 0.1^{10-n}$
  - *If the intervals are correct, the probability that at least 3 responses lie outside the intervals is* $\sum_{n=3}^{10}\binom{10}{n}0.1^n 0.9^{10-n}$ $\approx 7\%$ ➔ *The null hypothesis of not being overconfident can be rejected (at the 5 % confidence level)*

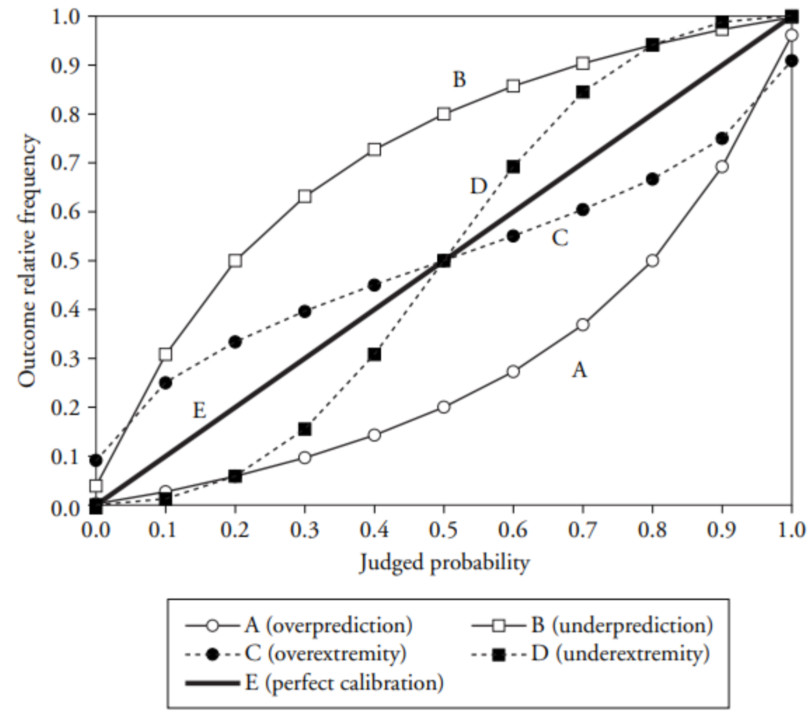
Overconfidence - debiasing

- Provide probability training
- Start with extreme estimates (low and high)
- Use fixed values instead of fixed probability in elicitations:
  - Do not ask: “What is the GDP growth rate x such that the probability of achieving this rate x or less x is 5 %”
  - Instead ask : “With what probability will the GDP growth rate be lower than -3%?”

- Based on empirical evidence, overconfidence is **difficult to correct**

Calibration curves

- ❑ People tend to assess probabilities best when they have frequent and concrete feedback
  - E.g., US weather forecasters
- ❑ Judged probabilities on *x*-axis
- ❑ Observed frequencies on *y*-axis

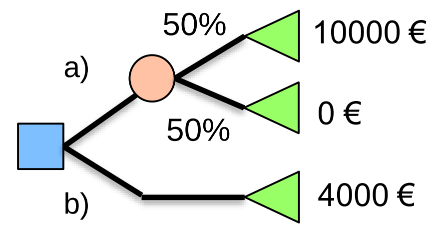


- ❑ Can be used for calibration
  - Instead of the judged probability, use the corresponding observed frequency
  - E.g., in the C case, the actual tail probabilities are more extreme than the judged ones

Risky or not (so) risky?

https://presemo.aalto.fi/riskattitude1/

- Which one would you choose:
  - *a) Participate in a lottery in which there is a 50 % chance of getting nothing and a 50 % chance of getting 10000 €*
  - *b) Getting 4000 € for sure*



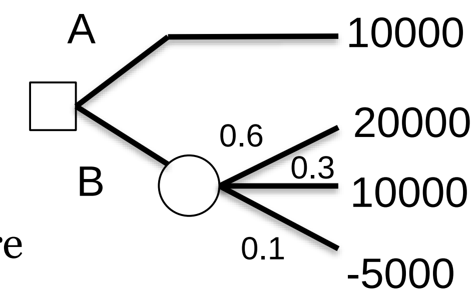
- Many choose the certain outcome of 4000 €, although the expected monetary value in alterantive a) is higher

Option b) involves less risk

How to compare risky alternatives?

❑ Last week

- We used decision trees to support decision-making under uncertainty assuming that the DM seeks to **maximize expected monetary value**
- This is valid if the DM is **risk neutral**, i.e., **indifferent** between
  - *obtaining x for sure and*
  - *a gamble with uncertain payoff Y such that x=E[Y]*
- Many DMs are **risk averse** = they **prefer** obtaining x for sure to a gamble with payoff Y such that x=E[Y]



Expectation = 14500

❑ Next

- We accommodate the DM's risk attitude (=preference over alternatives with uncertain outcomes) in decision models

Expected utility theory (EUT)

- John von Neumann and Oscar Morgenstern, *Theory of Games and Economic Behavior*, 1944
  - Axioms of rationality for preferences over alternatives with uncertain outcomes
  - If the DM follows these axioms, she should prefer the alternative with the highest expected utility

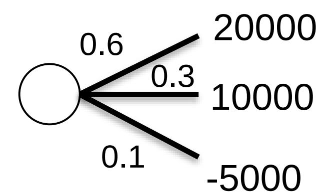
- Elements of EUT
  - Set of outcomes and “lotteries”
  - Preference relation over lotteries which satisfies four axioms
  - Representation of preference relation with expected utility

EUT: Sets of outcomes and lotteries

- Set of possible outcomes *T*:
  - E.g., revenue T euros / demand T
- Set of all possible lotteries *L*:
  - A lottery $f \in L$ associates a probability $f(t) \in [0,1]$ with each possible outcome $t \in T$
    - *Finite number of outcomes with a positive probability* $f(t) > 0$
    - *Probabilities add up to one* $\sum_t f(t) = 1$
    - *Lotteries are discrete probability mass functions (PMFs) / decision trees with a single chance node*
- Deterministic outcomes are modeled as degenerate lotteries

Lottery

Decision tree

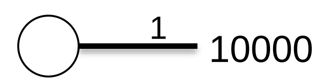


Probability mass function (PMF)

$$f(t) = \begin{cases} 0.6, t = 20000 \\ 0.3, t = 10000 \\ 0.1, t = -5000 \\ 0, elsewhere \end{cases}$$

Degenerate lottery

Decision tree



PDF

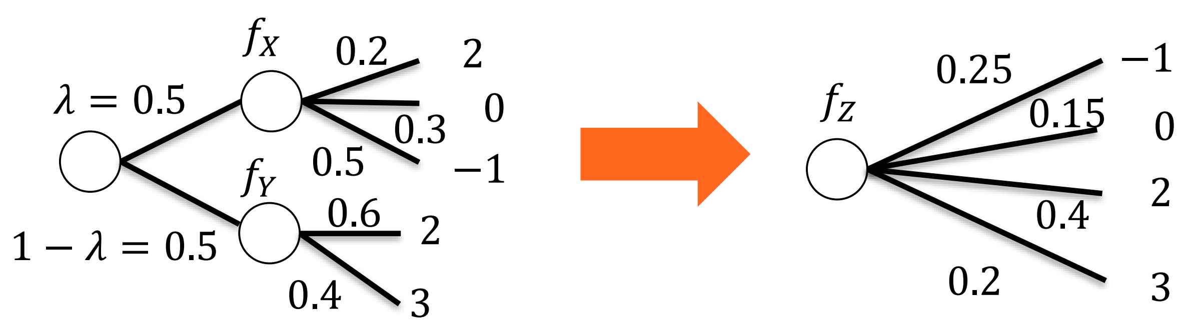
$$f(t) = \begin{cases} 1, t = 10000 \\ 0, elsewhere \end{cases}$$

EUT: Compound lotteries

- Compound lottery:
  - Get lottery $f_X \in L$ with probability λ
  - Get lottery $f_Y \in L$ with probability $1-\lambda$
- Compound lottery can be modeled as lottery $f_Z \in L$:

$$f_Z(t) = \lambda f_X(t) + (1-\lambda) f_Y(t) \;\; \forall t \in T \simeq f_Z = \lambda f_X + (1-\lambda) f_Y$$

- Example:
  - You have a 50-50 chance of getting a ticket to lottery $f_X \in L$ or to lottery $f_Y \in L$



Preference relation

- Let $\succcurlyeq$ be preference relation among lotteries in L
  - Preference $f_X \succcurlyeq f_Y$: f_X is at least as preferred as f_Y
  - Strict preference $f_X \succ f_Y$ defined as $\neg(f_Y \succcurlyeq f_X)$
  - Indifference $f_X \sim f_Y$ defined as $f_X \succcurlyeq f_Y \wedge f_Y \succcurlyeq f_X$

EUT axioms A1-A4 for the relation $\succcurlyeq$

- **A1:** $\succcurlyeq$ is complete
  - For any $f_X, f_Y \in L$, either $f_X \succcurlyeq f_Y$ or $f_Y \succcurlyeq f_X$ or both
- **A2:** $\succcurlyeq$ is transitive
  - If $f_X \succcurlyeq f_Y$ and $f_Y \succcurlyeq f_Z$, then $f_X \succcurlyeq f_Z$
- **A3:** Archimedean axiom
  - If $f_X \succ f_Y \succ f_Z$, then $\exists \lambda, \mu \in (0,1)$ such that

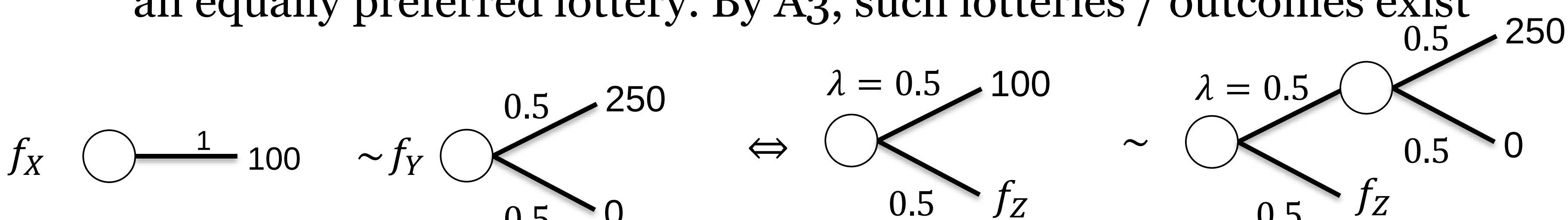
$$\lambda f_X + (1-\lambda) f_Z \succ f_Y \text{ and } f_Y \succ \mu f_X + (1-\mu) f_Z$$

- **A4:** Independence axiom
  - Let $\lambda \in (0,1)$. Then,

$$f_X \succ f_Y \Leftrightarrow \lambda f_X + (1-\lambda) f_Z \succ \lambda f_Y + (1-\lambda) f_Z$$

Equivalent formulations of A3 and A4

- **A3:** Archimedean axiom
  - If $f_X \succ f_Y \succ f_Z$, there then exists $p \in (0,1)$ such that $f_Y \sim p f_X + (1-p) f_Z$
- **A4:** Independence axiom
  - $f_X \sim f_Y \Leftrightarrow \lambda f_X + (1-\lambda) f_Z \sim \lambda f_Y + (1-\lambda) f_Z$
  - Any lottery (or outcome = a degenerate lottery) can be replaced by an equally preferred lottery. By A3, such lotteries / outcomes exist



  - NOTE: f_Z can be any lottery, it can have several possible outcomes

Main representation theorem for expected utility

- $\succcurlyeq$ satisfies axioms A1-A4 **if and only if** there exists a real-valued utility function *u(t)* over the set of outcomes *T* such that

$$f_X \succcurlyeq f_Y \Leftrightarrow \sum_{t \in T} f_X(t)u(t) \geq \sum_{t \in T} f_Y(t)u(t)$$

- Implication: **a rational DM** following axioms A1-A4 **selects the alternative with the highest expected utility**

$$E[u(X)] = \sum_{t \in T} f_X(t)u(t)$$

  – A similar result can be obtained for continuous distributions:
    - $f_X \succcurlyeq f_Y \Leftrightarrow E[u(X)] \geq E[u(Y)], where\ E[u(X)] = \int f_X(t)u(t)dt$

Computing expected utility

- Example: Joe's utility function for the number of apples is u(1)=2, u(2)=5, u(3)=7. Which alternative would he prefer?
  - X: Two apples for certain
  - Y: A 50-50 gamble between 1 and 3 apples

$$E[u(X)] = u(2) = 5$$

$$E[u(Y)] = 0.5u(1) + 0.5u(3)$$
$$= 0.5 \cdot 2 + 0.5 \cdot 7 = 4.5$$

- Example: Jane's utility function for money is $u(t) = t^2$. Which alternative would she prefer?
  - X: 50-50 gamble between 3M€ and 5M€
  - Y: A random amount of money from the uniform distribution over the interval [3,5]
  - What if her utility function was $u(t) = \frac{t^2-9}{25-9}$?

$$E[u(X)] = 0.5u(3) + 0.5u(5)$$
$$= 0.5 \cdot 9 + 0.5 \cdot 25 = 17$$

$$E[u(Y)] = \int_3^5 f_Y(t)u(t)dt = \int_3^5 \frac{1}{2}t^2dt$$
$$= \frac{1}{6}5^3 - \frac{1}{6}3^3 = 16.33333$$

Uniqueness up to positive affine transformations

- Let $f_X \succcurlyeq f_Y \Leftrightarrow E[u(X)] \geq E[u(Y)]$. Then $E[\alpha u(X) + \beta] = \alpha E[u(X)] + \beta \geq \alpha E[u(Y)] + \beta = E[\alpha u(Y) + \beta]$ for any $\alpha > 0$ and arbitrary β
- **Two utility functions $\boldsymbol{u_1(t)}$ and $\boldsymbol{u_2(t) = \alpha u_1(t) + \beta, (\alpha > 0)}$ establish the same preference order over lotteries**

$$E[u_2(X)] = E[\alpha u_1(X) + \beta] = \alpha E[u_1(X)] + \beta.$$

- Implications
  - Any linear utility function $u_L(t) = \alpha t + \beta, (\alpha > 0)$ that is a positive affine transformation of the identity function $u_1(t) = t \Rightarrow u_L(t)$ establishes the same preference order as the expected value
  - Utilities for two outcomes can be chosen freely:
    - *E.g., if utilities are represented by u_1, the normalized utility such that $u_2(t^*) = 1$ and $u_2(t^0) = 0$ can be derived through*

$$u_2(t) = \frac{u_1(t) - u_1(t^0)}{u_1(t^*) - u_1(t^0)} = \underbrace{\frac{1}{u_1(t^*) - u_1(t^0)}}_{=\alpha>0} u_1(t) - \underbrace{\frac{u_1(t^0)}{u_1(t^*) - u_1(t^0)}}_{=\beta}$$

Let's practice!

https://presemo.aalto.fi/drcuckoo

The utility function of Dr. Cuckoo is $u(t) = \sqrt{t}$. Would he

a) Participate in a lottery A with 50-50 chance of getting either 0 or 400 €?

b) Participate in a lottery B in which the probability of getting 900 € is 30% and getting 0 € is 70%?

$u(0) = 0$, $u(400) = 20$, $u(900) = 30$

a) $E[u(A)] = 0.5 \cdot 0 + 0.5 \cdot 20 = 10$

b) $E[u(B)] = 0.7 \cdot 0 + 0.3 \cdot 30 = 9$

NOTE! The expectation of lottery A = 200 € **is smaller than that of B** = 270€

Reference lottery revisited

- Assume that an expected utility maximizer with utility function *u* uses a reference lottery to assess the probability of event A

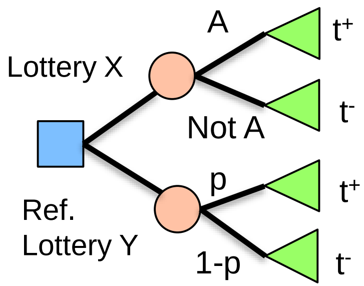
- She thus adjusts *p* such that she is indifferent between lottery X and the reference lottery Y

$$E[u(X)] = E[u(Y)]$$

$$\Leftrightarrow P(A)u(t^+) + \big(1 - P(A)\big)u(t^-) = pu(t^+) + (1-p)u(t^-)$$

$$\Leftrightarrow P(A)\big(u(t^+) - u(t^-)\big) = p\big(u(t^+) - u(t^-)\big)$$

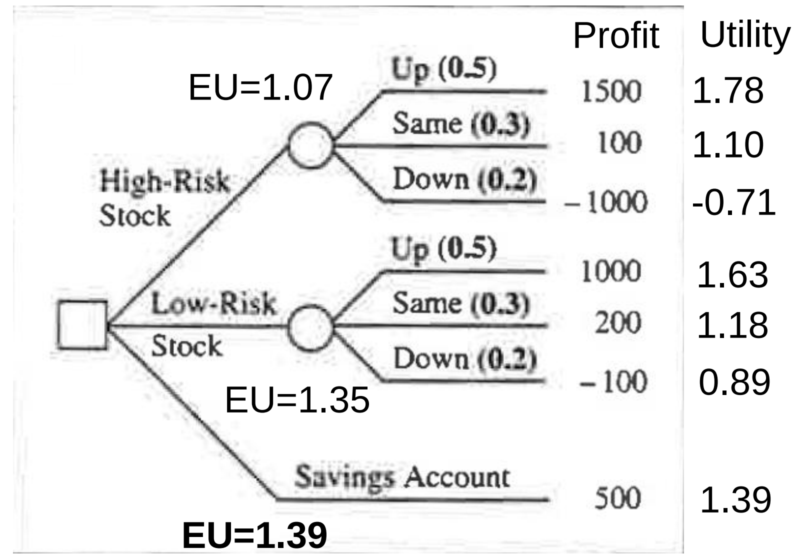
$$\Leftrightarrow P(A) = p$$



- The utility function *u* does not affect the result

Expected utility in decision trees

- Carry out everything as before, except:
  - Chance node: compute the expected <u>utility</u>
  - Decision node: select the alternative corresponding to maximum expected <u>utility</u>
  - Cf. the umbrella example, in which the 'magic numbers' represented preferences



$$u(t) = 2 - e^{\frac{-t}{1000}}$$

Expected utility in Monte Carlo

- Generate a sample $x_1, \dots, x_n$ of realizations from the probability density function
- Comput corresponding utilities for $u(x_i)$ for each x_i
- Mean of the sample utilities $u(x_1), \dots, u(x_n)$ provides an estimate for $E[u(X)]$

=2-EXP(-F12/1000)

C	D	E	F	G	H
		Col.mean	Col.mean	Col.mean	
		0.502964	990.3014	1.580972	
	Sample	u	x	Utility	
	1	0.464077	954.9167	1.615156	
1	2	0.704234	1268.308	1.718693	
	3	0.777865	1382.501	1.74905	
	4	0.534927	1043.831	1.647897	
	5	0.4426	927.8094	1.604581	
	6	0.916252	1690.147	1.815508	
	7	0.649453	1191.922	1.696363	
	8	0.65278	1196.418	1.697725	
	9	0.110887	389.0874	1.322325	
	10	0.189275	559.714	1.428628	
	11	0.902882	1649.073	1.807772	

Summary

- Probability elicitation is prone to cognitive and motivational biases
  - Some cognitive biases can be easy to correct, but…
  - Some other cognitive biases and all motivational biases can be difficult to overcome

- The DM's preferences over alternatives with uncertain outcomes can be described by a utility function

- A rational DM (according to the four axioms of rationality) should choose the alternative with the highest expected utility
  - This is **NOT** necessarily the alternative for which the utility associated with the expected monetary consequences is highest

EUT for normative decision support

- EUT is a normative theory: if the DM is rational (as defined by the axioms), she **should** select the alternative with the highest expected utility
  - Not descriptive or predictive: EUT does not describe or predict how people actually **do** select among alternatives with uncertain outcomes

- The four axioms characterize properties that can be associated with rational decision makers
  - E.g., if the transivity axiom A2 is violated so that $f_X \succ f_Y$, $f_Y \succ f_Z$, $f_Z \succ f_X$, one would be willing to pay in order exchange f_X for f_Z, then f_Z for f_Y and finally f_Y for f_X, thus becoming a "money pump"
  - If these rationality axioms are accepted, then the DM should abide by them

Question 1

https://presemo.aalto.fi/2135lecture2

- Which of the following alternatives would you choose?

1. A sure gain of 1 M€
2. A gamble in which there is a
   - *1% probability of getting nothing,*
   - *89% probability of getting 1M€, and*
   - *10% probability of getting 5M€*

Question 2

https://presemo.aalto.fi/2135lecture2

- A rare disease breaks out in a community, killing as many as 600 people. Which one of the following two programs for addressing the threat would you choose:
  - Program A: 200 people will be saved for sure.
  - Program B: There is a 33% probability that all 600 will be saved and a 67% probability that no one will be saved.

Which program will you choose?

1. Program A
2. Program B

Question 3

https://presemo.aalto.fi/2135lecture2

- Which of the below alternatives would you choose?

1. A lottery in which there is a
   - *89% probability of getting nothing*
   - *11% probability of getting 1M€*

2. A lottery gamble in which there is a
   - *90% probability of getting nothing*
   - *10% probability of getting 5M€*

Question 4

https://presemo.aalto.fi/2135lecture2

- Imagine that a rare disease is breaking out in some community and is expected to kill 600 people. Two different programs are available to deal with the threat.
  - Program C: 400 of the 600 people will die.
  - Program D: There is a 33% probability that nobody will die and a 67% probability that 600 people will die.

  Which program will you choose?

  1. Program C
  2. Program D