## Model Solutions 1

1. As an example, we can think about demand and supply for housing less than 5 kilometers away from Helsinki city center.

During the last 10 years, Helsinki has zoned land for housing in Jätkäsaari and Kalasatama that are both located near the city center. In a demand-supply framework, we can think that construction in these central locations represent a shift in the supply curve for apartments. Assuming that supply of housing close to the city center is driven only by city zoning policies (the availability of land for construction), we can think that supply curve is inelastic to changes in prices. In other words, whatever happens to housing prices, construction is still contained by city's planning decisions and therefore there are no supply responses to price changes. Therefore, we can plot the supply curve as a vertical line.

After the zoning of Kalasatama and Jätkäsaari, there is a shift in the supply curve due to the increase in the availability of lots for housing development. This is represented by a rightward shift in the supply curve.

We know that in 2012 (close to the beginning of construction in these areas), the average price per square meter in central Helsinki was 5,700 euros (Statistics Finland). The plan is that Kalasatama has 25.000 and Jätkäsaari has 21.000 residents when the areas are completed. Suppose that both areas are ready in 2030, the number of people closer than 5 kilometers away from city center increases from 240,000 to 286,000 because of the construction of these two areas (around 20 percent increase in population). Suppose that the number of apartments increases in the same proportion as population, then the number of apartments less than 5 kilometers away from city center increases from 126,000 to 150,000.

We can illustrate the shift in the following graph:


Figure 1: Demand and supply for housing less than 5 kilometers from Helsinki city center.

In the graph, supply shifts from $S$ to $S^{\prime}$ and equilibrium price changes from 5700 euros per square meter to a lower level $p^{\prime}$. In other words, with the usual assumptions related to demand-supply framework (e.g. perfect substitutes), the equilibrium price of housing near city center should go down due to the shift in supply. With everything else equal, a shift in housing supply in these locations shifts the supply curve downwards, which increases the number of people living near Helsinki city center (equilibrium quantity) and decreases the prices of these houses (equilibrium price).

Surely, in 20 years, not only supply shifts but there are shifts in the demand side as well. During this time, for instance, people's preferences has changed and more people prefer to live close to the city center. In addition, the population in Helsinki has increased, shifting the demand curve. These changes shift the demand curve so it is likely that during a 20 years' time, the prices could actually increase. However, one interpretation of this graph is that without this shift in supply, the increase would be even larger than in the absence of these supply changes.
2. In this exercise, all the quantities are in pallets per week and prices in euros per pallet. Consumer and supplier surpluses are measured in (pallets per week) $\times($ euros per pallet $)=$ euros per week.
(a) In the market equilibrium, the price is such that the amount producers are willing to supply equals the quantity the consumers are willing to purchase. That is, the equilibrium price $p^{*}$ solves

$$
\begin{aligned}
Q^{D}\left(p^{*}\right) & =Q^{S}\left(p^{*}\right) \Leftrightarrow \\
100-2 p^{*} & =4 p^{*}-20 \Leftrightarrow \\
p^{*} & =20 .
\end{aligned}
$$

Equilibrium quantity is found by plugging $p^{*}$ into either the demand or the supply curve as these are equal in the equilibrium:

$$
Q^{*}=Q^{D}\left(p^{*}\right)=Q^{S}\left(p^{*}\right)=60 .
$$

In order to calculate the surpluses it's easiest to first invert the supply and the demand curves. In this way we can easily see the difference in costs (marginal costs for producers, price for consumers) and the gains (price for producers, willingness to pay for consumers) for each unit traded in the market. The inverse demand and supply curves are given by

$$
\begin{aligned}
& p^{D}(Q)=50-\frac{Q}{2} \\
& p^{S}(Q)=5+\frac{Q}{4}
\end{aligned}
$$

Producer surplus therefore corresponds the triangular region between the supply curve and equilibrium price, whose area is given by

$$
\begin{aligned}
P S & =\frac{1}{2}(20-5) 60=450 \\
C S & =\frac{1}{2}(50-20) 60=900 .
\end{aligned}
$$


(b) The market supply is the sum of individual suppliers' quantities. Since all the individual suppliers are identical, we can simply multiply the individual supplies by the number of suppliers to obtain the market supply:

$$
\left.Q^{S}(p)=1000 \times((p-5) / 250)\right)=4 p-20
$$

Since the supply is identical to 2 a and the demand is unchanged, so are the equilibrium price and quantity as well surpluses.
(c) The market supply is now given by

$$
Q^{S}(p)=500 \times((p-5) / 250+(p-5) / 150)=\frac{16}{3}(p-5) .
$$

The market equilibrium is found by equating the market demand and the supply:

$$
\begin{aligned}
Q^{D}\left(p^{*}\right) & =Q^{S}\left(p^{*}\right) \Leftrightarrow \\
100-2 p^{*} & =\frac{16}{3}\left(p^{*}-5\right) \Leftrightarrow \\
p^{*} & =\frac{190}{11} \approx 17.3 \Longrightarrow Q^{*}=\frac{720}{11} \approx 65.5 .
\end{aligned}
$$

At the equilibrium price, higher productivity firms supply $500\left(\frac{190}{11}-5\right) / 150=\frac{450}{11}$ and therefore their share is $\frac{450}{11} / \frac{720}{11}=5 / 8=62.5 \%$.

3. In this exercise, all the quantities are MWh and prices in euros per MWh.
(a) We must first figure out the market supply. When the price is above 800, every plant can operate profitably and therefore the market supply is 8000 . Below that, when market price in increased by one euro, a constant $4000 / 800=5$ is gained in supply. Therefore the market supply is:

$$
\begin{aligned}
& Q_{N}^{S}(p)=\left\{\begin{array}{ll}
4000+5 p, & 0 \leq p<800 \\
8000, & p \geq 800
\end{array} \Leftrightarrow\right. \\
& p_{N}^{S}(Q)= \begin{cases}0, & Q \leq 4000, \\
-800+Q / 5, & 4000 \leq Q<8000 \\
\infty, & Q \geq 8000 .\end{cases}
\end{aligned}
$$

Note that there is no supply beyond 8000 regardless of the price so the inverse supply curve becomes a vertical line.

Market demand is obtained by aggregating the individual buyers' demands:

$$
\begin{aligned}
& \left.Q^{D}(p)=1000(10-p / 100)\right)=10000-10 p \Leftrightarrow \\
& p^{D}(Q)=1000-q / 10 .
\end{aligned}
$$

In the equilibrium, supply equals demand. Let's first try to find the solution from the non-flat part of the supply curve.

$$
\begin{aligned}
Q^{D}\left(p^{*}\right) & =Q^{S}\left(p^{*}\right) \Leftrightarrow \\
10000-10 p^{*} & =4000+5 p^{*} \Leftrightarrow \\
p^{*} & =400 \Longrightarrow Q^{*}=6000 .
\end{aligned}
$$

The equilibrium price actually lies on the upwards sloping part. If it didn't we should find the solution from the flat part starting at $p=800$. When supply and demand curves are defined piecewise it is often easiest to graph them first to see in which "piece" the equilibrium point is located.

(b) Depending on the day, the constant term (wind farms' supply) in the supply curve is altered while the supply form other plants remains the same. The market supplies on a low wind day and a high wind day are

$$
\begin{aligned}
& Q_{L}^{S}(p)=\left\{\begin{array}{ll}
2000+5 p, & 0 \leq p<800 \\
6000, & p \geq 800
\end{array} \Leftrightarrow\right. \\
& p_{L}^{S}(Q)= \begin{cases}0, & Q \leq 2000, \\
-400+Q / 5, & 2000 \leq Q<6000, \\
\infty, & Q \geq 6000 .\end{cases} \\
& Q_{H}^{S}(p)=\left\{\begin{array}{ll}
8000+5 p, & 0 \leq p<800 \\
12000, & p \geq 800
\end{array} \Leftrightarrow\right. \\
& p_{H}^{S}(Q)= \begin{cases}0, & Q \leq 8000, \\
-1600+Q / 5, & 8000 \leq Q<12000 \\
\infty, & Q \geq 1000 .\end{cases}
\end{aligned}
$$

Half of the consumers have a fixed price contract. Their demand is unaffected by the market price and constant at $\left.Q_{F P}^{D}(p)=500(10-400 / 100)\right)=3000$. Regardless of the market price, demand will be at least this amount. The other half has demand
dependent on the market price $\left.Q_{M P}^{D}(p)=500(10-p / 100)\right)=5000-5 p$ so the market demand is given by

$$
\begin{aligned}
Q^{D}(p)=Q_{F P}^{D}(p)+Q_{M P}^{D}(p) & =\left\{\begin{array}{ll}
8000-5 p, & 0 \leq p<1000 \\
3000, & p \geq 1000
\end{array} \Leftrightarrow\right. \\
p^{D}(Q) & = \begin{cases}\infty & 0 \leq Q<3000 \\
1600-Q / 5, & Q \geq 3000 .\end{cases}
\end{aligned}
$$

Now that we've derived the supply and demand curves, finding the equilibrium is straightforward. We equate demand and supply in low, normal and high wind day, respectively:

$$
\begin{aligned}
Q^{D}\left(p_{L}^{*}\right) & =Q_{L}^{S}\left(p_{L}^{*}\right) \Leftrightarrow \\
8000-5 p_{L}^{*} & =2000+5 p_{L}^{*} \Leftrightarrow \\
p_{L}^{*} & =500 \Longrightarrow Q_{L}^{*}=5000 . \\
Q^{D}\left(p_{N}^{*}\right) & =Q_{N}^{S}\left(p_{N}^{*}\right) \Leftrightarrow \\
8000-5 p_{N}^{*} & =4000+5 p_{N}^{*} \Leftrightarrow \\
p_{N}^{*} & =400 \Longrightarrow Q_{N}^{*}=6000 \\
Q^{D}\left(p_{H}^{*}\right) & =Q_{L}^{S}\left(p_{H}^{*}\right) \Leftrightarrow \\
8000-5 p_{H}^{*} & =8000+5 p_{H}^{*} \Leftrightarrow \\
p_{H}^{*} & =0 \Longrightarrow Q_{H}^{*}=8000 .
\end{aligned}
$$


(c) By similar reasoning as in 3a, the supply in a low wind day is given by

$$
Q_{N}^{S}(p)= \begin{cases}2000, & 0 \leq p<400 \\ 5 p, & 400 \leq p<800 \\ 4000, & p \geq 800\end{cases}
$$

That is, there's a capacity of 2000 which has zero marginal cost and therefore supplied to the market at any positive price. We now gain new suppliers only when price is increased in the interval $[400,800$ ), again at a constant rate due to uniform distribution. To find the requested equilibria, we equate our new supply with demand with fixed price contracts and market prices, respectively:

$$
\begin{aligned}
Q^{D}\left(p_{F P}^{*}\right) & =Q_{L}^{S}\left(p_{F P}^{*}\right) \Leftrightarrow \\
8000-5 p_{F P}^{*} & =5 p_{F P}^{*} \Leftrightarrow \\
p_{F P}^{*} & =800 \Longrightarrow Q_{F P}^{*}=4000 . \\
Q^{D}\left(p_{M P}^{*}\right) & =Q_{L}^{S}\left(p_{M P}^{*}\right) \Leftrightarrow \\
10000-10 p_{M P}^{*} & =5 p_{M P}^{*} \Leftrightarrow \\
p_{M P}^{*} & \approx 667 \Longrightarrow Q_{M P}^{*} \approx 3333 .
\end{aligned}
$$


4. (a) The market clears (i.e. the quantity demanded equals the quantity supplied) at any price $p \in[1880,1999]$ due to a vertical portion in the (inverse) demand curve. In accordance with the tie-breaking rules, we pick the lowest of these prices, $p^{*}=1880$ $€ / \mathrm{MWh}$. The equilibrium quantity is 10.97 GWh .
P ( $€ / \mathrm{MWh})$

(b) Now we have a counterfactual situation where supply is increased by 0.1 GWh for prices at or above $500 € / \mathrm{MWh}$. This upward shift increases the equilibrium quantity to circa 11.00 GWh (11.0027 GWh to be exact) and decreases the price quite dramatically to 700 €/MWh.



__ Demand —— Supply ----- Counterfactual supply
(c) Since the marginal costs of the already operative capacity is below the new market price $(200<700)$, all of it remains in operation. There's no change in the costs of the existing capacity, but the revenue decreases since the new market price decreases. The loss of revenue is is $500(1880-700) €$. Since the new market price is above the marginal cost of the additional capacity $(700>500)$, all of it gets sold in the market, yielding a revenue of $100 \times 700 €$. The cost of activation and production is $10000+400 \times 100 €$. Summing these up gives the change in profit: $-500(1880-700)+100 \times 700-(10000+400 \times 100)=$ $-570000 €$.

Note that the existing capacity brings more money than it costs. However, as the market price plummets, the loss of revenue from the existing capacity trumps this positive effect. Although 0.1 GWh is small compared to the total capacity on the market, even such a small shift can have a drastic effect on the equilibrium price if demand and supply are inelastic. Therefore, in this situation, if this supplier understands what is going on in the market it would not be activating its last 100MW power station.
One lesson of this part was that a seller with a very small market share (here about 5-6\%) can end up with significant market power at times when supply and demand curves are vertical (or close to vertical) near what would be the competitive market equilibrium. When a seller decides to withhold any capacity for the purpose of affecting the market price then the market is by definition not perfectly competitive.
5. In the first round the demand and supply curves in the classroom would have been consistent with any price between 7 and 8 in the equilibrium of a frictionless perfectly competitive market. The actual mean price was 7.3 (standard deviation 1.6). The number of trades in a frictionless market would have been 29, whereas in the classroom "pit market" 33 trades were made.

Supply and Demand (Pit market 1)


The "role reversal" between the first and second round amounts to negative shocks to both demand and supply. This had no effect on the frictionless market price, but it was certainly much harder to find a mutually profitable trading partner in round 2 ! In second round the average price was 6.9 (median 7), standard deviation 1.8, and 18 trades were made.

## Supply and Demand (Pit market 2)



Round 1 was worth 8 points and round 2 was worth 7 points.

A perfect score in a round required: i) writing down the Role either as buyer or seller, ii) a Valuation that has to be a whole number between 1 and 13 if seller, 2 and 14 if buyer. iii) either a surplus 0 (zero), or a "Trading price" that is a multiple of 0.5 together with a correctly calculated non-negative "surplus" (valuation-price for buyers, price-valuation for sellers).

A couple of trades were made where one trader made a negative surplus - which was the only way to make a serious mistake in this game. Regardless of how unlucky the draw, everyone can get at least zero surplus!

