ELEC-E8101 Digital and Optimal Control

Exercise 2

1. The considered process has first order dynamics and its static gain is one

$$\tau \dot{y}(t) + y(t) = u(t)$$

a. Discretize the process with a sampling time of h and assuming that the control signal $u(t)$ is piecewise constant between the sampling instants (ZOH). Use

i. discretization of the state-space model,
ii. discretization of the transfer function.

Compare the discretized models.

b.
i. Determine the unit step response of the continuous process.
ii. Determine the unit step response from the model 1a i. , assuming the initial values to be zero.
iii. Determine the unit step response from the model 1a ii., assuming the initial values to be zero.

Compare.

2. Let us consider the following scalar state-space model:

$$\begin{cases} \dot{x}(t) = ax(t) + bu(t) \\ y(t) = cx(t) \end{cases}$$

It is assumed that parameters c and b are positive, a non-zero.

a. For what parameter values the continuous process is stable?
b. Discretize the process (accurately, ZOH, sampling time h).
c. For what parameter values the discrete process is stable?
d. The continuous process is controlled with a continuous P-controller. For what values of gain K_P the controlled system is stable?
e. The continuous process is controlled with a discrete P-controller. For what values of gain K_P the controlled system is stable?
f. How does the stability region for K_P in **e.** change, if the sampling time is very small $(\lim h \to 0)$?

3. Consider the following difference equation:

$$y(k+2) - 1{,}3y(k+1) + 0{,}4y(k) = u(k+1) - 0{,}4u(k)$$

Determine the corresponding

a. state-space representation
b. pulse transfer function

Is the system stable?

∗ **4.** The pulse transfer function of a system with ZOH is

$$H\left(z^{-1}\right)=\frac{0{,}2z^{-1}}{1-0{,}8z^{-1}}\ .$$

Determine the step and pulse responses.

Theorems of Z-transformation

Definition: $F(z) = Z\{f(kh)\} = \sum_{k=0}^{\infty} f(kh) z^{-k}$	
z-transform	Sequence in time domain
$F(z)$	$f(k)$
$C_1F_2(z) + C_2F_2(z)$	$C_1f_2(k) + C_2f_2(k)$
$z^{-n}F(z)$	$q^{-n} f(k)$
$z^n \left(F(z) - \sum_{j=0}^{n-1} f(jh) z^{-j}\right)$	$q^n f(k)$
$F_1(z)F_2(z)$	$\sum_{n=0}^{k} f_1(n) f_2(k-n)$
If the limits of $f(kh)$ and $F(z)$ exist, they satisfy $\lim_{k\to\infty}\{f(kh)\} = \lim_{z\to 1}\{(1-z^{-1})F(z)\}$ $\quad f(0) = \lim_{z\to\infty} F(z)$	

Z-transformations and corresponding sequences

z-transform	Sequence in time domain
1	$\delta_k(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$
$\frac{z}{z-1}$	1
$\frac{z}{(z-1)^2}$	k
$\frac{z}{z-a}$	a^k
$\frac{az}{(z-a)^2}$	ka^k
$\frac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\sin(ak)$
$\frac{z(z-\cos(a))}{z^2 - 2z\cos(a) + 1}$	$\cos(ak)$
$\frac{bz\sin(a)}{z^2 - 2bz\cos(a) + b^2}$	$b^k \sin(ak)$
$\frac{z(z-b\cos(a))}{z^2 - 2bz\cos(a) + b^2}$	$b^k \cos(ak)$