Mathematics for Economists

Problem Set 2
Due date: Wednesday 28.9 at 13.15

Exercise 1

a) Consider the following system of equations

$$\begin{aligned} zx + ay + 10c + d &= 0 \\ cx + by - 5d &= 0 \end{aligned}$$

Find the matrix presentation of the system (i.e. "$A\mathbf{x} = \mathbf{b}$" with suitable A, $\mathbf{x}$, and $\mathbf{b}$), when the endogenous variables are

(1) x ja y,

(2) c, z ja y,

b) Consider the following quadratic function: $16x^2 + 9y^2 + 24xy$. What is the matrix A if we want to express this function as $\mathbf{x}^T A\mathbf{x}$, i.e., as an inner product of $\mathbf{x} = (x, y)$ and $A\mathbf{x}$?

Exercise 2

Find the inverse of the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{pmatrix}.$$

Exercise 3

a) How does the number of solutions of the below pair of linear equations depend on parameters a and b?

$$\begin{aligned} y - ax &= b \\ x + y &= 2 \end{aligned}$$

b) Determine whether or not the following set of vectors forms a basis of $\mathbb{R}^3$.

$$\begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 8 \end{pmatrix}.$$

Exercise 4

a) Assume that $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$, and "a rule" f is defined as $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$.
i) What do we have to assume about numbers a, b, c, d for f to be a function from A to B?
ii) Assume that A (or numbers a, b, c, d) are defined such that f is a function. Is it a surjection or injection?
b) Consider the function $f : \mathbb{R} \mapsto \mathbb{R}$, $f(x) = x + 1$. Argue why the function is a bijection and find its inverse.
c) Is the function $f(x) = x + y$ injective, surjerctive, or bijective?

Exercise 5

(a) Show that

$$\lim_{n \longrightarrow \infty} \frac{3n + 1}{4n} = \frac{3}{4}.$$

(b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be such that

$$f(x) = \begin{cases} x^2 & \text{if } x \neq 2 \\ 0 & \text{if } x = 2. \end{cases}$$

Show that f is *not* continuous at $x = 2$. [*Hint*: think what is the limit of f at $x = 2$ and what is its value.]

Exercise 6

Calculate all the partial derivatives of the following functions:

(a) $f(x, y) = ax^b y^c$

(b) $f(x, y) = a \ln(1 - x) + b \ln(y)$

(c) $f(x, y) = \dfrac{ay^d}{bx^c}$

(d) $f(x, y, z) = e^{ax-by} - z$

(e) $f(x, y, z) = \sqrt{x^{\frac{1}{2}} + y^{\frac{1}{3}} + 5z^2}$.