Model Solutions 2

1. (a) When the monopolist cannot set different prices across markets, it will consider market demand as the aggregate demand across the two regions in setting its price. The market demand is obtained by summing up the demanded quantities, across the two regions, for a given price level: $Q^d(p) = Q^d_N(p) + Q^d_S(p)$.

In doing this, it is important to consider the reservation prices for each market. These are easily obtained, for instance, by inverting the demand functions. In particular, $P^D_N(q) = 60 - 2q, P^D_S(q) = 24 - q$. Here, we see that reservation prices are 60 and 24, respectively. Taking these into account, the market demand function can be expressed as:

$$Q^d(p) = \begin{cases} 0, & p \geq 60 \\ 30 - p/2, & 60 > p \geq 24 \\ 54 - p3/2, & 24 > p \geq 0 \end{cases}$$

By inverting these equations, we can also write the market price function:

$$P^d(q) = \begin{cases} 60 - 2q, & 18 \geq q \geq 0 \\ 36 - q2/3, & 54 \geq q > 18 \end{cases}$$

In setting the optimal price, the monopolist sets q such that $MR(q) = MC(q) = 12$. From here, we can proceed by trial and error in considering both segments of the market demand curve. For the part in which only customers in North buy the product, we have $MR(q) = 60 - 4q = MC(q) = 12 \leftrightarrow q^* = 12 \rightarrow p^* = P^d(q^*) = 60 - 2 \times 12 = 36$. We see that the price is indeed within the bounds, and so the profit from this price is $\pi_1 = q^*(p^* - 12) = 288$.

Proceeding in a similar manner for the segment in which both markets are buying, we have $MR(q) = 36 - q4/3 = MC(q) = 12 \leftrightarrow q^{**} = 18 \rightarrow p^{**} = P^d(q^{**}) = 36 - 18 \times 2/3 = 24$. Note that at this price, no consumers in South are in fact buying anything, since this is their reservation price. This means the profit is necessarily smaller than in the first case, as this price-quantity pair was available in the above case (it is fact the kink of the market demand curve). Calculating the profit for this price-quantity pair shows that this is indeed the case: $\pi_2 = 18(24 - 12) = 212 < \pi_1$.

The above analysis directly implies that the common price for both regions is $p^* = 36$, with demands $q^*_N = 12, q^*_S = 0$. Since no consumer in South buys anything, the consumer surplus for South is zero, $CS_S = 0$. For North, we have trade, and consumer surplus is then calculated in the usual manner. It equals the triangular area bounded by the demand curve and the market price: $CS_N = 1/2[(60 - 36) \times 12] = 144$.

(b) When the monopolist can set different prices for the two regions, it will simply apply the $MC(q) = MR(q)$-rule separately for the two regions. This example is particularly simple in that constant marginal costs imply that the production choices can be treated as independent; the quantity produced in one region will not affect the cost of producing in the other region.

For South, we have $MR(q) = 24 - 2q = MC(q) = 12 \leftrightarrow q_S^* = 6 \rightarrow p_S^* = P_S^d(q_S^*) = 18$. Proceeding similarly for North, we have $MR(q) = 60 - 4q = MC(q) = 12 \leftrightarrow q_N^* = 12 \rightarrow p_N^* = P_N^d(q_N^*) = 36$. As we should expect, the price for North is the same as in the previous subsection, because we found that the monopolist chose the price so that only consumer is North buys the product.

This means that the consumer surplus for North is unchanged, that is, $CS_N = 144$. For South, we get $CS_S = 1/2[(24 - 18) \times 6] = 18$.

For profits, we note that the change is driven by the new trade in South, so profit increases by $\pi_S = 6(18 - 12) = 36$.

2. (a) To find the market demand function, we should first note that identical distributions in valuations across consumers means that we can simply multiply the individual demand functions to obtain the market demand function:

$$Q^d(p) = N \times Q_i^d(p)$$

In this example, we can think of the individual consumers demand function as giving a probability of purchasing one unit for a given price level. That is,

$$Q_i^d(p) = Prob(v_i > p),$$

where $v_i \in [0, 20]$ denotes the valuation of consumer i. Clearly, this probability should be zero at $p = 20$, and one at $p = 0$. This means we can express the individual demand function as $Q_i(p) = 1 - p/20$. Knowing this, we can easily obtain the market demand function (which is an arbitrary good approximation of the realized demand, given that we have a large number of customers). That is,

$$Q^d(p) = 4000 \times (1 - p/20) = 4000 - 200p. \leftrightarrow \\ P^d(p) = 20 - q/200,$$

where the price function was obtained by inverting the demand function.

(b) Here, the monopolist Acme Inc has to incur a fixed cost $FC = 2000$, which means it will only operate if it profitable to do so. Conditional on operating, it will set the quantity to equate marginal cost with marginal revenue, as usual.

To see if if will operate, we first proceed by calculating the optimal quantity, and then verify whether this is profitable. That is, $MR(q) = 20 - q/200 = 10 = MC(q) \leftrightarrow q^* = 1000 \rightarrow p* = P^d(q^*) = 20 - 1000/200 = 15$.

This implies the following profits $\pi = 1000(15 - 10) - 2000 = 3000 > 0$. As it is indeed profitable to operate at these prices, we conclude by noting that the optimal price is $p^* = 15$.

(c) We can proceed as in part as in the previous subsection by equating marginal revenue and marginal costs. For an unknown $N > 0$, market demand is given by $Q^d(p, N) = N(1 - p/20)$, and hence the price function is $P^d(q, N) = 20 - q(20/N)$. Equating marginal cost with marginal revenue: $MR(q, N) = 20 - q(40/N) = MC(q) = 10 \leftrightarrow q^* = N/4$. By substituting this into price function as usual, we get $p* = P^d(N/4) = 20-(N/4)(20/N) = 15$. One way to see why the optimal price is independent of N, is to note that the demand elasticity is independent of N, as this quantity merely indicates percentage changes in demand.

As in part b), we should still verify whether it is profitable to operate. Here, N plays a role in that it directly impacts the size of demand, and hence, revenues. We can solve for N to find the lower bound of customers that makes it profitable to operate: $\pi(N) = N(1 - 15/20)(15 - 10) - 2000 \geq 0 \leftrightarrow N \geq 1600$.

To conclude, provided $N \geq 1600$, Acme Inc operates and sets $p^* = 15$.

(d) The monopolist either faces a demand function with $N = 4000$ with some probability λ, and no demand with some probability $1 - \lambda$. In maximizing its profits, the monopolist now has to take into account the uncertainty of demand, while cost is deterministic. We can express the expected profit for a given quantity as follows:

$$E(\pi(q)) = \lambda q(20 - q/200) - q10 - 2000$$

The monopolist maximizes expected profits, which means that the optimal quantity will be a function of probability λ. So we solve for optimal q:

$$\frac{\partial E(\pi(q))}{\partial q} = 0$$
$$\leftrightarrow q^*(\lambda) = 2000(1 - 1/\lambda)$$

We then solve for the λ at which it the monopolist is indifferent between producing or not. This is done by substituting the optimal quantity (expressed as a function of λ), and setting profits to equal zero:

$$\mathbb{E}(\pi(q^*(\lambda))) = 0$$
$$\rightarrow \lambda \approx 0.78.$$

That is, if $\lambda \geq 0.78$, the monopolist produces, and it produces $q^*(\lambda) = 2000(1 - 1/\lambda)$.

Note: a tempting mistake to make here is to assume that the monopolist would produce a quantity such that $P^d(q) = 15$ when flu hits, as we found that the optimal price is independent of N, conditional on production. This is incorrect, as we are now considering uncertain demand, which is relevant for evaluating expected revenues.

(e) Without the bias, raising the price from the optimal level 15.00 to 15.99, would cause demand to decrease to 802. With the bias, demand only drops by half of this, that is, by 99 units to 901. This is because 50% of consumers don't make any difference between 15.00 and 15.99. However, further increasing the price by one cent to 16.00 causes a discrete drop in quantity demanded down to 800, with the bias.

It suffices to examine values nearby our initial equilibrium. Charging price 14.99 attracts 101 new customers and yields profits of $1101 \times (14.99 - 10) - 2000 = 3494$. Again, only 99 customers are lost if price is increased to 23.99, yielding profits of $(15.99 - 10) \times 901 - 2000 = 3397$. Therefore, $p^* = 14.99$.

Why is it sufficient to check only these specific values? In fact it exactly isn't but it's considered good enough for the purposes of this exercise. More generally, profit at price $p + .99$ is given by $(p + .99 - 10)(4000 - 200p - 99)$. From here one can obtain the same result as before easily. If even more rigorous, one could ask whether setting $p + d$, $d \in [0, 0.99)$, would be optimal instead. $(p + d - 10)(4000 - 200p - 100d)$ has either solution 14 or 15 depending on d, so the optimum must lie in the interval $[14, 16)$. Then it's easy to show that 14.99 beats any other price in the interval.

3. (a) Here, the monopolist sets quantity to equate marginal marginal revenue. That is, $MR(q) = 200 - 2q = MC(q) = 20 \leftrightarrow q^* = 90 \rightarrow p^* = P^d(q*) = 200 - 90 = 110$.

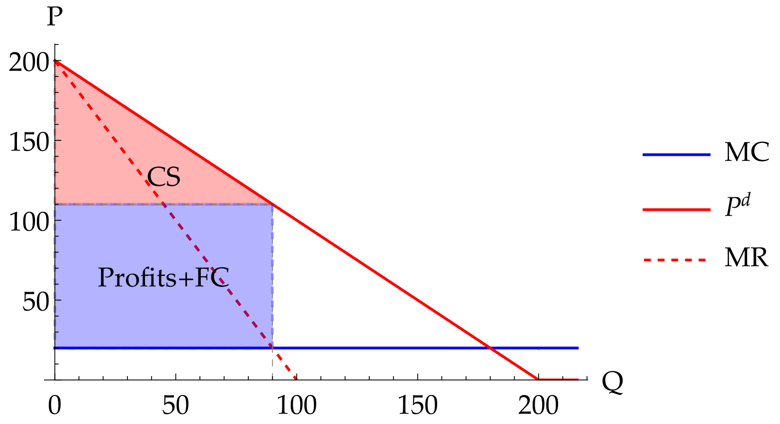

Figure 1: Monopoly pricing with fixed costs.

The monopoly operates only if it profitable to do so; $\pi = 90(110-20)-7200 = 900 \geq 0$. Knowing that the monopolist will indeed produce, the consumer surplus is given by $CS = 1/2[(200 - 110)90] = 4050$.

(b) As total consumer surplus is by definition the area between the demand curve and the horizontal line corresponding to the price level, it is clear that this quantity is increasing when the price decreases, provided that the monopolist produces. This is because a lower price will both increase the surplus for any consumer initially buying the product, and it will induce additional consumers to buy the product.

From the above, it directly follows that the price cap $\bar{p}$, should be so low as to barely make it profitable for the monopoly to produce: $\pi = Q^d(\bar{p})(\bar{p}-20)-7200 = 0 \rightarrow \bar{p} = 80$ or $\bar{p} = 140$. There are two solutions to this polynomial, so the lower value is the desired price cap, that is, $\bar{p} = 80$. Note that at this price level, the price exactly equals average costs, as indicated in figure 2.

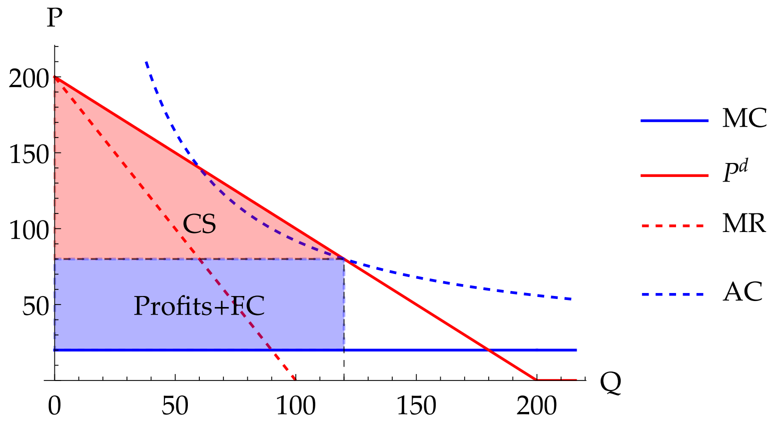

Figure 2: Consumer surplus maximizing price cap.

(c) The error in the estimation of the fixed cost of the monopolist implies that the true value takes values as follows: $FC \in [0, 14400]$. Noting that the monopolist only produces if $FC = 7200 + x \leq 7200$, it follows that the monopolist will not produce if $x > 0$. Provided that the monopolist produces, fixed costs do not matter in its price-quantity decision, as this is dictated only by the decision to equate marginal revenue with marginal costs. This means that if $x < 0$, the monopolist produces $Q^d(\bar{p}) = 200 - 80 = 120$. From this, we can calculate the Consumer surplus: $CS = 1/2[(200 - 80)120] = 7200$.

For profits, the impact is straightforward. If $x < 0$, so that the monopoly produces, any decrease in x will be directly passed into profit of the monopolist, as price and quantity are independent of the fixed cost, provided production takes place. If $x \geq 0$, nothing changes as the cap was initially set to induce the monopolist to make zero profits, and not producing also implies zero profits.

We can summarize the above observations formally as follows:

$$\Delta\pi(x) = \begin{cases} -x, & x < 0 \\ 0, & x \geq 0 \end{cases}$$

$$\Delta\text{CS}(x) = \begin{cases} -7200, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

4. (a) To find the efficient amount of cleaning hours q, we start by constructing the aggregate demand for cleaning by summing up the valuations for each individual in the household:

$$P^d(q) = \sum_i P_i^d(q), i = K, J, H$$

In doing this, we need to pay attention to kinks that may appear in the aggregate demand function, in particular, Hanna and Jaska do not value more cleaning at all at $q \geq 24$. Noting this, the aggregate demand function becomes:

$$P^d(q) = \begin{cases} 64 - q8/3, & 20 \geq q > 0 \\ 24 - q2/3, & 24 \geq q > 20 \end{cases}$$

This quantity also indicates the marginal benefit to the household of incrementally increasing the number of cleaning hours. Hence, the efficient number of hours is such that it equates the total marginal benefit with total marginal costs, that is, $P^d(q) = MC(q)$. We could proceed by trial and error, but by inspecting figure 3 we see that the marginal cost line intersects the marginal benefit curve at the upper part, where each individual values additional cleaning. Hence, we can solve for the efficient number of hours as follows: $P^d(q) = 64 - q8/3 = MC(q) = 16 \rightarrow q^* = 18$.

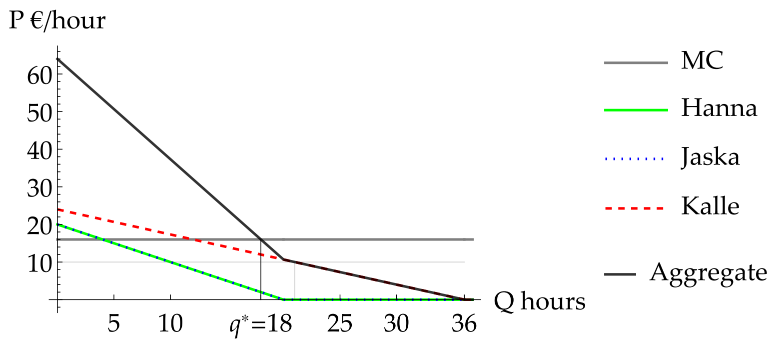

Figure 3: Household aggregate demand for cleaning.

Consumer surplus for each individual in the household is given by the area below the demand curve, subtracted by the cost paid by the consumer, a rectangular area. That is,

$$CS_i(q^*) = \int_0^{q^*} P_i^d(q)dq - TC_i(q^*)$$

Rather than evaluating the integral, we can also proceed by simply calculating the area below the demand curve for each individual, and then subtract the total cost from this:

$$\begin{aligned} CS_{J,H}(18) &= 1/2(20 \times 20 - (20 - 2)^2) - 18 \times 16/3 \\ &= 102, \\ CS_K(18) &= 1/2[24 \times 36 - (24 - (2/3)18)(24 - 18)] - 18 \times 16/3 \\ &= 228. \end{aligned}$$

(b) Given that Hanna and Jaska have identical preferences, their vote will obviously win, and hence the amount of cleaning will be decided by them. How will they vote? In voting, they will consider the amount of cleaning that maximizes their own consumer surplus, that is,

$$q^M = \arg\max_q CS_{J,H}$$

We can find q^M by taking the derivative of the consumer surplus function of Hanna and Jaska, and solve for the maxima:

$$\begin{aligned}\frac{\partial CS_{J,K}(q)}{\partial q} &= 0\\ \leftrightarrow 1/3(44-3q) &= 0\\ \leftrightarrow q^M &= 44/3 \approx 14.67.\end{aligned}$$

To find the resulting surpluses, we again evaluate the consumer surpluses at the relevant amount of cleaning:

$$\begin{aligned}CS_K(q^M) &= 1/2[(36\times 24)-(36-44/3)(24-(2/3)(44/3)]-(44/3)(16/3)\\ &\approx 202.07,\\ CS_{H,J}(q^M) &= 1/2(20^2-(20-44/3)^2)-(44/3)(16/3)\\ &\approx 107.56\end{aligned}$$

(c) The availability of a professional cleaner that is twice as effective as the individuals in the household means that the household can effectively purchase one hour of their own cleaning output for $10e$ (that is, half an hour of cleaning service from the professional). What matters here is how they value the cost of cleaning in terms of money. Since this option is cheaper, this is the new marginal cost of cleaning.

In finding the efficient amount of cleaning, the household should again equate the total marginal benefit with the marginal cost. In looking for the point of intersection between the aggregate demand curve and the cost curve, we can for instance proceed by trial and error. At the upper part of the curve, we have $64-q8/3=10 \leftrightarrow q=81/4\approx 20.25>20$. We see that this quantity is not feasible, given the shape of the demand curve and the the implied bounds. For the lower part, we have $24-q2/3=10 \leftrightarrow q* = 21$. This quantity is feasible and implies that only Kalle gets additional utility from the last hour of cleaning.

Results of the public good game

The objective in the game was to maximize one's own total consumption, which for individual i in a peer group $\{1, \ldots, 5\}$ was $\mathrm{C}_i = (10 - x_i) + (2/5)(x_1 + x_2 + x_3 + x_4 + +x_5)$, where x_i was the contribution by i on the public good. The distributions of individual contributions are shown in Figure 1. For the purpose of the classroom answer sheets the peer group of 4 other individuals was a random sample of participants (different sample in each round). However, for the calculation of final results, we use as everyone's peer group 4 times the average over all participants, so the public good contribution used to calculate total consumption of participant i is $x_i + 4E[x]$. Resulting total consumption can be interpreted as its expected value, over all possible randomly selected peer groups from the classroom. The distributions of resulting consumption levels are depicted in Figure 2.

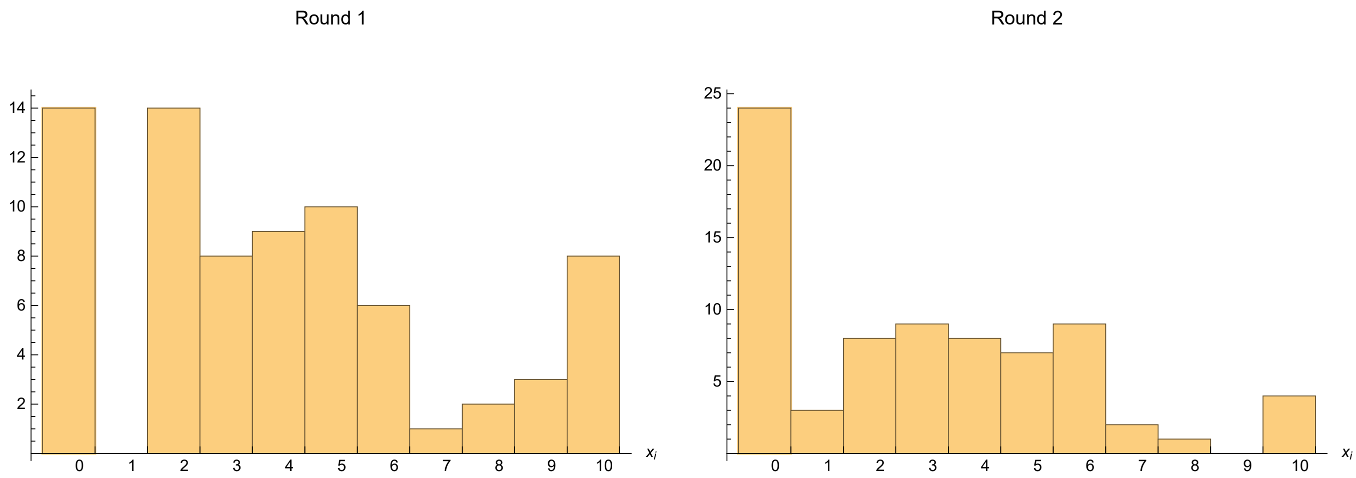

Figure 1: Distribution of public good contributions

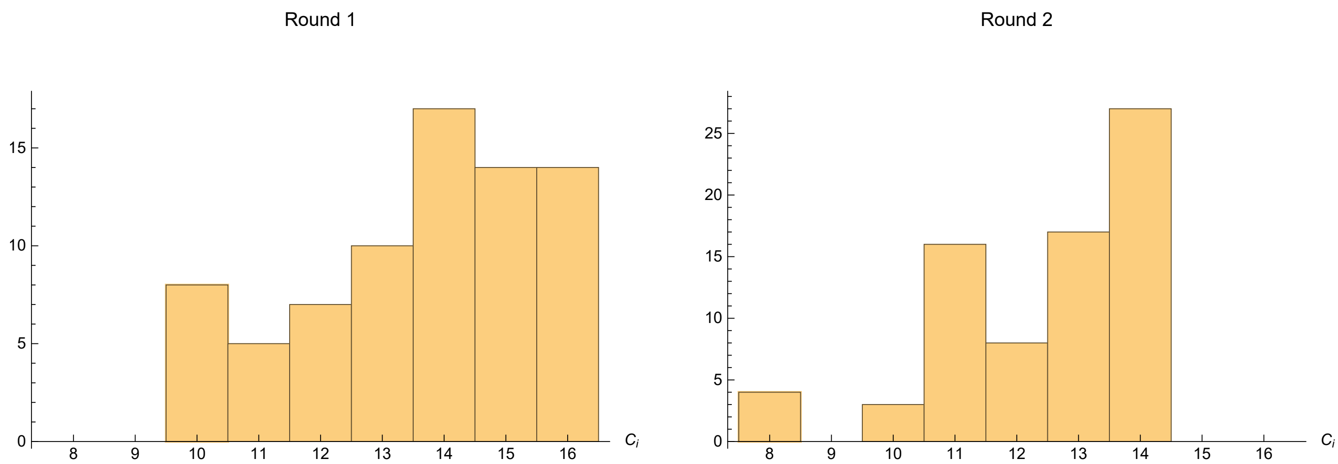

Figure 2: Distribution of total consumption

Table Summary statistics

	Round 1	Round 2
N	75	75
Mean x_i	4.05	3.05
SD x_i	3.13	2.86
Median x_i	4	3
4E[x]	16.21	12.21
Mean C_i	14.1	13.1
25th percentile C_i	12.9	11.9
Median C_i	14.1	13.1

In each round, full points were awarded if the answer sheet was correctly filled and consistent with the public good contributions of the classroom sample.

Later a monetary reward was raffled between all participants who had correctly filled their sheet. The prize was in euros the expected value of total consumption for the winner of the raffle (in a randomly selected round) over all possible groups of four peers in class. This implies a total contribution to the public good that is the sum of the winner's own contribution plus four times the average public good contribution of all participants.

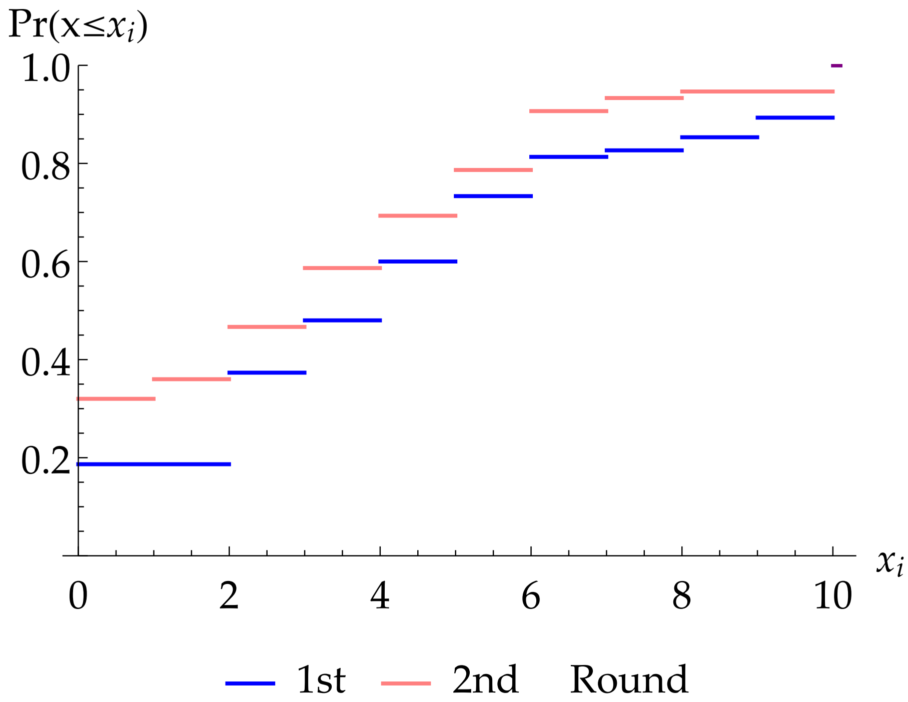

Figure 3: Distribution of public good contributions (CDFs)