Model Solutions 3

1. (a) Some goods are such that many economic agents can derive utility from consuming it simultaneously. Opera house, sevage system and railway connection are examples of goods that are still of value to others after someone has used them. An example of a good that does not have the property is a chocolate bar.

   The benefit of providing such good is the sum of the individual users' or consumers' benefits, which are calculated below for the projects considered in the exercise:

Project	(O) Opera	(S) Sewage	(A) Airport railway
Total cost	300	250	200
Total benefit	345	190	235

   Whenever the cost of provision is less than the total benefit, provision is efficient. Therefore opera and rail connection should be invested in.

   (b) Costs are shared evenly between the five municipalities. Under majority rule there must be at least three municipalities for which the share of costs is less than the municipality's gross benefit for the project to be executed. For opera, the cost per municipality is 60 M€. Rosicruce, Uqbar and Orbis Tertius have greater gross benefits and the metropolitan area will get its opera house under majority rule. Only Uqbar and Tlön would support sewage upgrade while Macondo and Tlön would stand behind railway connection by the same logic. These projects will not materialize.

   Since each project had opposition with a maximum of three municipalities supporting a single project, none of the projects will pass unanimity rule.

   (c) We have three different amalgam projects, whose costs and benefits are:

Project	O&S	O&A	A&S
Total cost	550	500	450
Macondo City	15	115	140
Rosicruce	160	150	70
Uqbar	155	115	70
Tlön	115	120	135
Orbis Tertius	90	80	10

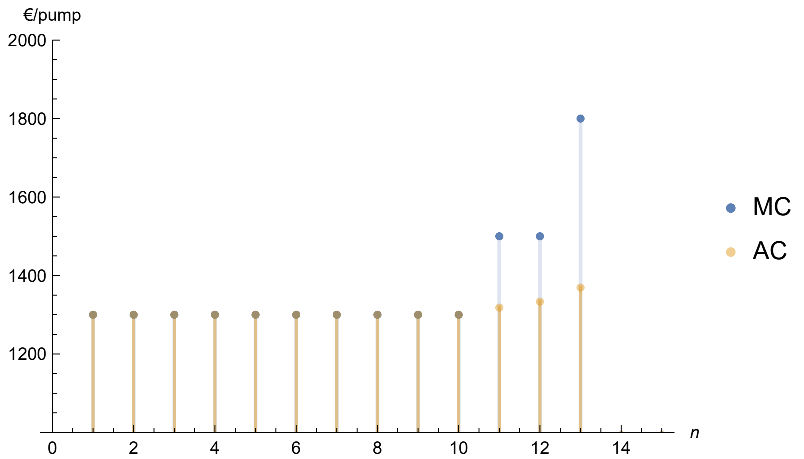
   For the projects, costs pre municipality 110, 100 and 90, respectively. Both sewage upgrade and railway connection will pass majority vote when coupled with opera.

2. (a) The firm has $4 \times 5 = 20$ working days at its disposal on weekdays, 4 on Saturday and 2 on Sunday, which translate into 10, 2, and 1 pumps, respectively. Since the firm can only install 13 pumps a week, the cost of purchasing a pump is always 1000 €.

Given the different wages for different days, the cost of installing nth pump is the sum of wage enxpenses and purchase price,

$$c(n) = \begin{cases} 2 \times 150 + 1000, & 1 \leq n \leq 10 \\ 1500, & 11 \leq n \leq 12 \\ 1800, & n = 13. \end{cases}$$

Average costs equals total cost divided by the number of pumps, $\sum_{i=1}^{n} c(i)/n$, and is plotted along with marginal costs below.



(b) The firm will only install units whose marginal cost $c(n)$ is less or equal to marginal revenue. Denote the market price for the installed pumps by p. The firm's supply (i.e. number of units it can install at a marginal cost below p) is given by

$$n(p) = \begin{cases} 0, & 0 \leq p < 1300 \\ 10, & 1300 \leq p < 1500 \\ 12, & 1500 \leq p < 1800 \\ 13, & 1800 \leq p. \end{cases}$$

$n(1600) = 12$. The firm will generate a revenue of $1600 \times 12 = 19200$ at a cost of $1300 \times 10 + 1500 \times 2 = 16000$, yielding a profit of $19200 - 16000 = 3200$. Total earnings are given by $10 \times 300 + 2 \times 500 = 4000$.

(c) Answered in 2b.

3. Macondo City has a power plant that supplies both electricity and district heating. Keeping the plant operational costs 1 M€ a month. The revenue from electricity is 900 k€ and from heating 200 k€ a month. The plant decides to allocate the fixed cost aka shared overhead proportionally to power output, which is 70 GWh of electricity and 30 GWh of heating a month.

The share of costs for heating is 300 k€ and 700k€ for electricity. It might seem like producing heating is unprofitable as $300 > 200$. If the plant stops district heating production on this basis and reallocates shared overhead solely to electricity, also electricity productions looks like bad business as 1000 > 900k€. Folllowing this logic, the plant should be shut down even though it was producing positive profit $-1000 + 900 + 200 = 100$ k€ a month.

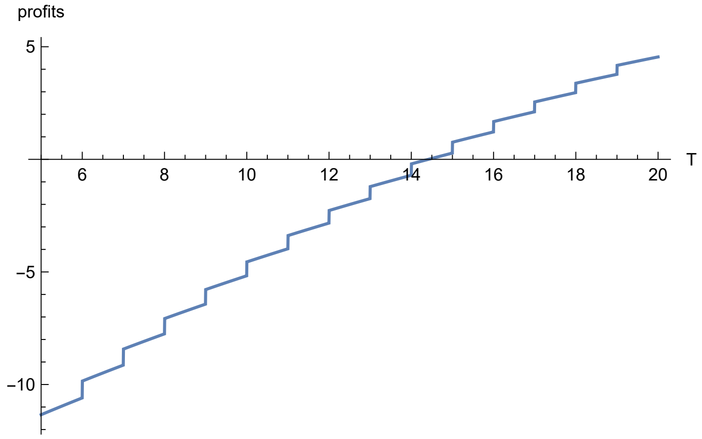
4. The gains (in billions of euros) for the firm in the first T years must be greater or equal to the losses from year $T+1$ onwards:[1]

$$\sum_{t=1}^{T}(1 \times \frac{1}{(1+r)^t}) \geq \sum_{t=T+1}^{\infty}(1 \times \frac{1}{(1+r)^t}) \Leftrightarrow$$
$$T \geq \frac{log(2)}{log(1+r)}.$$

A good way to arrive to the solution is to first calculate the sum of the infinite series starting from $t = 1$. The RHS is that sum postponed by T periods while the LHS is the infinite sum minus the RHS.

Plugging in $r = 0.05$ yields $T \geq 14.20... \implies T^* = 15$ years. Mephisto should engage the cost-cutting program if and only if the savings last for 15 years or more.

However, writing down the equation and arriving to the correct number via other means than solving the equation explicitly suffices. This can be done, for example, graphically:



5. (a) Before drawing the decision tree, let's shave off redundant branches.

- One-day ticket at 8€ is clearly worse than two single tickets at 6€ on the first day.
- A one day ticket at 8€ is clearly better than a two-day ticket at 12€ on the second day.
- If you bought a 2-day ticket on the first day, you don't need another ticket.

Furthermore, once you have a serial ticket on day two, you take all the trips that give you a positive gross benefit. On the second day, the benefit one gets from holding a one or two-day ticket is $5 + 4 + 3 + 2 + 1 = 15$, 10 or 6 when experience was *good*, *ok* and

[1] A one-year offset in the discount rates is completely fine.

bad, respectively. When buying single tickets, the net benefits are $(5-3)+(4-3)=3$, $(4-3)=1$ and 0 for the respective experiences.

Note that in the payoffs in the end of the branches, the first day ticket price enters each node as a sunk cost: whatever the tourist does, she's already paid that and therefore yesterday's price doesn't affect the decision.

(b) The tourist will buy a two-day ticket on day one as $-5/3 > -3$. On day two, she'll take all the trips that give her positive net benefit. The optimal actions are highlighted in the graph.

(c) If the tourist knew that the experience would be bad, she'd be indifferent between the two relevant ticket options on the first day. Both would give a payoff of -6 under the optimal course of actions as seen from the graph. For the other two experiences, it would still be optimal to take the two-day ticket as $-2 > -4$ and $3 > 1$.

The tourist would take the same decision on day one whether or not she knows the experience. On the second day, there's no more uncertainty anyway so reading won't affect day two decisions. As the decisions would be the same regardless, reading won't provide any valuable information and the tourist will put zero effort in it.