ELEC-E8101 Digital and Optimal Control

Exercise 4

1. (*) Derive the discrete-time system corresponding to the following continuous-time system when ZOH circuit is used.

$$\frac{d\mathbf{x}(t)}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}(t)$$

2. The double integrator is a common process in mechanical models. Its differential equation form is

$$\frac{d^2 y(t)}{dt^2} = u(t)\,.$$

a. Determine the state-space representation.

b. Sample the state model with sampling time *h*, assuming ZOH and determine the discrete state-space representation of the form:

$$\mathbf{x}\big((k+1)h\big) = \mathbf{\Phi}\mathbf{x}\big(kh\big) + \mathbf{\Gamma}u(kh)$$
$$y(kh) = \mathbf{C}\mathbf{x}(kh)$$

c. Simulate the step response with continuous-time and discrete-time models. Use Simulink.

3. Examine the stability of the following pulse transfer functions using Jury's stability test.

$$G(z) = \frac{B(z)}{A(z)}$$

a. $A(z) = z^2 - 1.5z + 0.9$

b. $A(z) = z^3 + 5z^2 - 0.25z + 1.25$

4. Consider a discrete time system

$$\begin{cases} x(k+1) = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ b \end{bmatrix} u(k) \\ y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) \end{cases}$$

where a_{11}, a_{12}, a_{22} and b are constants.

a) For which values of parameters is the system asymptotically stable?
b) By using an example, verify the fact that asymptotic stability implies input-output stability (BIBO stability).
c) It is well-known that BIBO stability doesn't necessarily imply asymptotic stability. Verify this truth by choosing the values of parameters such that the system is BIBO stable but not asymptotically stable.

Hint:

Jury's test gives information whether a polynomial of form $P(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n$ has roots outside the unit disc or not. Form the table:

$$\begin{array}{ccccc} a_0 & a_1 & \cdots & a_{n-1} & a_n \\ a_n & a_{n-1} & \cdots & a_1 & a_0 \\ \hline a_0^{n-1} & a_1^{n-1} & \cdots & a_{n-1}^{n-1} & \\ a_{n-1}^{n-1} & a_{n-2}^{n-1} & \cdots & a_0^{n-1} & \\ \hline \vdots & & & & \\ a_0^0 & & & & \end{array}$$

$$\alpha_n = \frac{a_n}{a_0}; \; a_0 = a_0^n, a_1 = a_1^n, \ldots, a_n = a_n^n$$

$$\alpha_{n-1} = \frac{a_{n-1}^{n-1}}{a_0^{n-1}}$$

$$a_i^{k-1} = a_i^k - \alpha_k a_{k-i}^k$$
$$\alpha_k = a_k^k / a_0^k$$

Let $a_0 > 0$ (if not, you can always multiply the equation $P(z) = 0$ by -1). If $a_0^k > 0$ for all k the polynomial does not have roots outside the unit circle. If no a_0^k is zero, then the number of negative terms of a_0^k is equal to the number of roots outside the unit circle.