Exercise and Homework Round 4

These exercises (except for the last) will be gone through on **Friday, October 7, 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Friday, October 14 at 12:00**.

Exercise 1. (Gradient descent for scalar linear model)

Let us consider a scalar linear model

$$y = g\,x + r \tag{1}$$

and the corresponding least squares cost function

$$J(x) = (y - g\,x)^2. \tag{2}$$

(a) Write down the gradient descent algorithm for minimizing $J(x)$.

(b) Compute the optimal step size γ by miniming J with respect to it.

Exercise 2. (Gauss–Newton for scalar linear model)

Consider the Gauss–Newton method for the model in the previous exercise.

(a) Show that the Gauss–Newton method converges for this model in a single step.

(b) What is the relationship of the Gauss–Newton method with the optimal step size that we computed in the previous exercise?

Exercise 3. (Implementation of Gauss–Newton)

Consider the model

$$y_n = g(\mathbf{x}) + r_n, \tag{3}$$

where $n = 1, \ldots, N$, $r_n \sim \mathcal{N}(0, R)$, and

$$g(\mathbf{x}) = \begin{bmatrix} \alpha \sqrt{x_1} \\ \beta \sqrt{x_2} \end{bmatrix}. \tag{4}$$

(a) Write down the corresponding weighted least squares problem for this model.

(b) Derive the Jacobian of g and write down the pseudo-code for the Gauss–Newton algorithm.

(c) Simulate data from this model with suitable parameters and implement Gauss–Newton algorithm for minimizing the cost function.

Homework 4 (DL Friday, October 14 at 12:00)

Implement gradient descend algorithm to minimize $J(x) = (1.1 - \sin(x))^2$. Also empirically test the effect of the step size to the convergence speed.